# Bell Nonlocality

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# Contents

# 1

# First encounter with Bell nonlocality

Misunderstandings of Bell's theorem happen so fast
that they violate locality.
R. MUNROE, *XKCD*

This chapter serves both as an introduction to the book, and as a self-contained first presentation of Bell nonlocality.

## 1.1 Three roles for Bell nonlocality

Few scientific statements are more radical than one of the core tenets of quantum physics: *there is indeterminacy in nature.* It has accompanied quantum theory since its earliest moments: only a few months after Heisenberg and Schrödinger independently defined the definitive formalism, Max Born suggested that the laws of the new theory should be seen as intrinsically statistical. This was to become the orthodox view. Sensing the danger, Einstein quickly wrote to Born his conviction that a theory with statistical laws could only be a temporary fix, and that determinism should ultimately be recovered. The debate continued for decades with a few flares, notably the celebrated EPR paper (Einstein, Podolsky and Rosen, 1935) and Bohr's immediate reply, but in an atmosphere of overall indifference among physicists at large. In those years, the excitement about quantum theory was not found in debating its meaning, but in its almost boundless predictive power. It has become commonplace to refer to the attitude of those years by Mermin's dictum "shut up and calculate".

Ultimately, the statistical language became the standard to which generations of physicists conformed out of inertia. If asked for evidence of indeterminacy, still today many would refer to Heisenberg's uncertainty relations, that however can only voice for indeterminacy in quantum theory, not in nature (see Appendix **??**). This is surprising because direct evidence has been compelling since 1964, thanks to the work of John Bell (Bell, 1964). He showed that the possibility of recovering a deterministic model is amenable to experimental falsification, through the observation of a phenomenon that we shall call *Bell nonlocality.* In a first approach, Bell's argument is mathematically simple (see sections 1.3-1.4); because of its importance, it has been submitted to a thorough scrutiny, from which it has emerged unscathed and actually strengthened by more solid foundations (see section 1.5 and chapter **??**).

In 1964 there was already a huge amount of experimental evidence supporting the validity of quantum theory. Nevertheless, none of those data could be used to check Bell's criterion: dedicated experiments had to be designed. The work of Alain Aspect and coworkers is credited as the first conclusive evidence of Bell nonlocality (Aspect *et al.*, 1982*b*; Aspect *et al.*, 1982*a*). The evidence has been steadily growing since then; eventually, three independent experiments reported in 2015 are considered definitive (Hensen *et al.*, 2015; Giustina *et al.*, 2015; Shalm *et al.*, 2015). The main text of this book does not describe experiments; to facilitate reading the experimental literature, a quick guide is provided as Appendix **??**.

Discovered thanks to quantum theory, indeterminacy has been vindicated as a physical fact, independent of the theory itself. It can be circumvented only at the price of adopting even more radical postures about physics and nature themselves (see section 1.6). *This direct vindication of indeterminism is the original motivation of Bell nonlocality.* For a few decades, it was held to be its sole role too: those who, for various reasons, were already won to the indeterministic cause had taken note of it and moved on. A series of works that started around 2005 have uncovered a second role: *Bell nonlocality provides the most compelling certification of the correct functioning of some quantum devices*, like those required to perform quantum cryptography and quantum computation. The fabrication of these devices and the development of certification tools based on nonlocality still constitute technical challenges, but we'll have to get there. Far from being an exercise in scientific archaeology, this book contains material that future quantum engineers will have to master — *in nuce* at least, this is a treatise in applied physics.

Finally, as a phenomenon independent of quantum theory, Bell nonlocality is not merely an instrument for a negative task (falsifying determinism): it has a right to citizenship in physics. As its third role, *Bell nonlocality can be used as a principle constraining possible candidates for physical theories.* Barring a few pioneering insights, this approach was also started after the year 2000. It has already contributed several new ideas and notions to the field of foundations of physics but is still very open to future developments.

These three roles of Bell nonlocality — evidence for indeterminism, certification tool for devices, and guideline for foundations — correspond to the three parts into which this book is divided, but pervade the whole text. With them in mind, we can enter the core of the subject.

## 1.2 Introducing Bell nonlocality

### 1.2.1 Setting Bell tests: laboratories and games

Tests of Bell nonlocality, or *Bell tests* for short, are currently *experimental setups in physics laboratories.* Since some years, theorists have rather chosen to present Bell tests as *games* that bear some analogy with TV quizzes, polls, exams, judicial trials and other familiar situations[1]. The game setting is definitely better to bring up the

[1]The reader may come back to this list after becoming familiar with Bell test, to find analogies and differences. For instance, in exams, the content of the answer matters, and the verifier will evaluate the performance of each player without any concern for correlations. In judicial trials, the players's

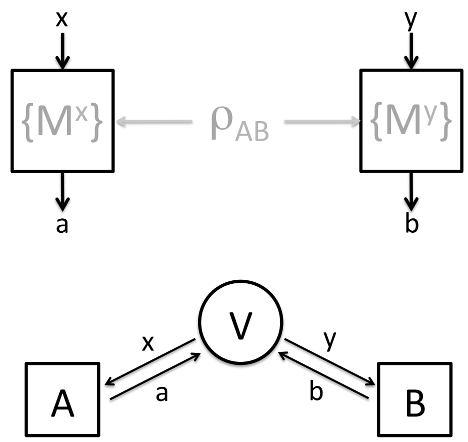


**Fig. 1.1** Sketch of a Bell test for two players (the generalisation to more players is straightforward) in the *laboratory setting* (top) and in the *game setting* (bottom). After having agreed on a process for that round, each player receives an input and has to provide an output; the data of several rounds are then sorted to establish the correlations between the outputs $a$ and $b$ for any pair $(x, y)$ of inputs. In the laboratory setting, we give in grey the usual representation of the process in quantum theory: a quantum state $\rho$ is prepared and some measurements are chosen; which measurement is actually performed in each round is determined by the input. None of this enters the definition of Bell nonlocality. In the game setting, we introduce a verifier V that queries the players Alice and Bob and collects their answers.

essence of nonlocality, and we shall mostly follow it in this book. Nonetheless, as we shall also see, many important discussions cannot be fully appreciated without going back to the lab. The two settings are sketched and compared in Figure 1.1.

In a Bell test game, the *players*, referred to alphabetically as Alice, Bob, Charlie etc., are all on the same team. The game consists of many *rounds*. In each round, the players will be separated: each will receive a query (*input*) and will have to provide an answer (*output*). It is useful to think of a *verifier* distributing the inputs[2] and collecting the outputs.

The rules of the game and the list of possible queries are known in advance. The

goal is to provide a consistent version of the story (which may not be the truth), but they don't know in advance the set of questions that they may be asked; etc.

[2]In actual experiments, the inputs are usually generated at each player's location by a "random number generator": the image of the verifier allows us to postpone the delicate discussion about randomness and its generation with physical means (subsection 1.5.3).

players are allowed to prepare a common *strategy* before the game, which consists in deciding which *process* they will use in each round of the game. We shall also speak of the *resources* that are used in these processes. If the players were allowed to communicate among each other during the game, they would actually not be separated and could easily win any game of this type: the most powerful resources are *signaling* ones. The case is more interesting with *no-signaling resources*. The most elementary example of a no-signaling resource is a list of pre-determined outputs, one for each possible input (that is, the process consists in producing the output by reading the list). It is no-signaling because, if Alice does something to her list, the other players obviously won't notice anything: in other words, Alice can't send a message to others by manipulating her list.

For instance, consider three games, each defined by one of the following rules:

(i) The players must produce the same answer if they receive the same query.
(ii) The players must produce the same answer if and only if they receive the same query.
(iii) The players must produce different answers if both receive query "1", the same answer otherwise.

A game based on rule (i) is trivially won by the players agreeing on a fixed common output. A game based on rule number (ii) can be similarly won by agreeing on a pre-determined output for each input, provided that the number of inputs is not larger than the number of outputs. If there were more inputs than outputs, the game cannot be won with a list of pre-determined answers. Finally, no strategy based on pre-determined answers can win a game based on rule (iii).

### 1.2.2 The definition of Bell nonlocality

*Bell locality* means that *the process by which each player generates the output does not take into account the other player's input.* In other words, all correlations between the players' outputs is due to the shared resource, on whose nature no assumption is made: it can be anything, from a list of numbers on a piece of paper to two jointly programmed quantum computers. When Bell locality does not hold we speak of *Bell nonlocality.*

This notion of locality can be formalised as follows. Denote by $\lambda$ the process. It does not need to be deterministic, so we can say that Alice generates $a$ by sampling from a probability distribution $P_\lambda(a|x)$. What is crucial is that this does not take Bob's input $y$ into account. Similarly, Bob generates $b$ locally by sampling from a probability distribution $P_\lambda(b|y)$. If this is the case, the statistics observed by the verifier (who is not privy to $\lambda$) will be described by

$$P(a,b|x,y) = \int d\lambda\, Q(\lambda)\, P_\lambda(a|x)\, P_\lambda(b|y) \tag{1.1}$$

where $Q(\lambda)$ is the probability distribution that describes the strategy, i.e. how often a specific process $\lambda$ is used. Bell locality is clearly a restriction: not all conceivable $P(a,b|x,y)$ can be written in this form. The extreme counterexample is a strategy that wins the game in which each player is suppose to output the other player's input.

Also, any winning strategy for the game based on rule (iii) above requires one of the players to sample from a distribution that depends on the other player's input.

Statistics will be called *local* if they can be written in the form (1.1), *nonlocal* if they cannot. A *Bell test* is a game whose winning strategy is described by nonlocal statistics.

### 1.2.3 On resources and semantics

If nonlocal statistics are observed, the verifier knows that the players have shared a *nonlocal resource*. If local statistics are observed, we can't say much about the resource: the players might have shared a potentially nonlocal one but have used it poorly. This sounds like elementary logic, but it triggers two crucial remarks:

- The definition of Bell locality does not rely on a prior characterisation of the class of "local resources"; even less one needs to assume that there *exist* in nature resources that are intrinsically local in this sense[3]. The opposite is the case: from the definition of Bell locality, that stands its ground, one can *define* "local resources" as hypothetical resources that could only lead to local statistics. The traditional name for such local resources is *"local hidden variables" (LHVs)* or simply *"local variables" (LVs)*, the word "hidden" being a relic of the discussions on quantum theory.
- We'll see in chapter **??** that every local statistics (1.1) can even be realised with a strategy based on pre-determined outputs. This result, known as Fine's theorem, is the basis for the mathematical tools of the field. But we cannot infer from this theorem that all observed local behaviors are actually generated with pre-determined outputs, nor that the definition of Bell locality assumes determinism.

Now, were it not for quantum theory, the definition of Bell nonlocality would sound both uninteresting and uncontroversial: nonlocal resources would be communication devices. Quantum theory[4] however forces us to enlarge, at least in principle, the list of possible nonlocal resources. Let us then consider that the players *share physical systems* in a state that quantum theory describes as $\rho_{AB}$, and let's assume that the process that produces the outputs is *performing local measurements* on this state. Specifically, upon receiving her input $x$, Alice performs a measurement on her system, with the output $a$ of that measurement is associated to the positive operator $\Pi_a^x$. Bob acts similarly. After several rounds, all played with this process, quantum theory predicts that the statistics collected by the verifier are given by

$$P(a,b|x,y) = \mathrm{Tr}(\Pi_a^x \otimes \Pi_b^y \, \rho_{AB})\,. \tag{1.2}$$

[3] In the same vein, notwithstanding the frequent replacement of "local" with "classical" in the field's jargon, the definition of Bell nonlocality does not rely on a definition of classicality, and even less on assuming the existence of intrinsically classical physical systems. Overall, we shall avoid speaking of classical/quantum systems or phenomena. It is correct to speak of classical theory and quantum theory, because these are well-defined (see Appendix **??** for the essentials). It is also customary to speak of classical/quantum *information* to refer to the resources, insofar as described within each theory, but we won't do it.

[4] Familiarity with elementary quantum theory is given for granted in this book; more advanced topics and specific aspects of quantum information theory are summarized in Appendix **??**.

In general, these statistics cannot be cast in the form (1.1): this is the content of *Bell's theorem* (Bell, 1964). Explicit examples will fill this book, but for the time being let us accept that *some shared quantum states are nonlocal resources.* However, it is also well known[5] that shared quantum states *are not communication channels*: by acting only on her system, Alice cannot learn anything about what Bob has done with his — he could have measured it, kept it, discarded it, and Alice does not see any change in her statistics. In this sense, quantum states are *no-signaling resources* just as shared lists of numbers. *Bell nonlocality is interesting and intriguing because it can be demonstrated by sharing no-signaling resources.*

Or can it? Famously (or notoriously), quantum theory does not provide any recipe for the generation of each round's output. Would it be possible that what quantum theory describes as no-signaling resources are actually signaling ones? Einstein dubbed this possibility "spooky action at a distance": as we shall see in section 1.6, it is one possible interpretation. Among those who oppose it, some think that the wording "nonlocality" evokes too closely this unwelcome interpretation. What we called "locality" in subsection 1.2.2, they'd rather call *local realism* or *local causality*. These are elegant expressions with philosophical appeal: they remind us that we are not merely dealing with operations and observations, but with a prejudice in our *Weltanschauung* that has been shattered. However, they are also not exempt from the danger of being over-interpreted[6].

With all their potential limitations, the wordings "nonlocality", "local realism", and "local (hidden) variables" have already enjoyed a few decades of tradition and are most probably here to stay. I hope I have said enough to prevent their misuse, and I shall use them freely. As for interpretations, we shall return to them in section 1.6.

## 1.3 My first Bell test: Clauser-Horne-Shimony-Holt (CHSH)

To put these general considerations on concrete grounds, we proceed to describe some specific examples of Bell tests. For this introductory chapter, I have chosen to present five classic examples: one in this section and four in the next; several others will be presented later in the book. An elementary proof that each is a indeed Bell test is given, exploiting Fine's theorem (subsection **??**) that allows considering only strategies based on pre-established answers. The relevance of each test for the certification of quantum entanglement is merely stated, leaving all the calculations for chapters **??**-**??**.

[5]Even if this should be elementary knowledge, given the centrality of the claim for the content of this book, the explicit proof is given in Appendix **??**.

[6]It has become commonplace to split the prejudice of "local realism/causality" into two separate prejudices, "locality" and "realism" (or "causality"). To be at peace with the fact of a violation, it would then be enough to abandon either. *Abandoning locality* may legitimately mean signaling: in (1.1), one would have $P(a|x,y,\lambda)$, $P(b|x,y,\lambda)$, or both; and this modification is indeed sufficient to generate Bell nonlocality. But the meaning of *abandoning realism/causality* is by far less clear (Norsen, 2007; Gisin, 2012). It cannot mean "abandoning determinism": determinism is not assumed in (1.1), so abandoning it does not generate Bell nonlocality. Neither should it mean "abandoning any connection with reality", reducing physics to unfounded speculations: if we abandon "local realism" it's because we accept the verdict of observation. Probably, "abandoning realism/causality" is a way of saying that only statistics are speakable, see subsection 1.6.3. But then, this alternative is at a different level than signaling: it is not a mechanism, but the statement that no mechanism should be looked for.

It should be obvious that a Bell test requires *at least two players*, otherwise there is no notion of locality. For each player, there must be *at least two possible inputs*: if some players could only receive one query, those inputs would be known to the other players. Finally, for each input, there must be *at least two possible values for the output.* We start with this simplest scenario.

The inputs of Alice are labelled $x \in \{0, 1\}$, her outputs $a_x \in \{-1, +1\}$ (labelling is of course arbitrary, this choice is convenient for the calculation to come). The inputs of Bob are labelled $y \in \{0, 1\}$, his outputs $b_y \in \{-1, +1\}$.

The rule of the game prescribes that Alice and Bob should aim at giving the same answer whenever $(x, y) \in (0, 0), (0, 1), (1, 0)$, but opposite answers when $(x, y) = (1, 1)$. We consider the score

$$S = \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \tag{1.3}$$

where the average is taken over an arbitrarily large number of rounds. The maximal score is obviously $S = 4$.

To prove that this game is a Bell test, we need to find what score can be achieved with local resources. Invoking Fine's theorem, it is enough to see what happens when Alice and Bob have shared a pre-determined quadruple $(a_0, a_1; b_0, b_1)$ in each round. The existence of these four numbers entails the existence of a well-defined value for the derived quantity $s = a_0 b_0 + a_0 b_1 + a_1 b_0 - a_1 b_1$. Since the average of a sum is the sum of the averages, $S = \langle s \rangle$ holds. Now, for every quadruple, either $s = +2$ or $s = -2$. This is readily seen by rewriting $s = a_0(b_0 + b_1) + a_1(b_0 - b_1)$: indeed, if $b_0 = b_1$ the second term is zero, if $b_0 = -b_1$ the first term is zero. In table 1.1 we list explicitly all sixteen possibilities: eight of them give $s = +2$ and the other eight $s = -2$. Notice how three out of four pairs of inputs contribute in the same way, but the last pair pulls the sum down (or up if it was negative). This observation will become handy later.

In each round, the verifier sees only the pair $(a_x, b_y)$ corresponding to the inputs $(x, y)$ he has sent, so he cannot estimate $s$. However, by performing statistics conditional on each pair of inputs, he can estimate the four $\langle a_x b_y \rangle$ and obtain $S$. Now, if $S = \langle s \rangle$ and each instance of $s$ can only take the values $\pm 2$, it follows that

$$|S| \overset{LV}{\leq} 2\,. \tag{1.4}$$

Suppose now that the verifier finds $S > 2$: he will have to admit that the players were not sharing pre-established quadruples, nor any resource that can be simulated with them – in other words, he will have to admit that *they share a nonlocal resource.*

This Bell test is called CHSH from (Clauser, Horne, Shimony and Holt, 1969). Notice how the mathematical expression of the test is an *inequality*, here Eq. (1.4). The players will convince the verifier that they have a nonlocal resource if they manage to *violate the inequality.* John Bell's original inequality (Bell, 1964) is basically the same as (1.4), but derived under the assumption that one of the $\langle a_x b_y \rangle$ is exactly equal to 1 (Appendix **??**). Since perfect correlations can be predicted but cannot be observed, that inequality is sufficient to prove that quantum theory predicts nonlocal resources but is untestable in experiments.

| $a_0, a_1; b_0, b_1$ | $a_0b_0$ | $a_0b_1$ | $a_1b_0$ | $a_1b_1$ | $s$ |
|---|---|---|---|---|---|
| $+1,+1;+1,+1$ | $+1$ | $+1$ | $+1$ | $+\mathbf{1}$ | $+2$ |
| $+1,+1;+1,-1$ | $+1$ | $-\mathbf{1}$ | $+1$ | $-1$ | $+2$ |
| $+1,+1;-1,+1$ | $-1$ | $+\mathbf{1}$ | $-1$ | $+1$ | $-2$ |
| $+1,+1;-1,-1$ | $-1$ | $-1$ | $-1$ | $-\mathbf{1}$ | $-2$ |
| $+1,-1;+1,+1$ | $+1$ | $+1$ | $-\mathbf{1}$ | $-1$ | $+2$ |
| $+1,-1;+1,-1$ | $+\mathbf{1}$ | $-1$ | $-1$ | $+1$ | $-2$ |
| $+1,-1;-1,+1$ | $-\mathbf{1}$ | $+1$ | $+1$ | $-1$ | $+2$ |
| $+1,-1;-1,-1$ | $-1$ | $-1$ | $+\mathbf{1}$ | $+1$ | $-2$ |
| $-1,+1;+1,+1$ | $-1$ | $-1$ | $+\mathbf{1}$ | $+1$ | $-2$ |
| $-1,+1;+1,-1$ | $-\mathbf{1}$ | $+1$ | $+1$ | $-1$ | $+2$ |
| $-1,+1;-1,+1$ | $+\mathbf{1}$ | $-1$ | $-1$ | $+1$ | $-2$ |
| $-1,+1;-1,-1$ | $+1$ | $+1$ | $-\mathbf{1}$ | $-1$ | $+2$ |
| $-1,-1;+1,+1$ | $-1$ | $-1$ | $-1$ | $-\mathbf{1}$ | $-2$ |
| $-1,-1;+1,-1$ | $-1$ | $+\mathbf{1}$ | $-1$ | $+1$ | $-2$ |
| $-1,-1;-1,+1$ | $+1$ | $-\mathbf{1}$ | $+1$ | $-1$ | $+2$ |
| $-1,-1;-1,-1$ | $+1$ | $+1$ | $+1$ | $+\mathbf{1}$ | $+2$ |

**Table 1.1** All sixteen quadruples of pre-established values, and derivative quantities for the CHSH test. In boldface, the term that pulls the sum $s$ in the opposite direction as the other three. Notice that the second half of the table is the mirror image of the first half, since flipping all the signs does not change the products.

The CHSH test is the workhorse of the field, we'll study it in great detail. Let's just mention that the maximal score $S = 4$ can be reached of course by signaling[7], but also with a hypothetical no-signaling resource called a *PR-box* (Popescu and Rohrlich, 1994) that we'll encounter in chapter **??**. However, PR-boxes exist in mathematics but don't seem to exist in nature: with quantum entanglement, the maximal score is $S = 2\sqrt{2} \approx 2.8284$. This value can be achieved by suitable measurements on the maximally entangled state of two qubits[8]

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle\,|0\rangle + |1\rangle\,|1\rangle\,\big) \tag{1.5}$$

and in fact, in a sense to be made precise in chapter **??**, *only* by that state and those measurements.

[7]Though rather obvious, here is one possible way in which the players can score $S = 4$ with signaling. Alice and Bob agree on a bit $a = b$. Upon being queried, Alice outputs $a$ and sends $x$ to Bob; Bob outputs $(-1)^{xy}b$. Notice that Alice's answer is pre-determined, but Bob's is not: he has to wait for $x$ before producing it. So, the conclusion that both outputs could not have been pre-determined holds.

[8]The notation $|0\rangle\,|0\rangle$ stand for $|0\rangle\otimes|0\rangle$. Here and in the rest of the book, the tensor product symbol is usually implicit when writing quantum states, while it is often explicit when writing operators. I find that this is the choice that facilitates the reading.

## 1.4 Four more classic Bell tests

This section introduces four other examples of Bell tests that should help gaining further familiarity with these notions.

### 1.4.1 Mermin's outreach criterion

In his effort to explain Bell nonlocality in a simple way, David Mermin (1981) conceived a Bell test that has become popular. There are two players, each with three inputs ($x, y \in \{1, 2, 3\}$) and two outputs ($a, b \in \{+1, -1\}$). Let's assume that the observation shows that $a_i = b_i$ in all cases where the same input was chosen. We are interested in $O = \sum_{x,y} P(a_x = b_y) = 3 + \sum_{x \neq y} P(a_x = b_y)$. If the outputs are pre-determined, $(a_1, a_2, a_3) = (b_1, b_2, b_3)$ can take eight values, namely $(+1, +1, +1)$, $(+1, +1, -1)$ etc. For $(+1, +1, +1)$ and $(-1, -1, -1)$, one finds $O = 9$; the other six triples give $O = 5$. Thus, certainly $O \geq 5$ for local resources. Quantum theory predicts that one can go down to $O = 4.5$. In particular, not even with quantum resources one can win perfectly the game based on rule (ii) defined in subsection 1.2.1, because that would correspond to $O = 3$.

The inequality $O \geq 5$ defines a Bell test only if $a_i = b_i$. If this assumption is removed, the inequality may be violated with LVs. For instance, the choice of pre-determined outputs $(a_1, a_2, a_3) = (+1, +1, +1)$ and $(b_1, b_2, b_3) = (-1, -1, -1)$ gives $O = 0$. This is the same weakness of Bell's original criterion, which had to be transformed into CHSH to become robust. Robust Bell tests for two players, three inputs and two outputs will be discussed in section **??**.

### 1.4.2 Greenberger-Horne-Zeilinger (GHZ) test

The original nonlocality test by Daniel Greenberger, Michael Horne and Anton Zeilinger (1989) considers four players, but the argument can be made for any number of players larger than two. Nowadays, we refer to the three-players version as the "GHZ test" without further qualifiers (Mermin, 1990); this is the one we present here. As in the CHSH case, each player has two inputs and two outputs, those of Charlie being labelled $z$ and $c$. Assume now that the verifier observes the following perfect correlations:

$$\langle a_0 b_0 c_1 \rangle = \langle a_0 b_1 c_0 \rangle = \langle a_1 b_0 c_0 \rangle = +1 \,. \tag{1.6}$$

Then

$$\langle a_1 b_1 c_1 \rangle \overset{LV}{=} +1 \,, \tag{1.7}$$

Indeed, $\langle a_x b_y c_z \rangle = +1$ means that $a_x b_y c_z = +1$ in each single round. If the outputs are pre-determined, they must be such that $a_0 b_0 c_1 = a_0 b_1 c_0 = a_1 b_0 c_0 = +1$ in each round. By multiplying the three conditions and noticing that $a_0^2 = b_0^2 = c_0^2 = 1$, we get $a_1 b_1 c_1 = +1$. In quantum theory, however, one can have the correlations (1.6) alongside

$$\langle a_1 b_1 c_1 \rangle = -1 \,. \tag{1.8}$$

This is obtained with suitable measurements on the state

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle\,|0\rangle\,|0\rangle + |1\rangle\,|1\rangle\,|1\rangle\,\big) \tag{1.9}$$

and, just as for CHSH, this quantum realisation is unique (see chapter **??**).

As presented, the GHZ test relies on the observation of perfect correlations or anti-correlations. To cope with unavoidable imperfect situations, its score can be turned into the so-called *Mermin inequality*:

$$M \;=\; \langle a_0b_0c_1\rangle + \langle a_0b_1c_0\rangle + \langle a_1b_0c_0\rangle - \langle a_1b_1c_1\rangle \;\overset{LV}{\leq}\; 2 \tag{1.10}$$

The proof that inequality (1.10) holds is left as Exercise 1.1. Thus, contrary to what happens with CHSH, the maximal score $M = 4$ of the Mermin inequality can in principle be attained in quantum theory. We shall complete the study of this inequality and its generalisations for more parties in section **??**.

### 1.4.3 Hardy's test

At this point, one may ask if one can build a Bell test for *two* players on extreme correlations that quantum resources can in principle achieve. There are indeed such examples. The first one was found by Lucien Hardy (1992, 1993). The extreme probabilities that are enforced are

$$P(a_0 = +1, b_0 = +1) = 0 \tag{1.11}$$
$$P(a_0 = -1, b_1 = +1) = 0 \tag{1.12}$$
$$P(a_1 = +1, b_0 = -1) = 0\,. \tag{1.13}$$

The following inferences are then obvious:

- From (1.12): $b_1 = +1$ implies $a_0 = +1$;
- From (1.11): $a_0 = +1$ implies $b_0 = -1$;
- From (1.13): $b_0 = -1$ implies $a_1 = -1$.

By enchaining these three inferences we are led to a fourth one, namely "$b_1 = +1$ implies $a_1 = -1$", and thus in particular to the prediction

$$P(a_1 = +1, b_1 = +1) \overset{LV}{=} 0\,. \tag{1.14}$$

Another way of reaching the same conclusion is suggested in Exercise 1.2.

In subsection **??**, we shall see that in quantum theory one may have the three constraints (1.11)-(1.13) and nonetheless $P(a_1 = +1, b_1 = +1) > 0$ with a maximal value of approximately 0.11. This looks absurd, since enchaining the three inferences above looks innocuous — but it is not: just as in the derivation of the CHSH inequality, the enchaining assumes that one can speak of both $a_0$ and $a_1$, and of both $b_0$ and $b_1$. Rigorously, the first inference should read: if $b = +1$ was found for $y = 1$, we know that $a = +1$ will be found *if the input* $x = 0$ *is called*; and similarly for the others.

### 1.4.4 The Magic Square

The rules of the Magic Square test[9] (Cabello, 2001*b*; Cabello, 2001*a*; Aravind, 2002) are slightly more complex. The test has three possible inputs per player, $x, y \in \{1, 2, 3\}$; the players are asked to output three bits each: $a_x = (a_x^1, a_x^2, a_x^3)$, $b_y = (b_y^1, b_y^2, b_y^3)$. These outputs should ideally satisfy the following conditions:

$$\prod_j a_x^j = +1\,,\ \prod_k b_y^k = -1\ ,\ \text{and}\ a_x^{j=y} = b_y^{k=x}\,. \tag{1.15}$$

To see that these conditions are impossible to fulfil perfectly with pre-determined outputs, let us arrange the nine pre-determined bits in a $3 \times 3$ square. Upon being queried, Alice outputs the $x$-th line of her square, and Bob the $y$-th column of his. The third condition says that the value at the intersection should be the same for every call $(x, y)$ of the verifier, which means that the squares must be identical. However, the first condition implies $\prod_{x,j} a_x^j = +1$, the second $\prod_{y,k} b_y^k = -1$. If these are to be enforced, Alice's and Bob's 9-bit squares should differ in at least in one bit — and then, if the verifier calls precisely those inputs, he will see that the third condition fails.

All in all, with pre-determined outputs, Alice and Bob can satisfy the three conditions (1.15) for at most 8/9 of the rounds on average; but there exists a quantum state and measurements that can fulfil them perfectly, as we shall show in subsection **??**.

## 1.5 A closer scrutiny: addressing loopholes

As we have just seen, Bell tests can be described in very elementary terms. But is this not too elementary, especially given the strong conclusions that are reached? Over the years, Bell tests have been submitted to tight scrutiny, searching for flaws, or *loopholes*, in the reasoning or in the implementations.

The four possible loopholes that have been identified turn out to be very different from each other: some are mere technical fixes (that have been fixed), others border on philosophy and can be closed only under reasonable assumptions (in other words, there is a price to pay if one wants to believe that they are still open). We review them here in this order; their working will be illustrated with the CHSH test.

### 1.5.1 The "memory loophole", or doing proper statistics

The memory loophole is related to statistics. Basic statistics assumes that rounds of a test are *independent and identically distributed (i.i.d.)*, but this i.i.d. assumption is obviously unwarranted when it comes to such fundamental tests. Would it be possible for the players to give a false positive in a Bell test, i.e. violate a Bell inequality with local resources, by adopting a non-i.i.d. strategy, that is, by choosing the process to be

[9] The square made its first appearance as a test for single-player "contextuality" (see Appendix **??** for this notion) in works of N. David Mermin and Asher Peres. Because of this, it is often called Mermin-Peres Magic Square. Here I cite the subsequent works that introduced it as a two-player nonlocality test.

| $a_0, a_1; b_0, b_1$ | $a_0 b_0$ | $a_0 b_1$ | $a_1 b_0$ | $a_1 b_1$ |
|---|---|---|---|---|
| $+1, +1; +1, [+1]$ | $+1$ | $N$ | $+1$ | **N** |
| $-1, -1; -1, [-1]$ | $+1$ | $N$ | $+1$ | **N** |
| $+1, +1; +1, [-1]$ | $+1$ | **N** | $+1$ | $N$ |
| $-1, -1; -1, [+1]$ | $+1$ | **N** | $+1$ | $N$ |
| $+1, -1; [+1], +1$ | $N$ | $+1$ | **N** | $-1$ |
| $-1, +1; [-1], -1$ | $N$ | $+1$ | **N** | $-1$ |
| $+1, -1; [-1], +1$ | **N** | $+1$ | $N$ | $-1$ |
| $-1, +1; [+1], -1$ | **N** | $+1$ | $N$ | $-1$ |

**Table 1.2** A strategy exploiting the detection loophole for the CHSH test. In each round, Alice and Bob choose one of the eight quadruples of pre-established values that would give $s = +2$ (c.f. Table 1.1). Alice answers always, while Bob declines to answer to the input indicated by brackets, in such a way that the problematic output (boldface) is never produced.

used in round $r$ based on all that has happened in the previous rounds? The answer is no.

To appreciate why, consider one round of the CHSH test. The players must choose the pre-established quadruple of values to be used in that round. They can base their choice on whatever piece of information from the past: there will always be one pair of inputs which pulls the sum in the wrong direction (see Table 1.1), and the verifier might have picked precisely that pair; so $|S| \leq 2$ still holds. This simple argument shows that there is only one way for the players to generate a false positive: avoid the wrong pair of inputs, either by refusing to answer or by colluding with the verifier. These are, respectively, the fair-sampling loophole and the free-will loophole described below.

Thus, even if we initially derived our Bell inequality thinking in i.i.d. terms, we have proved that *there is no memory loophole* if the Bell test is infinitely long and the verifier can extract perfect statistics. In a real test, when only finitely many rounds are possible, the Bell test must be phrased as hypothesis testing: how likely is the observed string of outputs assuming that the players are using a local resource? For such likelihood bounds, non-i.i.d. estimators must indeed be used instead of the familiar i.i.d.-based Gaussian standard deviations. We'll get back to this point in section **??**.

### 1.5.2 The "detection loophole(s)", or the dangers of post-selection

If in an exam the students were allowed to decline to answer till they are asked a question of their liking, the average score would be certainly increased. The same happens for nonlocality. In the CHSH test played with local strategies, we have seen that only one pair of inputs $(x, y)$ pulls the value of $S$ in the wrong direction: if the players were allowed to decline answering when they receive that specific query, the verifier would never catch them at fault. There is only a subtlety: neither of the players knows the pair $(x, y)$, so the decision to decline must be made locally, based on $x$ or on $y$ alone. A possible strategy in which Alice answers always while Bob is in charge of declining is given in Table 1.2. We refer to Exercise 1.3 for a thorough study of this strategy, and to Appendix **??** for a more rigorous quantitative approach.

The fix for this loophole is clear: *the verifier must elicit an answer in every round.* This does not sound to be a big deal, but it may be. Let us look at this loophole from the perspective of a honest experimentalist who possesses a very good nonlocal resource (say, a source of photons entangled in polarisation) and very accurate measurement devices, but whose detectors have poor efficiency. If she is obliged to produce outputs in every round, in most of the rounds she'll have to produce a dummy output because the detectors won't have fired: nonlocality is quickly washed down, and likely the observed data will be compatible with a local resource.

The situation is all the more annoying because this loophole has a very conspiratorial character. As John Bell himself stressed and many after him, quantum theory provides a very accurate description of such an experiment: it's very hard to believe that this accuracy is just an accident due to detectors being inefficient. Besides, the mechanism of the loophole assumes that a detector's firing depends on the input of the Bell test chosen in each round (to continue with the example: the polarisation basis chosen for the measurement). But experimentalists know that the detector's firing depends on internal parameters: with respect to the inputs of the Bell test, the detector is performing a *fair sampling.*

This is why, rather than claiming failure, all the early experiments have reported the observation of nonlocality under the fair-sampling assumption. Most likely, several future experiments will legitimately continue to do so. That being said, it's also important to put to record that nonlocality without the fair sampling assumption has been observed, first in an experiment with entangled ions (Rowe *et al.*, 2001), then in several other platforms, including of course the three "loophole-free" experiments of 2015 cited above.

Other loopholes related to the process of detection have been identified for some specific implementations: we refer the interested reader to the comprehensive review by Larsson (2014). Ultimately, they can all be closed by a rigorous implementation of the rule "one output for every input".

In summary, although they may prove challenging for some platforms, *the detection loopholes can be closed* and are therefore not a threat for the certification of nonlocality.

### 1.5.3 The "free-will loophole", or measurement independence

Instead of allowing the players to decline answering, the verifier could reveal some information about the inputs he is going to send out in every round. In the CHSH test, it is enough to inform the players about a pair of inputs that won't be used in a given round (in fact, more than enough: see Exercise 1.4).

This possibility sounds as artificial as the previous one, because the verifier has no reason to reveal that information. But there is a crucial difference. In the case of the detection loophole, it is easy to enforce an answer in every round (if the players refuse to comply, the verifier can fill the answer himself and too bad for them). Here, the verifier has to *ensure that no information leaks out to the players.* As frequent reports of leakage and hacking confirm, this is notoriously much more difficult to check, and ultimately impossible to guarantee in an absolute way.

In the laboratory setting, this loophole is open if the preparation of the system to be measured is correlated with the measurements that are going to be performed. This possibility is called *measurement dependence*; though less frequently than hacking, it has also been in the news[10]. The ultimate form of measurement dependence is superdeterminism; leaving metaphysics for section 1.6, let us assume that measurement independence is possible in principle, and ask how can one try and enforce it.

For some, the ultimate enforcement of measurement independence would be to choose the inputs using human free will; whence the name of "free-will loophole". I prefer to leave such a delicate notion as free will out of the picture[11]. At any rate, all that is required is to choose the inputs with *a process that is very unlikely to be correlated* with the resources shared by the players[12]. Some have gone as far as to generate the inputs of a Bell test from fluctuations of radiation coming from very distant stellar objects, i.e. produced by matter that has not been in contact with earthly matter since inflation, if ever (Handsteiner *et al.*, 2017). Impressive for a physicist, this choice of process may not be convincing for a technological skeptic: who guarantees that those telescopes and electronics are really producing stellar randomness? This is why others have argued that the best way to approach the free will loophole is to generate the inputs from the letters of one's favorite book or the Geneva phonebook (Pironio, 2015). Surely there are several ways in which these inputs may not be called random: there is a structure in the text, and the information has been available in the universe for quite some time — still, in order to refuse the evidence of Bell nonlocality, our skeptics must now believe that the behavior of some physical systems is correlated to the text of a book. If someone does not find this insane, there is little chance that they can be convinced anyway.

In summary: whether viewed as leakage of information from a verifier, or as hidden correlations among the devices in a laboratory, we are in the presence of a loophole that *can only be closed under reasonable assumptions*. In this book, *we shall always assume that measurement independence holds* till section **??**, where we shall see that one can relax it partially and still be able to certify Bell nonlocality. Finally, for interpretational matters it is crucial to stress that *measurement independence is compatible with determinism*: it requires that there exist several uncorrelated chains of events, but each chain can be deterministic. In other words, by assuming measurement independence, we are not introducing indeterminism by *fiat*, just as we are not requiring to accept any modality of human free will in order to certify nonlocality.

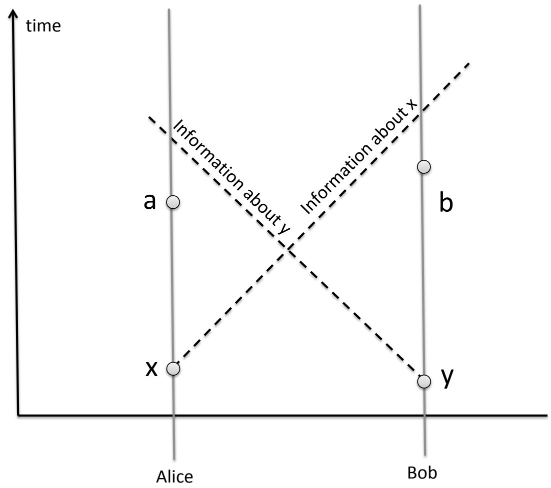


**Fig. 1.2** The space-time configuration in which the locality loophole is closed, with the dotted lines representing the light cones. Information about $x$ may arrive at the location of Bob only after the output $b$ has been produced, and information about $y$ may arrive at the location of Alice only after the output $a$ has been produced. There is no absolute way of determining when the information about the inputs was created, nor when a definite output is produced: the events that are relevant to close the locality loophole can only be defined under reasonable assumptions.

### 1.5.4 The "locality loophole", or hidden communication channels

The last loophole has a different flavor than the previous ones. It does not aim at generating a false positive, but at a trivial positive: if the nonlocal resource could be *communication*, one says that the "locality loophole" is open. Closing the loophole would mean to design a Bell test, in such a way as to certify nonlocality while guar-

[10] In 2015 it was discovered that a car manufacturer had programmed some models to detect the specific procedures of an anti-pollution test. The cars' emissions would then be lowered in order to pass that test, before returning to the normal, environment-unfriendly settings. As we see, measurement dependence exists; but it is rarely the result of an accident and is usually taken as the evidence of conscious tampering.

[11] I cannot resist referring the reader to an introductory book that reviews the debate on free will, accessible to beginners like us physicists (Griffith, 2013).

[12] If the process that is used to choose the inputs is pseudo-random, it is a finite-length algorithm and ultimately the players would be able to learn it (Bendersky *et al.*, 2016). I cite this in a footnote because, important as it is conceptually, this possibilty makes no difference in practice: real Bell tests achieve excellent statistical significance in far fewer rounds than would be needed to guess even a moderately short algorithm.

anteeing that no communication was happening. This seems to be possible using a physical fact: information propagates at a speed bounded by that of light in vacuum.

Recall that the information that needs to be communicated are the inputs. For every player $p$, we denote by $\vec{x}^{\,(p)}$ their position, by $t_i^{(p)}$ and $t_o^{(p)}$ the times at which the input is received, respectively at which the output is produced. The verifier wants to enforce that each player produce their output before any information about the other players' inputs may have arrived at their location: in jargon, *space-like separation* between the event "output" of each player and the events "input" of all the other players. This requirement reads[13] $|\vec{x}^{\,(p)} - \vec{x}^{\,(p')}| > c(t_o^{(p)} - t_i^{(p')})$ for all pairs of players $(p, p')$ (Figure 1.2).

Can one find flaws in this argument? Some may question the physical assumption: maybe some information can propagate faster than light. We shall discuss this option as interpretation in subsection 1.6.1, then as attempted models in chapter **??**. But there is a more subtle possible flaw that went unnoticed for decades: in order to run an argument based on space-like separation, one must also be able to *identify the relevant events*. Alice can say when and where the input was fed into her devices: but she can't say when and where the information about the input was created in the universe. Similarly, Alice can estimate when an electric current left the detector to convey the information to the computer: but is *that* the time at which the actual result is created, the time of the "collapse"?

In summary, *the locality loophole can be closed for known communication channels* between the players, or by adopting operational definitions of the events, as several experiments did[14]. Ruling out any unknown form of communication seems to be impossible: even if the speed of communication is believed to be bounded, we wouldn't know which are the relevant events.

### 1.5.5 The unknown loophole: skepticism

It should be clear that Bell nonlocality has been scrutinised with great rigour. Some die-hard skeptics are not convinced: every now and then, someone claims to have found the flaw in the argument. These claims are usually based on very convoluted arguments and tend to fall into three categories: utterly wrong (for instance, the alleged counterexample is just a variation of the detection loophole); exegeses of Bell's papers (whether Einstein, Bell or anyone else was right or wrong is interesting for the history of science, but science should be judged without reference to their authority); deep discussions on the meaning of probability (some of which may hit home but would apply to every statistical statement and not only to the certification of Bell nonlocality:

[13] For notational simplicity, we assume that the relative positions of the players are fixed, which is usually the case in implementations. Also notice that, since the condition of space-like separation is Lorentz-invariant, we did not need to specify in which frame events are parametrised.

[14] The definition of the events is related to electric signals: for the input, when a signal leaves the "random number generator" to reach the measurement device; for the output, when the electric signal leaves the detector to propagate to the computer where the information will be stored. The locality loophole was first addressed in (Aspect, Dalibard and Roger, 1982*a*), but the first experiment using a more proper random number generator was performed years later in the group of Anton Zeilinger (Weihs *et al.*, 1998). A few months earlier, the group of Nicolas Gisin had put the emphasis on the distance rather than on the timing of the random number generation, observing Bell nonlocality between players separated by 10km (Tittel *et al.*, 1998).

after all, the philosophy of probability and the cogency of statistical conclusions are still debated).

To be sure, by definition one cannot exhaust the list of possible loopholes, and science should always be open to revision. But at this stage, I strongly believe that the burden of the proof should be on the deniers. If anyone has found the flaw, they should be able to write the corresponding algorithm, take two computers that have been pre-programmed together but do not communicate during the rounds of the game, and exhibit a statistically significant violation of a Bell inequality [see e.g. section 9 of (Gill, 2014)]. In the presence of such evidence, all physicists of the "establishment" will be ready to reconsider the matter.

## 1.6 Experimental metaphysics?

There is abundant observational evidence for nonlocality with all the loophole closed – the memory and detection loopholes, indisputably; the free-will and locality loopholes, up to assumptions that go unquestioned in virtually all the rest of science, and have been noticed in this context only because of the strength of the claim. So, we are in the presence of a phenomenon that calls for interpretation.

We may want to start by refocusing on the dilemma. It is the same dilemma for any form of nonlocality, but let us just refer to the GHZ test: given the outputs of two of the players, there is only one possible output for the third — and nevertheless, that output was not predetermined. When was it determined then, and how?

Many positions have been put forward, often overlapping with one's interpretation of quantum theory. For the purpose of this book, I have chosen to classify the options in four groups. This being my own classification, to avoid both canonisations and imprecise attributions I have decided not to insert any citation in what follows. For further reading, one can start with some essays of very different, often opposite flavor published with the occasion of the 50th anniversary of Bell's theorem (Fuchs *et al.*, 2014; Maudlin, 2014; Werner, 2014; Wiseman, 2014; Żukowski and Brukner, 2014). There are also two systematic treatises on the meaning of Bell nonlocality, by Tim Maudlin (2011) and Jeffrey Bub (2015); and two popular books by Anton Zeilinger (2010) and Nicolas Gisin (2014) — and of course, a famous collection of John Bell's reflections (Bell, 2004).

### 1.6.1 Group 1: nonlocal hidden variables

The easiest position to describe is that of those who infer from Bell nonlocality the existence of *nonlocal hidden variables*. With this position, one recovers determinism by staying within a *mechanical paradigm*: that is, one can simulate of how nature processes information in order to produce the outputs.

What would these nonlocal hidden variables be? A form of communication (superluminal or even retrocausal, i.e. propagating to the past), the infinitely rigid quantum ether of Bohmian mechanics, a connection in an unknown dimension... Whatever they may be, in our $(3+1)$-dimensional space-time they would appear as "influences" carrying information from one location to the other, which Einstein famously dubbed "spooky action at a distance" in his debates with Bohr[15].

Now, these hypothetical influences would carry information, only to tweak it in such a way that it looks no-signaling to us. With a positive wink, Shimony called it "peaceful coexistence with relativity"; more negative critics rather highlight the conspiratorial flavor: why would nature use a signal while hiding its use from us? Both this fine tuning and the relation will relativity will be discussed in detail in chapter **??**. In particular, there we shall see a quantitative result: in order to remain "hidden", these influences must propagate at an *infinite* speed in their preferred frame.

### 1.6.2 Group 2: superdeterminism and its friends

In the second group, I shall put superdeterminism and stances that (at least in my view) are akin to it.

In subsection 1.5.3, we have seen that Bell nonlocality can be demonstrated as soon as the processes that choose the inputs and those chosen by the players are independent. The strongest way to deny this measurement independence is *superdeterminism*: all the events in the universe constitute a single, deterministic process. There could be somewhat milder ways of *denying measurement independence*: for instance, worldviews à la *The Matrix* in which our universe is a big simulation, maybe not deterministic in origin (it could be run by aliens using their true free will). Needless to say, the consequences of adopting such a worldview extend far beyond solving the conundrum of Bell nonlocality.

Another option, directly inspired by quantum theory, are the so-called *many-worlds interpretations* that deny that a definite output is ever singled out. These interpretations say that the reversible dynamics of quantum theory is an accurate depiction of the deepest reality, which we do not perceive with our senses but have discovered with our investigations. What is usually deemed irreversible in an elementary reading of the quantum formalism, namely the act of measurement, is nothing else than getting entangled with the apparatus, and then with the environment, with the hard disk that stores the data, with the consciousness of the players... The players will perceive definite outputs, and the verifier will certify Bell nonlocality after many rounds, because that's how the rules are set: in every world, there is indeterminacy and nonlocality. But nature is playing all the options, and this deployment of correlations is fully deterministic.

### 1.6.3 Group 3: only statistics are speakable, a.k.a. the "orthodox"

Like the many-worlds interpretations, the third group also asserts the correctness of quantum formalism, but in a very different way. Here, quantum theory is the correct way of computing probabilities in the (only) physical world — and there is nothing else we should talk about. Individual rounds of a Bell test (or of any other experiment: interferometers, Stern-Gerlach...) are "unspeakable".

As some of my colleagues like to say, this is "just standard quantum mechanics": indeed, it is what people identify as the orthodox interpretation. But what should be

[15] The first record of this expression seems to date from the 1927 Solvay conference.

mentioned, is that it implies a significant epistemological discipline. Physics is generally understood as a representation of nature, a study of the constituents of matter and their dynamics — loosely speaking, it should describe "how nature does it". However, a long list of philosophers may find this stance too naive, and the intrinsically statistical character of quantum theory has won several physicists over to the idea that *a law of nature may rather be a way of organising our knowledge.* Bayesianism becomes the proper language: probabilities capture someone's degree of belief. A law of physics does not prescribe the belief itself, which is subjective, but how beliefs should evolve given new information (and it is perfectly acceptable that these updating rules be not subjective).

If someone adopts this approach to physics and knowledge, the "intrinsic indeterminacy" of quantum theory adds only a minor element of discomfort: at some point, agents have to give up the possibility of a more refined description, one leading to stronger beliefs. As for the description of Bell tests, contrary to many-worlds, the definite outputs are real and the agent has a special role.

The strength and weakness of this position are both simultaneously evident in the way it deals with the GHZ test: one denies to explain how the third player manages to give that unique answer, but stresses than an agent should definitely bet on that unique answer, if informed about the other two. This position makes pragmatically correct statements and avoids all the problems: for some it's wisdom, for others escapism.

### 1.6.4 Group 4: hoping for collapse

This last position, in a sense, closes the circle. In this view, the outputs are real (contrary to some in Group 2), they are produced through some intrinsically random process (contrary to Groups 1 and 2) and through Bell nonlocality we are learning about this process that does happen in nature (contrary to Group 3).

The challenge here is to describe the process, usually called "collapse", by which the output of each round is generated. There have been several attempts, but none seems to be fully convincing. Because of Bell nonlocality, any collapse model will have to be nonlocal to describe bipartite or multipartite statistics: the process that generates one player's output must take into account other players' inputs. But it seems inescapable that collapse models exhibit some form of nonlocality for single-player processes too: in a measurement of position, the particle must localise itself somewhere, whence the possibility of finding it elsewhere should fall to zero; if a single photon is sent through a beam-splitter and found in one of the beams, it must become impossible to find it in the other beam.

### 1.6.5 Additional remarks

I have just tried a simple systematisation of the current state of interpretations. This matter is complex and can be approached from several angles. I'll go through a few more viewpoints here.

Let us first address the question of whether Bell nonlocality is *experimental metaphysics* that shapes our *Weltanschauung*. Group 1 would certainly claim so. For Groups 2 and 3, Bell nonlocality is just one of the rules that have been set up and does not play a foundational role (when it comes to Group 2, very few facts can claim to play a foundational role in a deterministic worldview). For Group 4, collapse models have first been studied to explain the appearance of a classical macroscopic world, but Bell nonlocality is something that such models are urged to explain too. At any rate, whatever position one reaches after reflection, Bell nonlocality must have entered that reflection: nobody brushes it off as irrelevant *a priori*.

Next, I find it interesting to compare these interpretations in terms of *resources and information*. We don't have a recipe to describe how the outputs are generated if we stick only to resources that we can control. Group 4 hopes that this is still possible, at least with a special recipe of collapse. Group 1 favours a mechanistic recipe, at the price of introducing unobserved resources, the nonlocal hidden variables. Groups 2 and 3 stick to the resources that we have: for Group 2, all the outputs are generated according to the rules and it's sheer chance that we end up perceiving one rather than another alternative; for Group 3, how nature generates the outputs is not our business as long as our predictions are correct.

Further, let us consider the issue of *whether quantum theory is complete*, which was the title of the EPR paper. Both Groups 2 and 3 would definitely answer that quantum theory is complete; but we have noticed that they differ in what they call "quantum theory": for the ones it's the kinematics and reversible dynamics, for the others the recipe for computing probabilities. Representatives of Group 4 would like to complete the theory with a model of collapse, probably hinting that collapse has always been a desired feature of quantum theory, a statement with which the others would vehemently disagree. Group 1 advocates for the need of quantum theory to be completed, at least in its ontology (our predictive power may well remain the same).

*Philosophical labels* are also worth mentioning. Based on observation, I can certify that most of my colleagues would like to be called "realist" in the philosophical sense of believing in the existence and intelligibility of an external reality. Conversely, when the debates get heated, it is frequent to hear insults like "idealist", "pragmatist" or "solipsist" thrown to the representatives of the other camp. One may wonder, for instance, where is the realism in Group 3: they would answer that the laws for updating our beliefs come to us from observation. All in all, this kind of labels should be avoided, also because few of us physicists would be able to pass an exam of philosophy on their exact meaning — philosophers themselves may not agree!

Another favorite topic of speculation is *where great figures of the past would stand in this debate*. The usual names that come up are of course Einstein, Bohr and Bell himself. Assuming that he would stick to his aversion to a dice-playing God and to action-at-a-distance, Einstein would either think that we don't have yet enough evidence (something like Group 4) or lean towards determinism at higher levels (Group 2). For Bohr, it is clear that he would despise Group 1; if I'd have to bet, his sympathy would go to Group 3. We know more for Bell. He set out to construct a local hidden variable model, probably thinking that it was possible. When he found it is not, he shifted towards Bohmian mechanics, and when collapse models started to be studied he clearly looked at them with great hope. Where he would stand today, given the evidence that collapse models have not delivered much, is only guesswork.

Surely there is much more to it, and the readers will find further inspiration in reading more or in their own reflections. For the purpose of this book, it is time to put an end to this general introduction and to move on to the formalisation of Bell nonlocality.

## Exercises

**Exercise 1.1** *Prove that the Mermin inequality* (1.10) *holds indeed for deterministic local variables. Hint: either* $c_0 = c_1$*, or* $c_0 = -c_1$.

**Exercise 1.2** *Re-derive Hardy's LV prediction* (1.14) *by ticking out from Table 1.1 the quadruples of pre-established values that do not comply with the constraints (1.11)-(1.13).*

**Exercise 1.3** *We consider a modification of the detection loophole strategy for CHSH described in Table 1.2. At every round, Alice and Bob choose one of the eight quadruples listed in the Table with probability* $\frac{1}{8}$. *Then Bob applies that strategy of declining with probability* $1-p$*, whereas with probability* $p$ *he produces the agreed output to whatever input he receives. The verifier computes* $S$ *using only the rounds in which both Alice and Bob replied.*

1. *Prove that the verifier will observe* $S = 4\frac{3-p}{3+p}$ *(hint: what is the fractions of rounds in which Bob replies?). Deduce that this strategy gives a false positive for every* $p > 0$.
2. *According to the verifier, Bob replies to either input with "efficiency"* $\eta_B = \frac{1+p}{2}$. *Deduce the efficiency threshold* $\eta_B^*(S)$ *for this strategy as a function of* $S$.
3. *In a Bell test, the verifier has observed* $S_{obs}$, $\eta_{A,obs} = 1$ *and* $\eta_{B,obs}$*: what can be said based on the above calculation? (a) The detection loophole will be certainly closed if* $\eta_{B,obs} > \eta_B^*(S_{obs})$ *(b) The detection loophole will certainly not be closed if* $\eta_{B,obs} \leq \eta_B^*(S_{obs})$. *(c) Both of the above. (d) None of the above.*

**Exercise 1.4** *We consider a false positive for the CHSH test based on the free will loophole.*

1. *In every round, the verifier informs the players that one specific pair of inputs will be sent out with probability* $q$*, while the three other pairs will be equally probable. Find the value of* $S$ *that the players can achieve with local strategies for every* $q \in \left[0, \frac{1}{4}\right]$. *Deduce that this leads to a false positive for every* $q < \frac{1}{4}$*, and that the quantum maximum* $S = 2\sqrt{2}$ *can be reached without having to set* $p = 0$.
2. *Consider now a different situation: the verifier informs the players that in every round the pair* $(x, y) = (1, 1)$ *is drawn with probability* $q$ *and the other three pairs with equal probability. Does this open any loophole? Hint: the probabilities that enter a nonlocality test are conditional on the inputs.*

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