VII. Applications of indefinite temporal orders

1) Query complexity (Araujo et. al 2014)

Task: (special case)
- 2 black-box unitaries U_A, U_B with promise that $[U_A, U_B] = 0$ (C) or $\{U_A, U_B\} = 0$ (AC) surely holds.
- With min. queries to U_A & U_B determine whether C or AC holds.

QS: One query per unitary. Fixed order: atleast one U queried twice.

$$\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C)|\psi\rangle_T \xrightarrow{QS} \frac{1}{\sqrt{2}}(|0\rangle_C U_B U_A|\psi\rangle_T + |1\rangle_C U_A U_B|\psi\rangle_T)$$

Hadamard on C.

$$= \frac{1}{2}\left(|0\rangle_C \{U_A, U_B\}|\psi\rangle_T + |1\rangle_C [U_A, U_B]|\psi\rangle_T\right)$$

⇒ measure control in Z basis to get deterministic answers.

More generally: n-switch permutes n unitaries.
& we can consider $n!$ different properties.
- n-switch: $O(n)$ queries ← worst case
- fixed orders: $\Omega(n^2)$ queries ← best case

⇒ polynomial advantage

But can get exponential adv. in communication Complexity!

2) Communication complexity (Guerin et. al 2016)

Set up (causally ordered): 3 parties A, B, C.

- A, B given inputs $x \in X$, $y \in Y$.
- C must calculate $f(x,y)$ while minimizing communication between parties.

Only one-way communication allowed!
(i.e either $A \to B \to C$ or $B \to A \to C$)

Exchange evaluation game (EE_n, $n \in \mathbb{Z}^+$)

A, B given inputs $(\vec{x}, f), (\vec{y}, g) \in \mathbb{Z}_2^n \times F_n$
(n bit string encoded as vectors)

$$F_n := \{f : \mathbb{Z}_2^n \to \mathbb{Z}_2 \mid f(\vec{0}) = 0\}$$

C must output: $EE_n(\vec{x}, f, \vec{y}, g) = f(\vec{y}) \oplus g(\vec{x})$
($\oplus$: mod 2 addition)

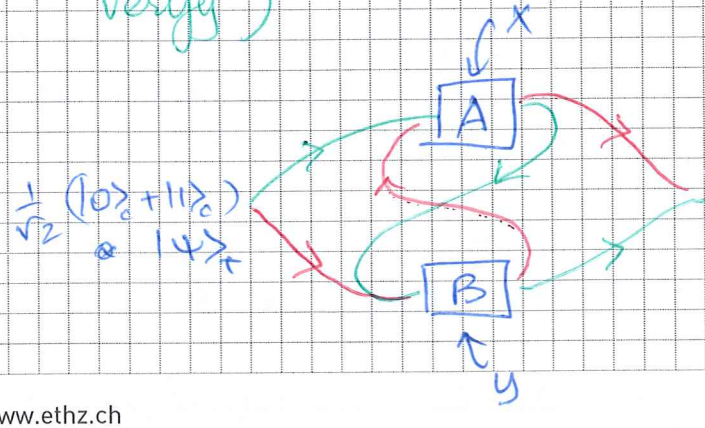
→ Can encode all possible $(\vec{x}, f)$ in a set of n-qbit unitaries $U_n = \{U_n^i\}$ such that any 2 Us in this set either satisfy C or AC.

Then apply QS to solve this!

- communication exchanged b/w A & B is target system: n-qbits
- Fixed orders ~ 2^n qbits required.

→ exp. advantage

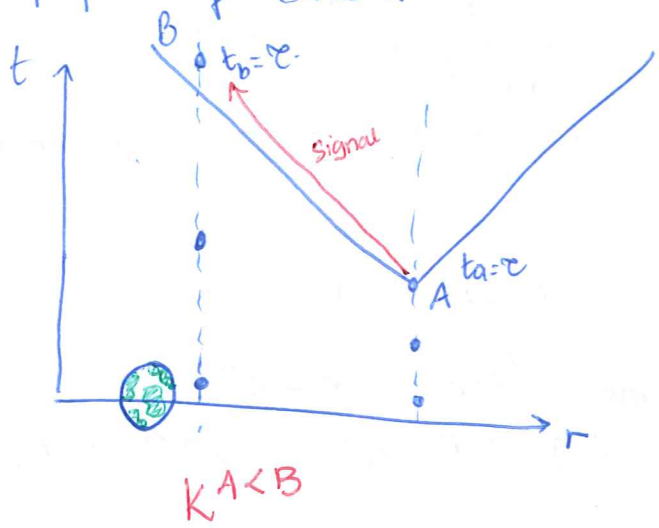
★ controlled superpos of $A \to B \to C$ & $B \to A \to C$ is **not** 2 way communication!
(Can introduce counters at output ports of A, B to verify)



VIII | Gravitational Quantum Switch (GS) | (Zych et. al 2017)

↑ if there is time!

Spatial Superpos. of gravitating mass ⇒ superpos. of spacetime geometries ⇒ superpos. of causal order.



- Initially synchronised clocks at A & B.
- t: time co-ord. of far away agent C
  t_a, t_b: proper times of A & B.
- Similarly $K^{B<A}$ when mass is next to A's lab. (B can signal to A)

Thought experiment: What if we prepare the mass M in a superpos of the 2 configurations?

⇒ can use it as control system to send target S from A to B or B to A depending on it coherently.

$$\left(\alpha |K^{A<B}\rangle_M + \beta |K^{B<A}\rangle_M\right) \otimes |\psi\rangle_S \xrightarrow{GS} \alpha |K^{A<B}\rangle_M U_B U_A |\psi\rangle_S + \beta |K^{B<A}\rangle U_A U_B |\psi\rangle_S$$

PM for QS & GS are the same but they are physically very different situations! QS is a CB but GS is not!

GS	QS
1) From Alice's & Bob's point of view, the system enters their labs at time τ for both alternatives.	1) System enters each lab at different times depending on the control. E.g if $\|0\rangle_c$, A gets system at $t=1$ & B at $t=3$ & reverse for $\|1\rangle_c$ (CB see rep.)
2) Causal structure indefinite in neighbour hood of mass; labs.	2) Temporal order indefinite only for the target system
3) Cant be modelled as CB	3) Can be modelled as CB.

IX. Conclusions

1) Quantum causality fundamentally different from classical

2) Many frameworks, important insights can be gained by comparing them.

3) Quantum switch claimed to be an indefinite causal structure but comparing it in PM & CB frameworks shows that it implements an indefinite temporal order

4) QS still interesting: computational advantage over fixed orders, can't be written as quantum circuit.

5) Superposition of gravitating mass can result in a truly indefinite causal order.

6) Many open questions! (see thesis)

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