Interpretations of Quantum Mechanics

Thursday, June 20, 2019

Some Obligatory Feynman Quotes

“I think I can safely say that nobody understands quantum mechanics” – ”The Character of Physical Law”, chapter 6, p. 129

“We always have had … a great deal of difficulty in understanding the world view that quantum mechanics represents. At least I do, because I'm an old enough man that I haven't got to the point that this stuff is obvious to me. Okay, I still get nervous with it. And therefore, some of the younger students … you know how it always is, every new idea, it takes a generation or two until it becomes obvious that there's no real problem. It has not yet become obvious to me that there's no real problem. **I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem.**” - "Simulating Physics with Computers", *International Journal of Theoretical Physics*, volume 21, 1982, p. 467-488

Outline of the Lectures

1. **Defining the Problem Precisely**
2. **Orthodoxy and the Measurement Problem**
3. **An Aside on Decoherence**
4. **De Broglie-Bohm Theory**
5. **Spontaneous Collapse Theories**
6. **Everett/Many-Worlds**
7. **Copenhagenish Interpretations**

1. Defining the Problem Precisely

The standard postulates of quantum theory involve heavy use of the concept of "measurement" or "observer".

This is unlike previous fundamental theories, which gave a story of what things exist and how they behave independently of our interventions, e.g. in classical electromagnetism there are particles with charge and mass, there is the electromagnetic field, and they obey Maxwell's equations and the Lorentz force rule.

Is this a problem? Depends on how you think about scientific theories in general.

1.i Realism vs. Anti-Realism

Realism

There exists an objectively real physical world, independent of observers.

The job of a physical theory is to attempt to describe it.

Successful physical theories are approximately correct descriptions of the objectively real physical world.

It is more accurate to think of theoretical entities, e.g. electrons, quarks, as referring to things that actually exist than to do otherwise.

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Realism does not necessarily mean:

- All quantum observables must have definite values at all times.
- The quantum state is a description of reality.
- Nature must be deterministic.
- Our theories are literally true.

Warning: “Realism” is often used in these ways in the literature on quantum foundations. It is much harder to deny “real” realism.

Antirealism

Varieties: idealism, logical positivism, empiricism, instrumentalism, operationalism.

The only things we have direct access to are our own perceptions and/or the records of results from our experimental apparatuses.

Theories are simply systems for organizing/predicting regularities in those perceptions/results.

Theoretical entities, e.g. electrons, are a convenient fiction used in our calculations.

Operationalism: Every statement of a theory should boil down to a list of instructions for what to do in the lab and what will be seen as a result.

Putnam's No Miracles Argument

"When they argue for their position, realists typically argue against some version of idealism - in our time, this would be positivism or operationalism. (…) And the typical realist argument against idealism is that it makes the success of science a miracle

(…)The modern positivist has to leave it without explanation (the realist charges) that ‘electron calculi’ and ‘space-time calculi’ and ‘DNA calculi’ correctly predict observable phenomena if, in reality, there are no electrons, no curved space-time, and no DNA molecules. If there are such things, then a natural explanation of the success of theories is that they are partially true accounts of how they behave. And a natural account of the way scientific theories succeed each other (…) is that a partially correct/incorrect account of a theoretical object (…) is replaced by a better account of the same object or objects. But if those objects don’t really exist at all, then it is a miracle that a theory which speaks of gravitational action at a distance successfully predicts phenomena; it is a miracle that a theory which speaks of curved space-time successfully predicts phenomena; and the fact that the laws of the former theory are derivable ‘in the limit’ from the laws of the latter theory has no methodological significance."

H. Putnam, Meaning and the Moral Sciences, Routledge (1978)

Eddington's Fishy Allegory

"Let us suppose that an ichthyologist is exploring the life of the ocean. He casts a net into the water and brings up a fishy assortment. Surveying his catch, he proceeds in the usual manner of a scientist to systematise what it reveals. He arrives at two generalisations: (1) No sea-creature is less than two inches long. (2) All sea-creatures have gills. These are both true of his catch, and he assumes tentatively that they will remain true however often he repeats it.

In applying this analogy, the catch stands for the body of knowledge which constitutes physical science, and the net for the sensory and intellectual equipment which we use in obtaining it. The casting of the net corresponds to observation; for knowledge which has not been or could not be obtained by observation is not admitted into physical science.

An onlooker may object that the first generalisation is wrong. "There are plenty of sea-creatures under two inches long, only your net is not adapted to catch them." The icthyologist dismisses this objection contemptuously. "Anything uncatchable by my net is *ipso facto* outside the scope of icthyological knowledge. In short, "what my net can't catch isn't fish." Or — to translate the analogy — "If you are not simply guessing, you are claiming a knowledge of the physical universe discovered in some other way than by the methods of physical science, and admittedly unverifiable by such methods. You are a metaphysician. Bah!""

A. Eddington, The Philosophy of Physical Science (1938)

Realism vs. Anti-Realism

You don't have to be a realist to realize that realist explanations are often useful.

- If I have a story about what exists and how it behaves then I have a framework for reasoning about novel physical situations and for generalizing the theory.

It is interesting that all of our physical theories prior to quantum theory admit a realist account, even if you don't believe they are literally (approximately) true.

We might ask why we cannot find a realist account that we all agree upon for quantum theory.

You don't have to be an operationalist to realize that stepping back from a realist account and temporarily defining things in terms of things we can do in the lab is often useful.

- E.g. Einstein's derivation of special relativity, the development of thermodynamics prior to statistical mechanics.

Since the operational implications of quantum theory are the only part we all agree upon, it may be useful to reformulate the theory operationally and come back to the realism question later.

1.ii Four Criteria for an Interpretation of Quantum Theory

Four criteria for an interpretation of quantum theory:

1. **Ontology**
2. **Save the Phenomena**
3. **Consistency**
4. **Progress**

Ontology

If quantum theory is an approximately correct theory of the universe:

What kinds of things exist and how do they behave?

- We are asking what *would* exist in a world described by quantum mechanics, not what actually exists in our world. It is more about the explanatory structure of the theory than the real world.
- We are not assuming realism: only outcomes exist is an acceptable answer, but you still have to deal with the other three criteria.

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Save The Phenomena

What we see in experiments should be explained *in terms of* our ontology.
Two parts:

- Explain what occurs in quantum theory (e.g. why do probabilities obey the Born rule?)
- Explain the emergence of classicality

This constrains our answer to the ontology problem. A bunch of green aliens on Mars is a possible ontology, but it would not save the phenomena.

Local Beables vs. Emergence

In classical mechanics we have particles and fields that are localized in spacetime. It will obviously be easier to derive this if we assume that the theory underlying quantum theory also has an ontology of entities that are localized in spacetime, e.g. particle trajectories or fields that are functions of spacetime. These were called **local beables** by John Bell. Also known as **primitive ontology** in the literature. de Broglie-Bohm and spontaneous collapse theories take this option.

The alternative is that there are no local beables. Instead, localized entities emerge in the classical limit, in the same sort of way that there is no notion of temperature in Newtonian mechanics, but it emerges in the thermodynamic limit. Everett/many-worlds and Copenhagenish interpretations take this option.

Consistency

It should not be possible by analysing an experiment in two different ways to arrive at two contradictory conclusions about:

- what will be observed in the experiment, or
- what the state of reality is

Progress

The correct interpretation of quantum mechanics should lead to progress in physics.

I don't want to dwell on this, and many would disagree, but I believe there is a *correct* interpretation of quantum mechanics, and that scientific truths should have pragmatic value, so this is the way we will identify that we have arrived at the correct interpretation.

2. Orthodoxy and the Measurement Problem

Macroscopic superpositions and the measurement problem are often thought to be the most pressing problems in the foundations of quantum theory.

But they have been solved multiple times. They are not problems with quantum theory per se, but rather with the interpretation of quantum theory usually given in textbooks.

- This is why I defined the criteria differently, in an interpretation neutral manner.

This is known as the *Orthodox, Textbook, or Dirac-von Neumann* interpretation.

It is often mislabeled as the Copenhagen interpretation, but it differs drastically from the views of Bohr, Heisenberg, etc. that it is not even in the same category.

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2.i. The Orthodox Interpretation

1. Physical systems have objective properties:
   - The possible properties of a system are its observables. The possible values of those properties are the corresponding eigenvalues.
2. The **eigenvalue-eigenstate link**:
   - When the system is in an eigenstate of an observable M with eigenvalue m then M is a property of the system and it takes value m.
   - The system has no objective physical properties other than these.

The eigenvalue-eigenstate link is equivalent to saying that the quantum state $|\psi\rangle$ is an objective property of an individual quantum system and that it is the *only* objective property of the system.

Why?

- By e-e link $|\psi\rangle\langle\psi|$ is a property of the system with value 1.
- This uniquely determines $|\psi\rangle$ (up to global phase), so $|\psi\rangle$ is a property.
- All other objective physical properties are uniquely determined by $|\psi\rangle$.

2.ii. Schrödinger's Cat

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself, it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks." – J. Trimmer, "The Present Situation in Quantum Mechanics: A Translation of Schrödinger's 'Cat Paradox' Paper" Proc. Am. Phil. Soc. vol. 124 pp. 323-338 (1980).

If we interact a macroscopic system with a microscopic system in a superposition, then we can generate superpositions of macroscopically distinct states, e.g.

$$\frac{1}{\sqrt{2}}\left(|\text{Cat is alive}\rangle + |\text{Cat is dead}\rangle\right)$$

In orthodox interpretation this is physically distinct from

$$|\text{Cat is alive}\rangle \quad \text{or} \quad |\text{Cat is dead}\rangle$$

The macroscopic superposition does not correspond to anything in our direct experience.

2.iii. The Measurement Problem

A related problem is that there are two ways of handling measurements in quantum theory.

1. The measurement postulates.
2. A measurement device is a physical system, made of atoms, so we ought to be able to describe it as a quantum system, which interacts unitarily with the system being measured.

As an example, consider a qubit in state

$\alpha|0\rangle + \beta|1\rangle$

upon which we perform an ideal measurement in the basis $\{|0\rangle, |1\rangle\}$.

According to the measurement postulates, the system will either collapse to

$|0\rangle$ with probability $|\alpha|^2$

or $|1\rangle$ with probability $|\beta|^2$.

Now consider the measurement device as a physical system. Let $|R\rangle$ be the state in which it is ready to perform the measurement, i.e. initial state is

$(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle$

The measurement is an interaction between the system and the measuring device, described by a unitary operator U.

Let $|M_0\rangle$ be the state in which the measuring device registers 0.

Let $|M_1\rangle$ be the state in which the measuring device registers 1.

Then,

$U|0\rangle \otimes |R\rangle = |0\rangle \otimes |M_0\rangle$

$U|1\rangle \otimes |R\rangle = |1\rangle \otimes |M_1\rangle$

By the superposition principle, we should then have:

$U[(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle] = \alpha|0\rangle \otimes |M_0\rangle + \beta|1\rangle \otimes |M_1\rangle$.

On the orthodox interpretation, this is physically distinct from

$|0\rangle$ with probability $|\alpha|^2$

or $|1\rangle$ with probability $|\beta|^2$.

This is a violation of our consistency criterion, so the orthodox interpretation is not adequate.

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- In response to Schrödinger's cat and the measurement problem, John Bell said

  "Either the wavefunction, as given by the Schrödinger equation, is not everything or it is not right" - J. S. Bell, Are There Quantum Jumps? In Speakable and Unspeakable In Quantum Mechanics, 2nd edition, pp. 201-212 (Cambridge University Press, 2004)

- As we are not necessarily primitive ontologists, we can expand this to, either the unitary evolution of the wavefunction:

1. Is everything, but we deny the eigenvalue eigenstate link
   Everett/many-worlds
2. Is not everything
   de Broglie-Bohm theory
3. Is not right
   spontaneous collapse theories
4. Is not anything
   Copenhagenish interpretations

- These are the "big four" interpretations of quantum mechanics, i.e. the ones most often discussed in the literature. They are far from the only possibilities, but they do illustrate the main existing strategies for interpretation.

3. An Aside on Decoherence

Strategies 1, 2, and 4 work by denying that the measurement postulates represent a real physical process. To do this, we have to understand how a unitarily evolving quantum state can account for the fact that systems look, to us, *as if* they have been measured and undergone a collapse. Decoherence usually forms *part* of this story.

The basic idea is to recognize that quantum systems typically interact with their environments, e.g. photons and air molecules scatter off the system. As a toy model, consider a system in a superposition of two macroscopically distinct quantum states $|\langle\psi_1|\psi_2\rangle| \approx 0$

$$\alpha|\psi_1\rangle + \beta|\psi_2\rangle$$

Now imagine that a photon scatters off the system. We will assume a scattering limit, where the interaction with the photon has very little effect on the system, but does affect the photon. If the state of the photon before the interaction is $|\chi_0\rangle$ then we will have an interaction like

$$|\psi_1\rangle|\chi_0\rangle \to |\psi_1\rangle|\chi_1\rangle, \qquad |\psi_2\rangle|\chi_0\rangle \to |\psi_2\rangle|\chi_2\rangle$$

so the superposition will evolve into

$$\alpha|\psi_1\rangle + \beta|\psi_2\rangle \to \alpha|\psi_1\rangle|\chi_1\rangle + \beta|\psi_2\rangle|\chi_2\rangle$$

The scattering interaction may not be very strong, so we need not have $\langle\chi_1|\chi_2\rangle = 0$. Instead we generally have $0 < |\langle\chi_1|\chi_2\rangle| < 1$

Now let's look at the reduced density operator of the system in the $|\psi_1\rangle, |\psi_2\rangle$ basis. Before the scattering it is

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}.$$

After the scattering it is

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\chi_2|\chi_1\rangle \\ \alpha^*\beta\langle\chi_1|\chi_2\rangle & |\beta|^2 \end{pmatrix}$$

so the moduli of the off diagonal terms have been reduced by a factor $|\langle\chi_1|\chi_2\rangle|$.

Now suppose that N photons scatter off the system according to the same interaction. Then we will have

$$\alpha|\psi_1\rangle|\chi_1\rangle^{\otimes N} + \beta|\psi_2\rangle|\chi_2\rangle^{\otimes N}$$

and the reduced density operator will be

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\chi_2|\chi_1\rangle^N \\ \alpha^*\beta\langle\chi_1|\chi_2\rangle^N & |\beta|^2 \end{pmatrix}$$

Suppose that the number of scattered photons per unit time is n so that $N = nt$. Then if $\omega(t)$ is the modulus of the off diagonal terms at time t, we will have

$$\omega(t) = \omega(0)\left|\langle\chi_1|\chi_2\rangle\right|^{nt} = \omega(0)e^{-t/t_D} \quad \text{where} \quad t_D = -\frac{1}{n\ln|\langle\chi_1|\chi_2\rangle|} \quad \text{is called the } \textit{decoherence time}.$$

The off diagonal terms decay exponentially and the reduced density operator will quickly become indistinguishable from

$$\begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}.$$

This is the same density operator that the system would have if it were prepared in the state $|\psi_1\rangle$ with probability $|\alpha|^2$ or $|\psi_2\rangle$ with probability $|\beta|^2$, which is exactly what the measurement postulates say happens. Thus, if you do not have access to the environment, you will be unable to tell whether or not the measurement collapse has happened by measuring the system alone.

Comments on Decoherence

On its own, decoherence does not solve the measurement problem. The state of the universe is still

$$\alpha|\psi_1\rangle|\chi_1\rangle^{\otimes N} + \beta|\psi_2\rangle|\chi_2\rangle^{\otimes N}$$

which makes different predictions from $|\psi_1\rangle$ with probability $|\alpha|^2$ or $|\psi_2\rangle$ with probability $|\beta|^2$, so the eigenvalue-eigenstate link still leads to a contradiction.

If we discard the orthodox interpretation, decoherence on its own does not supply us with an adequate interpretation, as we have not specified an ontology. However, once an ontology is specified, decoherence does play an important role in saving the phenomena in many interpretations.

- Our toy example already shows that decoherence is not exact. The off diagonal terms are extremely small, but not zero.
- In more realistic models, there will be more than just the two states $|\psi_1\rangle, |\psi_2\rangle$. Since these have small, but nonzero overlap, these states and even the number of them, are not precisely defined.
- If you have limited precision measurements, then these effects will not be noticeable, but, in general, any model of decoherence involves coarse graining, and the branches it leads to are not precisely defined.

  For a more detailed treatment of decoherence, see M. Schlosshauer, Decoherence and the quantum to classical transition (Springer, 2007).

4. de Broglie-Bohm Theory

dBB theory is a primitive ontology theory. It satisfies the ontology criterion by postulating that every particle has a definite position.

4.i. Equations of the theory

Let's start with a single spinless particle in one dimension. Label the definite position of the particle X.

In addition to the position, the particle also has a wavefunction $\psi(x) = \langle x|\psi\rangle$. This is a *secondary ontology*. A table is made of particles with definite positions, the wavefunction is used to determine how they move.

The wavefunction obeys the Schrödinger equation

$$i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$$

In addition to this, the particle position obeys the *guidance equation*

$$\frac{dX}{dt} = \frac{1}{m}\frac{\mathrm{Im}\left(\psi^*(x,t)\dfrac{\partial\psi(x,t)}{\partial x}\right)}{\psi^*(x,t)\psi(x,t)}\Bigg|_{x=X}$$

Note: the wavefunction influences the motion of the particle, but the particle motion does not influence the evolution of the wavefunction. This is responsible for many of the counter-intuitive features of dBB theory, as we shall see.

Moving on to a spinless particle in 3D, we introduce the basis $|\vec{q}\rangle = |x\rangle \otimes |y\rangle \otimes |z\rangle$ and the wavefunction $\psi(\vec{q}) = \langle\vec{q}|\psi\rangle$.

The quantum state evolves according to the Schrödinger equation as before, but the particle also has a definite position vector $\vec{Q}$, and the guidance equation becomes

$$\frac{d\vec{Q}}{dt} = \frac{1}{m}\frac{\mathrm{Im}\left(\psi^*(\vec{q},t)\vec{\nabla}\psi(\vec{q},t)\right)}{\psi^*(q,t)\psi(q,t)}\Bigg|_{\vec{q}=\vec{Q}}$$

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To describe N particles, we need to specify a position vector for each of them

$\boldsymbol{q} = (\vec{q}_1, \vec{q}_2, \cdots, \vec{q}_3)$

Notation: $\vec{q}$ denotes a vector in $\mathbb{R}^3$. $\boldsymbol{q}$ denotes a vector in $\mathbb{R}^{3N}$, called a *configuration vector***.**

$\mathbb{R}^{3N}$ is called *configuration space*.

We can write a quantum state as a wavefunction on configuration space:

$\psi(\boldsymbol{q}, t) = \psi(\vec{q}_1, \vec{q}_2, \dots, \vec{q}_N, t) = \langle \boldsymbol{q} | \psi(t) \rangle = \langle \vec{q}_1, \vec{q}_2, \dots, \vec{q}_N | \psi(t) \rangle$

The wavefunction obeys the Schrödinger equation, but dBB also has an actual point in configuration space:

$\boldsymbol{Q} = (\vec{Q}_1, \vec{Q}_2, \dots, \vec{Q}_N)$

This obeys the guidance equation:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}\left(\psi^*(\boldsymbol{q}, t) \vec{\nabla}_k \psi(\boldsymbol{q}, t)\right)}{\psi^*(\boldsymbol{q}, t)\psi(\boldsymbol{q}, t)}\Bigg|_{\boldsymbol{q}=\boldsymbol{Q}}$$

Note: The form of the guidance equation shows that dBB is highly nonlocal. The motion of particle k depends on the current position of all the other particles.

4.ii. Saving the Phenomena

Having formulated the theory, we need to determine whether it reproduces the quantum predictions and accounts for the apparent collapse of the wavefunction. For this we need one additional postulate.

Equivariance and the Equilibrium Hypothesis

The *quantum equilibrium hypothesis* states that at time $t = t_0$, the probability density for the system to occupy the configuration space point $\boldsymbol{Q}$ is

$\rho(\boldsymbol{Q}, t_0) = |\psi(\boldsymbol{Q}, t_0)|^2.$

We will show that, if this holds at $t = t_0$ then it also holds at all other times. This is called *equivariance*.

There is controversy about what $\rho(\boldsymbol{Q})$ means as dBB is applied to the *entire universe*, which only has a single configuration space point.

> Roughly speaking, if we prepare many systems in the state $|\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle$, the probability density of configurations is $\rho(\boldsymbol{Q})$**.**

Note that the quantum state is playing two *independent* roles:

- It governs dynamics via the guidance equation.
- It is used to set the probability density.

Bell's Derivation of the Guidance Equation and Equivariance

Solutions of the Schrödinger equation satisfy the continuity equation:

$$\frac{\partial |\psi(\boldsymbol{q}, t)|^2}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}(\boldsymbol{q}, t) = 0$$

where $\boldsymbol{J}(\boldsymbol{q}, t)$ is the probability current:

$\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \dots, \vec{J}_N) \qquad \vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k} \mathrm{Im}(\psi^* \vec{\nabla}_k \psi)(\boldsymbol{q})$

Consider a large number of systems prepared in the state $|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle \otimes \cdots$

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We want to consider $\boldsymbol{J}(\boldsymbol{q},t)$ as a flow of particle density rather than probability.

The simplest way to do this is to assume the flow is generated by a velocity field $\boldsymbol{v}(\boldsymbol{q})$, i.e. a unique velocity for every point in configuration space as in hydrodynamics. Then, $\boldsymbol{J} = \rho \boldsymbol{v}$, so the equation for the velocity field should be:

$$\boldsymbol{v}(\boldsymbol{q}) = \frac{\boldsymbol{J}(\boldsymbol{q})}{\rho(\boldsymbol{q})} \qquad \vec{v}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\frac{\mathrm{Im}\left(\psi^* \vec{\nabla}_k \psi\right)}{\rho}(\boldsymbol{q})$$

which gives the dBB velocities if we set $\rho(\boldsymbol{Q}) = \left|\psi(\boldsymbol{Q})\right|^2$.

The continuity equation then guarantees that equivariance holds.

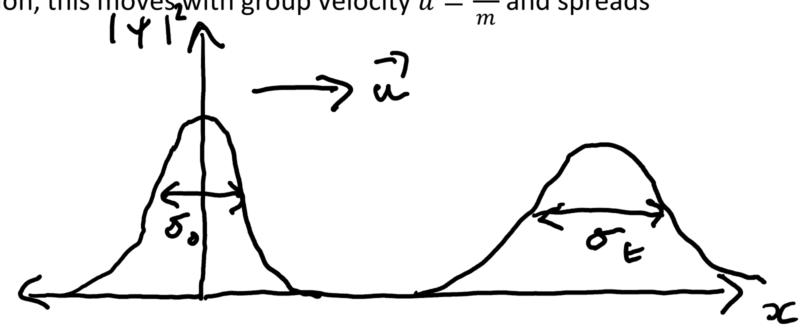
Trajectories for 1D Gaussian Wavepackets

Consider an initial Gaussian wavepacket moving towards the right

$$\psi(x,0) = \frac{1}{(2\pi\sigma_0^2)^{\frac{1}{4}}}\exp\left[-\frac{x^2}{4\sigma_0^2} + ikx\right]$$

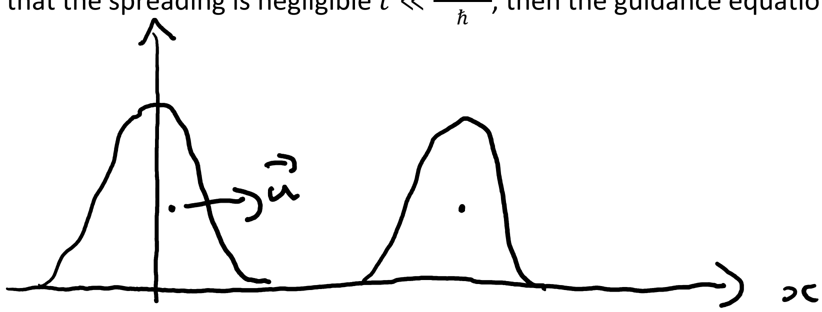
Under free particle evolution, this moves with group velocity $u = \frac{\hbar k}{m}$ and spreads

$$\sigma_t = \sigma_0\sqrt{1 + \frac{\hbar^2 t^2}{4m^2\sigma_0^4}}$$



If we consider a timescale such that the spreading is negligible $t \ll \frac{2m\sigma_0^2}{\hbar}$, then the guidance equation gives

$$\frac{\mathrm{d}X}{\mathrm{d}t} \approx u$$



As you were told in undergraduate QM, in dBB a Gaussian wavepacket really does correspond to a particle moving along with the group velocity of the wavepacket.

In general, a particle located at the center of the wavepacket moves with velocity u. Those ahead of the center move a bit faster and those behind move a bit slower, and this accounts for the spreading of probabilities.

See A. Pan, Pramana J. Phys. 74:867 (2010) for a detailed treatment of Gaussian wavepackets in dBB.

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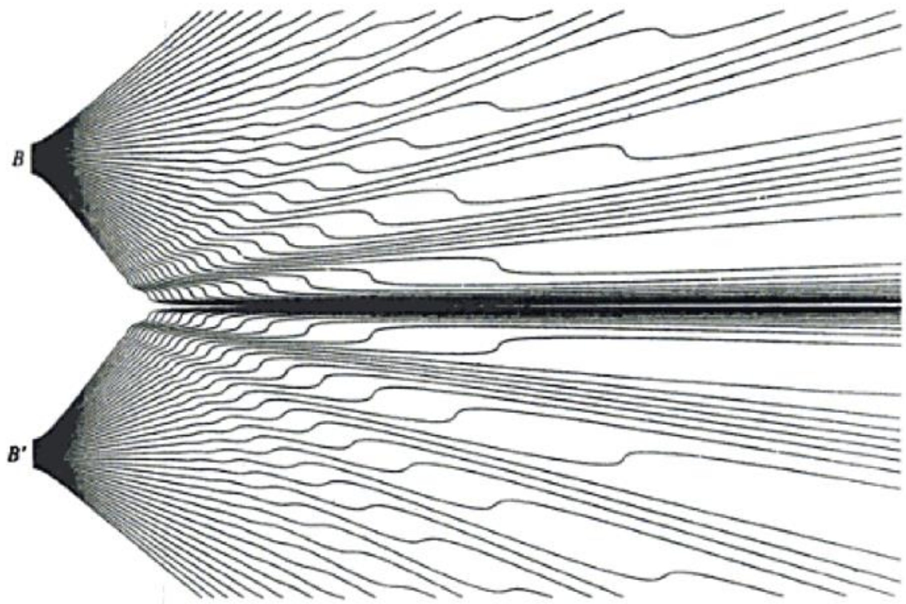
Double Slit Trajectories



C.Philippidis et. al. Il Nuovo Cimento, vol.52B, No.1 (1979)

Model with Gaussian slits

$\psi(\vec{q},t) = \psi_B(\vec{q},t) + \psi_{B'}(\vec{q},t)$

When ψ_B and $\psi_{B'}$ have approximately no overlap (close to the slits)

$\vec{J} \approx \vec{J}_B + \vec{J}_{B'}$
$\vec{J}_B = \frac{1}{m}\mathrm{Im}\left(\psi_B^*\vec{\nabla}\psi_B\right), \qquad \vec{J}_{B'} = \frac{1}{m}\mathrm{Im}\left(\psi_{B'}^*\vec{\nabla}\psi_{B'}\right)$

The trajectories are as in geometric optics, i.e. perpendicular to the wave-fronts.

When the two components overlap there are additional cross-terms (interference) in the current, causing deflections that give rise to the characteristic double-slit pattern.

Measurements in dBB theory

If we divide the universe into system S and environment E, with associated configurations $(\boldsymbol{Q}_S, \boldsymbol{Q}_E)$, we can define a pure state for the system called its *conditional quantum state*

$$\left|\psi_{\boldsymbol{Q}_E}\right\rangle_S = \langle \boldsymbol{Q}_E | \psi \rangle_{SE}$$

where $\boldsymbol{Q}_E$ is the actual configuration of the environment.

Generally, these do not evolve according to the Schrödinger equation, *except* if there is decoherence into environment states that are localized in configuration space.

For example, suppose we model a measurement device as a large number of particles, where the outcomes of the measurement correspond to macroscopically distinct states with very little overlap in their configuration space wavefunctions.

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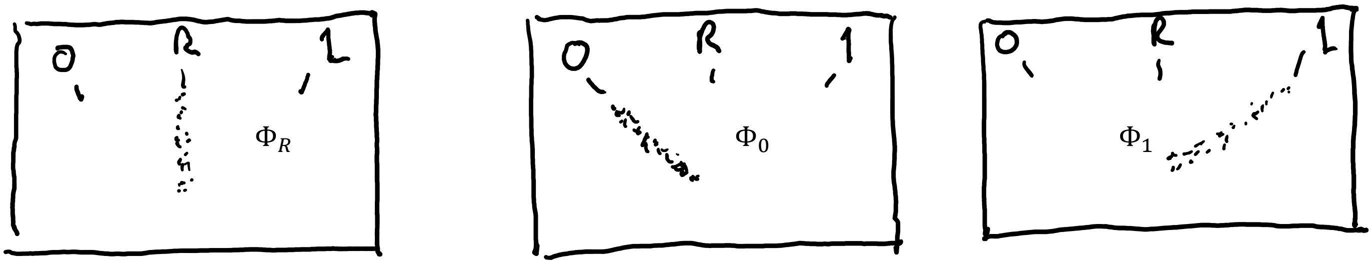

$$\Phi_0(\boldsymbol{q}_E)\Phi_1(\boldsymbol{q}_E) \approx 0$$

In a measurement interaction:

$$[\alpha\psi_0(\boldsymbol{q}_S) + \beta\psi_1(\boldsymbol{q}_S)]\Phi_R(\boldsymbol{q}_E) \quad \rightarrow \quad \alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

If the lack of position overlap between $\Phi_0(\boldsymbol{q}_E)$ and $\Phi_1(\boldsymbol{q}_E)$ persists in time then:

- The actual configuration of the environment $\boldsymbol{Q}_E$ is either in the support of $\Phi_0(\boldsymbol{q}_E)$ or the support of $\Phi_1(\boldsymbol{q}_E)$.
- By equivariance, it will be in the support of $\Phi_0(\boldsymbol{q}_E)$ with probability $|\alpha|^2$ and in the support of $\Phi_1(\boldsymbol{q}_E)$ with probability $|\beta|^2$.
- The conditional state of the system will either be $\propto \psi_0(\boldsymbol{q}_S)$ or $\propto \psi_1(\boldsymbol{q}_S)$.
- $\psi_0(\boldsymbol{q}_S)$ and $\psi_1(\boldsymbol{q}_S)$ each evolve according to the Schrödinger equation.
- The current breaks into two terms $\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{J}_1$, with $\boldsymbol{J}_0 = 0$ in the support of $\Phi_1(\boldsymbol{q}_E)$ and vice versa, i.e. no cross terms in the guidance equation.

We get an effective collapse into either $\psi_0(\boldsymbol{q}_S)$ or $\psi_1(\boldsymbol{q}_S)$ and we can use the corresponding current $\boldsymbol{J}_0$ or $\boldsymbol{J}_1$ in the guidance equation to compute subsequent evolution.

If the measurement is an (approximate) position measurement then also $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \approx 0$.

The initial configuration $\boldsymbol{Q}_S$ of the system is either in the support of $\psi_0(\boldsymbol{q}_S)$ with probability $|\alpha|^2$ or in the support of $\psi_1(\boldsymbol{q}_S)$ with probability $|\beta|^2$.

The measurement outcome is a deterministic function of $\boldsymbol{Q}_S$: position measurements simply reveal the pre-existing position.

However, for other observables, e.g. momentum, $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \neq 0$, i.e. the initial configuration does not necessarily "belong" to one of the two eigenstates.

Which measurement outcome occurs is a function of *both* $\boldsymbol{Q}_S$ and $\boldsymbol{Q}_E$.

Momentum measurement does not measure the dBB momentum $m_k \frac{d\vec{Q}_k}{dt}$.

The theory is *deterministic*: outcome uniquely determined by states of system and measuring device.

But not *outcome deterministic*: outcome uniquely determined by state of system on its own.

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Treatment of Spin

In the minimalist approach to dBB (favored by Bell), no observables apart from position are part of the primitive ontology.

Spin only appears in the wavefunction.

We can write a wavefunction including spin as a spinor, e.g. for a single particle:

$$\psi_0(\vec{q}) \otimes |\uparrow\rangle + \psi_1(\vec{q}) \otimes |\downarrow\rangle \quad \rightarrow \quad \bar{\psi}(\vec{q}) = \begin{pmatrix} \psi_0(\vec{q}) \\ \psi_1(\vec{q}) \end{pmatrix}$$

For N spin-1/2 particles, we would have a 2^N dimensional spinor vector.

The guidance equation is now:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}\left(\bar{\psi}^* \cdot \vec{\nabla}_k \bar{\psi}\right)}{\bar{\psi}^* \cdot \bar{\psi}} (\boldsymbol{Q}),$$

where $\cdot$ is spinor inner product, i.e. $\bar{\psi}(\vec{q}) \cdot \bar{\phi}(\vec{q}) = \psi_0(\vec{q})\phi_0(\vec{q}) + \psi_1(\vec{q})\phi_1(\vec{q})$

4.iii Counterintuitive Features of dBB trajectories

dBB trajectories display several features that violate classical intuitions about particle trajectories.

It is important to note that, if decoherence occurs in an environmental basis that is localized in position, dBB trajectories of the system will approximately follow classical trajectories.

dBB doesn't *owe* us anything more than that. So long as:

- it reproduces the predictions of quantum theory in measurements, and
- macroscopic systems typically have approximately classical trajectories,

then the theory saves the phenomena.

Since quantum and classical predictions are different, dBB trajectories *must* differ from classical ones in some situations.

The question is only if they are weirder than absolutely necessary to reproduce quantum theory, and whether that is a bad thing.

Real Stationary States are Really Stationary

Consider a stationary state: $\psi(\boldsymbol{q}, t) = \psi_n(\boldsymbol{q}) e^{-iE_n t/\hbar}$

The current is: $\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k} \mathrm{Im}(\psi_n^* \vec{\nabla}_k \psi_n)(\boldsymbol{q})$, i.e. is independent of t.

However, if $\psi_n(\boldsymbol{q})$ is also a real valued function then:

$$\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{2im_k} (\psi_n^* \vec{\nabla}_k \psi_n - \psi_n \vec{\nabla}_k \psi_n^*)(\boldsymbol{q}) = 0$$

The particles are also stationary, e.g. particle in an infinite well, harmonic oscillator, ground state of spherically symmetric potentials (hydrogen atom).

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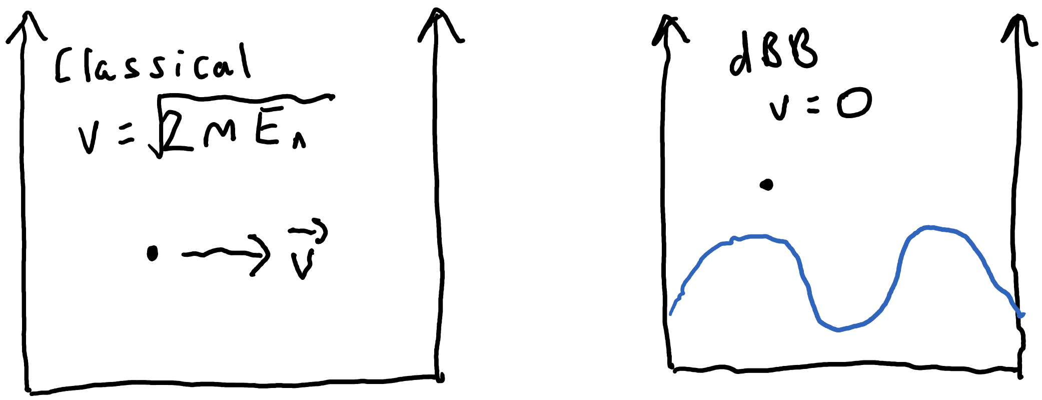


The No-Crossing Rule

In classical mechanics, phase space trajectories do not cross (except at singularities) because equations are 2 nd order and so $(\boldsymbol{q}, \boldsymbol{p})$ contains enough data to specify a unique trajectory.

In dBB the guidance equations is 1st order and there is no back action on the quantum state from the configuration space point:

$[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}(t_0)]$ and $[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}'(t_0)]$ specify unique trajectories.

Trajectories associated with the same wavefunction evolution cannot cross in *configuration space*.

This is responsible for almost all the weird features of dBB trajectories.

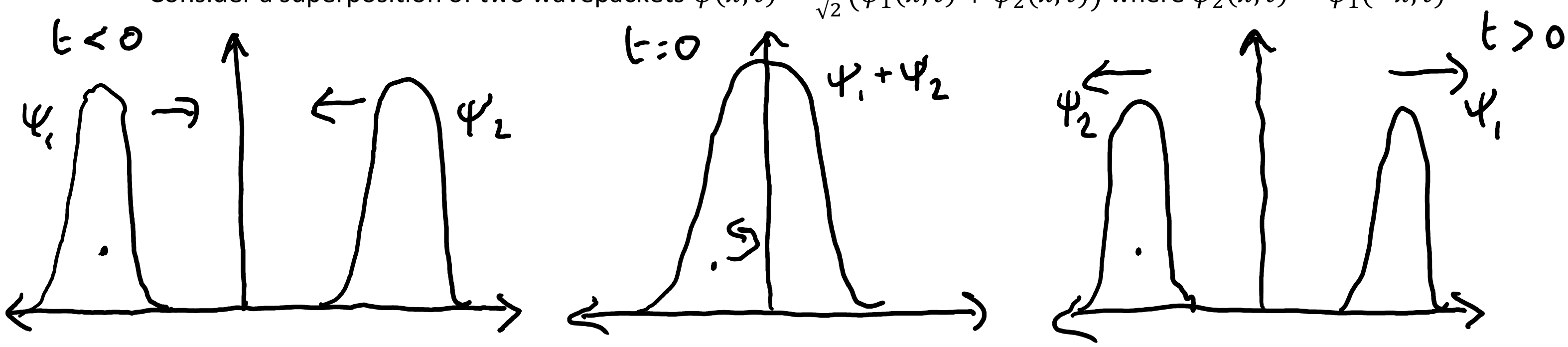
Note: with decoherence into localized environment states:

$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$

trajectories can cross in the *system* configuration space because $\boldsymbol{Q}_E$ is necessarily different in the two branches. This is needed to recover classical trajectories.

Empty Waves Steal the Particle

Consider a superposition of two wavepackets $\psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1(x,t) + \psi_2(x,t)\right)$ where $\psi_2(x,t) = \psi_1(-x,t)$



It is natural think that the two wavepackets correspond to two different trajectories for the particle as the two components do not interact.

However, the dBB particle must switch wavepackets during the interference due to the no crossing rule: the empty wave steals the particle.

Here, we can explicitly see that $J(0,t) = 0$ for all times so the particle can never cross $x = 0$.

To see this, define

$$J_{km}(x,t) = \frac{\hbar}{m}\mathrm{Im}\left(\psi_k^* \frac{\partial \psi_m}{\partial x}\right)$$

Then,

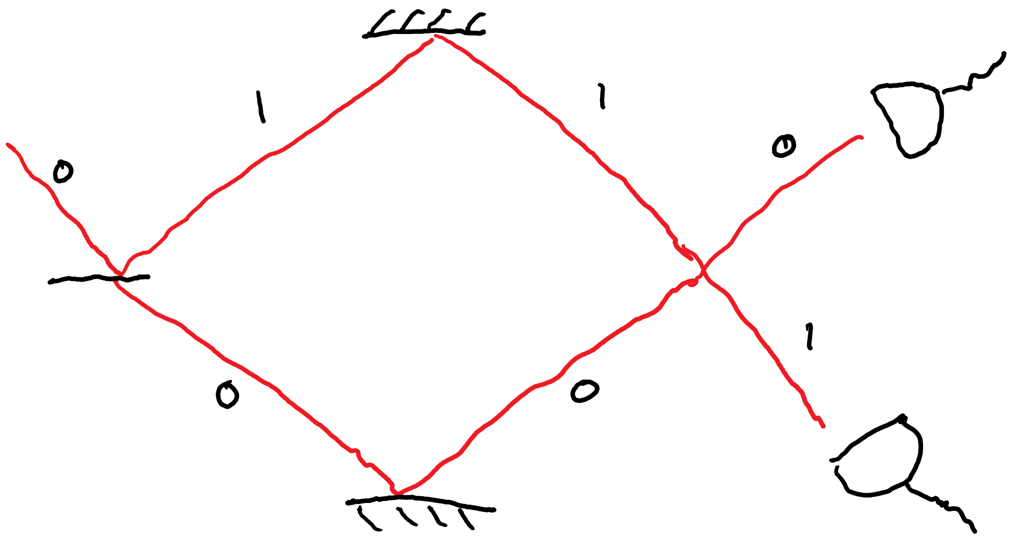
$$J(x,t) = \left[J_{11}(x,t) + J_{22}(x,t) + J_{12}(x,t) + J_{21}(x,t)\right]$$

Because $\psi_2(x,t) = \psi_1(-x,t)$ we have $J_{22}(x,t) = -J_{11}(-x,t)$ and $J_{21}(x,t) = -J_{12}(-x,t)$, so we get

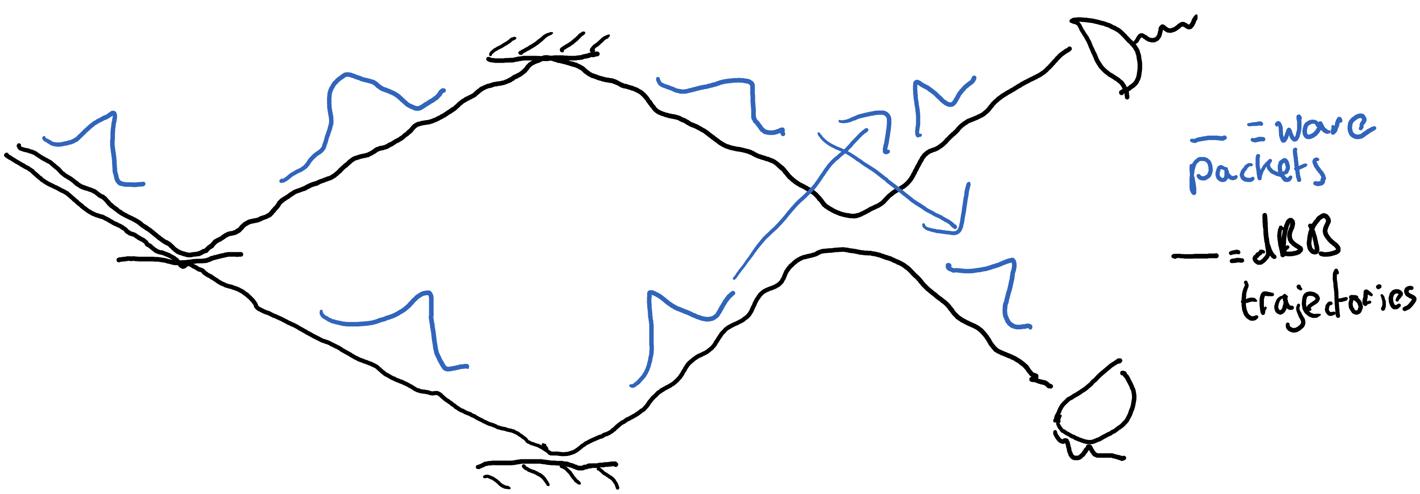
$$J(x,t) = \left[J_{11}(x,t) - J_{11}(-x,t) + J_{12}(x,t) - J_{12}(-x,t)\right],$$

so $J(0,t) = 0$.

Implications for the Mach-Zehnder Interferometer



If we remove the second beamsplitter then many people would be inclined to view the firing of detector 0 as evidence that the particle travelled along path 0, and the firing of detector 1 as evidence that the particle travelled along path 1. However, due to the no-crossing rule, the opposite is true in dBB theory.



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No-crossing ⇒ the empty wave steals the particle

Detector 0 clicks ⇒ the particle travelled along path 1

Surreal Trajectories

To make things more dramatic, we can place a localized spin-1/2 particle P in path 0, initially prepared in the $|\uparrow\rangle$ state and have the interaction

$$\psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \;\; \rightarrow \;\; \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\downarrow\rangle$$

$$\psi_1(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \;\; \rightarrow \;\; \psi_1(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle$$

Because $\vec{Q}_P$ is unaffected by this interaction, the current will still be zero in the interference region.

If we detect the particle at detector 0 and subsequently measure the spin, we will find it spin down.

You might view this as evidence that the particle travelled along path 0, but in dBB the trajectory is still path 1.

The spin has been flipped without the particle's position ever being near it.

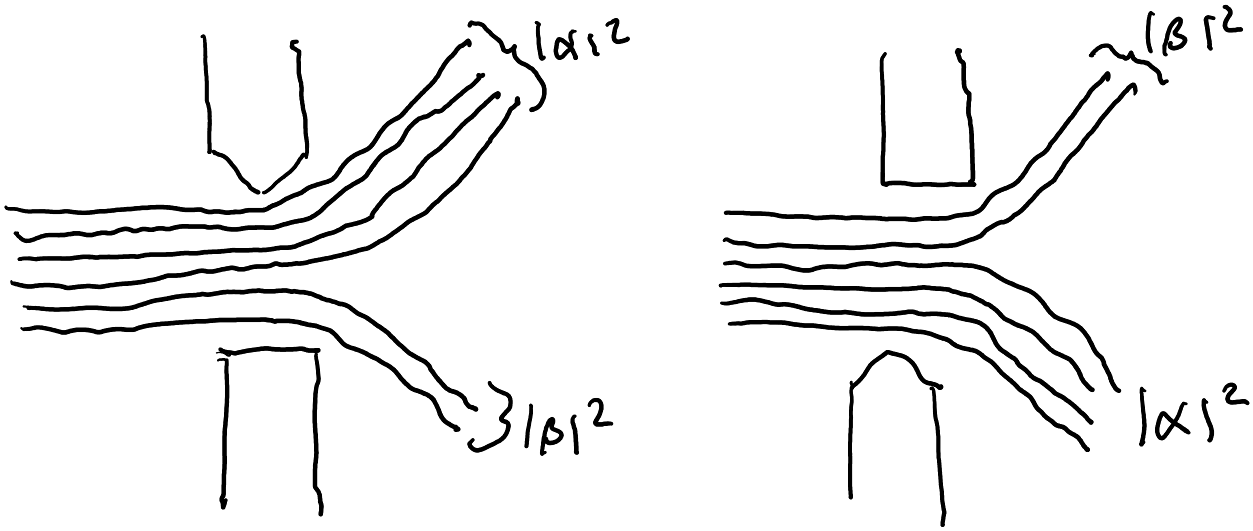
This can happen because the spin flip does not lead to decoherence that is localized in position.

Contextuality

We know from the Kochen-Specker theorem that dBB must be contextual. In fact, it is much more contextual than required.

KS contextuality occurs in dBB because the outcome of an experiment depends on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S), \boldsymbol{Q}_E, \Phi_R(\boldsymbol{q}_E)$, and the interaction Hamiltonian, and not on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S)$ alone.

Example: Stern-Gerlach measurement of $\psi(\boldsymbol{q}_S) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$



No-crossing rule ⇒ some $\boldsymbol{q}_S$ switch between giving spin up and spin down outcomes when we rotate the magnets by $180°$.

This is *more contextual* than implied by KS, which can only be proved in $d \geq 3$.

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4.iv. Underdetermination and the Equilibrium Hypothesis

The only property of the guidance equation needed to reproduce the quantum predictions is equivariance:
$\rho(\boldsymbol{Q}, t_0) = |\psi(\boldsymbol{Q}, t_0)|^2 \ \rightarrow \rho(\boldsymbol{Q}, t\,) = |\psi(\boldsymbol{Q}, t\,)|^2$ for all other t.

Any other equivariant dynamics would do just as well, e.g. (E. Deotto, G. Ghiradri, Found.Phys. 28:1-30 (1998))

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k}\frac{\mathrm{Im}\left(\psi^* \vec{\nabla}_k \psi\right)}{\psi^* \psi}(\boldsymbol{Q}) + \frac{\vec{J}_0(\vec{Q}_k)}{\psi^*\psi(\boldsymbol{Q})} \quad \text{with} \quad \vec{\nabla} \cdot \vec{J}_0 = 0$$

Further:

- If we allow a stochastic dynamics, we can use a discrete basis instead of/in addition to position, e.g. we could add primitive variables for spin.
- We could use a different basis, e.g. momentum.
- We could even use a POVM, e.g. coherent states.

We can remove the underdetermination if we drop the equilibrium hypothesis.

Recall that the quantum state plays two roles in dBB:

- Dynamical: it appears in the guidance equation.
- Probabilistic: We set $\rho(\boldsymbol{q}, t_0) = |\psi(\boldsymbol{q}, t_0)|^2$ as a postulate – *quantum equilibrium hypothesis.*

These two roles are independent, we could set the probability density to anything else.

There is evidence (analytic and numerical) that, under suitable coarse-graining, other densities relax to $|\psi(\boldsymbol{q}, t_0)|^2$ over time, akin equilibriation in statistical mechanics.

Valentini posits that nonequilibrium states may have occurred in the early universe.

- This would resolve the underdetermination, but leads to the bold hypothesis that superluminal signaling occurs in our universe.

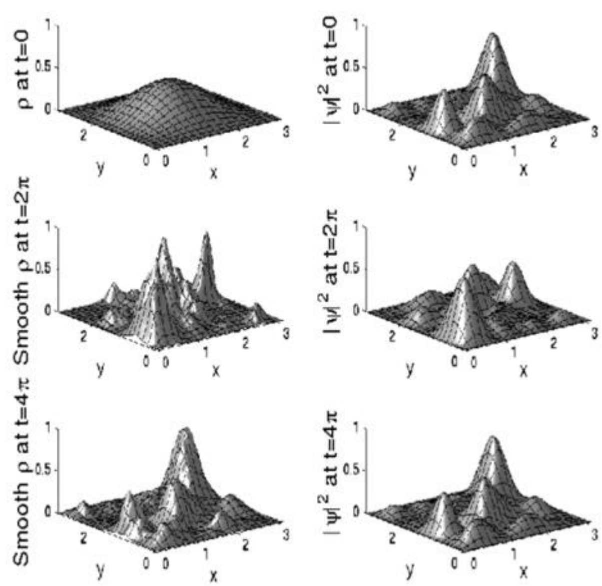

Figure 7: Smoothed $\bar{\rho}$, compared with $|\psi|^2$, at times $t = 0$, 2π and 4π. While $|\psi|^2$ recurs to its initial value, the smoothed $\bar{\rho}$ shows a remarkable evolution towards equilibrium.

A. Valentini, H. Westman, Proc. Roy. Soc. Lond. A 461:253-272 (2005)

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4.v. Relativistic Generalizations

Generalizations of dBB to relativistic QFT have been developed. There are various versions:

- Particle ontology vs. field ontology.
- An ontology with particle occupation numbers requires stochastic dynamics.
- A mixture of the two, e.g. particles for fermions and fields for bosons, only fermions and treat bosons like spin or vice versa.

These theories cannot be *fundamentally* Lorentz invariant:

- Under the equilibrium hypothesis, the operational predictions are Lorentz invariant.
- But the theories violate parameter independence – there is superluminal signaling at the ontic level.
- These effects would become observable in nonequilibrium states.

4.vi. Summary

dBB provides a coherent ontology with straightforward equations of motion, and saves the phenomena.

Trajectories do not obey common intuitions, but arguably this must be so if they are to reproduce quantum phenomena.

dBB arguably *more weird* than an interpretation has to be, i.e.

- Contextual in ways that QM does not require.
- Nonlocal in experiments that have local explanations.
- Surreal trajectories

Taking the equilibrium hypothesis as a postulate leads to underdetermination of the theory.

Viewing it as emergent removes the underdetermination, but leads to the bold hypothesis that we should expect to see explicit Lorentz violation, i.e. signaling, somewhere in nature.

dBB is nonetheless a good counterexample to many exaggerated claims about QM.

5. Spontaneous Collapse Theories

In orthodox quantum theory, the system evolves according to the Schrödinger equation, except if there is a "measurement" when the state randomly collapses.

The idea of spontaneous collapse theories is to modify the Schrödinger dynamics so that collapses are included as a natural dynamical process.

- **Microscopic** systems obey Schrödinger dynamics to a good approximation.
- **Macroscopic** systems quickly collapse to localized states with high probability.

This means that the predictions of a collapse theory will differ from those of standard quantum theory. They can in principle be empirically refuted.

5.i. Single Particle Girhardi-Rimini-Weber (GRW) Model

Consider a single particle in one dimension for simplicity.

Most of the time, the system obeys Schrödinger dynamics

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}|\psi(t)\rangle$$

There is a constant probability per unit time for a spontaneous localization to occur

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda$$

This will give rise to a Poisson distributed sequence of times $t_1, t_2, \cdots$ at which localizations occur. The average waiting time will be

$$\tau = \overline{t_{n+1} - t_n} = \frac{1}{\lambda}$$

GRW recommend $\lambda \approx 10^{-16}\ \mathrm{s}^{-1}$ or $\tau \approx 10^{16}\ \mathrm{s} = 3 \times 10^8$ years. Localizations occur extremely rarely.

When a localization occurs, the wavefunction is updated to

$$\psi(x,t) \to \psi'(x,t) = \frac{1}{p(X)} g_X(x)\psi(x,t)$$

where

$$g_X(x) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-(x-X)^2/4\sigma^2}$$

The value of X at which the localization occurs is chosen with probability density

$$p(X) = \int_{-\infty}^{+\infty} \left|g_X(x)\psi(x,t)\right|^2 \mathrm{d}x$$

This introduces a new parameter σ. GRW recommend $\sigma \approx 10^{-7}$ m.

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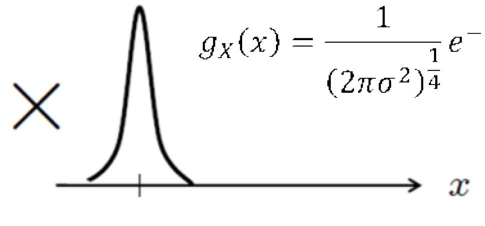
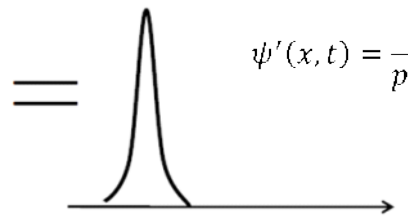
$$\psi(x,t) = \frac{\sqrt{3}}{2}\phi_1(x) + \frac{1}{2}\phi_2(x)$$

$$g_X(x) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-(x-X)^2/4\sigma^2}$$



$$\psi'(x,t) = \frac{1}{p(X)} g_X(x)\psi(x,t)$$



GRW in terms of a POVM

We can rewrite the spontaneous collapse in terms of a (continuous) POVM

$$E(X) = M^\dagger(X)M(X), \qquad M(X) = \int_{-\infty}^{+\infty} \mathrm{d}x\, g_X(x)\, |x\rangle\langle x|\,, \qquad \int_{-\infty}^{+\infty} \mathrm{d}X\, E(X) = I$$

Then

$$p(X) = \langle\psi(t)|E(X)|\psi(t)\rangle, \qquad |\psi'(t)\rangle = \frac{M(X)|\psi(t)\rangle}{\sqrt{p(X)}}$$

or, in terms of density operators

$$p(X) = \mathrm{Tr}\big(E(X)\rho(t)\big), \qquad \rho(t) \to \rho'(t) = \frac{M(X)\rho(t)M^\dagger(X)}{p(X)}$$

X is unknown to the experimenter, so they will observe the average state update

$$\rho(t) \to \int_{-\infty}^{+\infty} \mathrm{d}X\, M(X)\rho(t)M^\dagger(X')$$

Recall that a CPT map has the form

$$E(\rho) = \sum_j M^{(j)}\rho M^{(j)\dagger}$$

The GRW map is a continuous analogue of this. The spontaneous collapse process will look like an approximate position decoherence to an experimenter.

The same dynamics could be achieved by unitary interaction with the environment. Cannot tell GRW from decoherence via experiments.

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5.ii. Multi-Particle Generalization

Each particle experiences localizations at a rate λ.

The total rate of localizations for N particles will be $N\lambda$.

Average time between localizations is τ/N.

For a macroscopic system, $N \approx 10^{23}$, this gives

$$N\lambda \approx 10^{7}\ \mathrm{s}^{-1}, \qquad \frac{\tau}{N} \approx 10^{-7}\ \mathrm{s}$$

Collapses occur very frequently. For noninteracting unentangled particles

$$\psi(x_1, x_2, \cdots, x_N, t) = \psi_1(x_1, t)\psi_2(x_2, t) \cdots \psi_N(x_N, t)$$

this won't make a difference. Each particle collapses extremely rarely.

For entangled particles, it makes a big difference.

On average, every τ/N, one particle is selected at random (suppose it is particle 1). The whole wavefunction gets updated to

$$\psi'(x_1, x_2, \cdots, x_N t) = \frac{1}{p(X)} g_X(x_1)\psi(x_1, x_2, \cdots, x_N, t)$$

where $p(X) = \int_{-\infty}^{+\infty} \left|g_X(x_1)\psi(x_1, x_2, \cdots, x_N, t)\right|^2 \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_N$

Suppose

$$\psi(x_1, x_2, \cdots, x_N, t) = \alpha\phi_a(x_1)\phi_a(x_2) \cdots \phi_a(x_N) + \beta\phi_b(x_1)\phi_b(x_2) \cdots \phi_b(x_N)$$

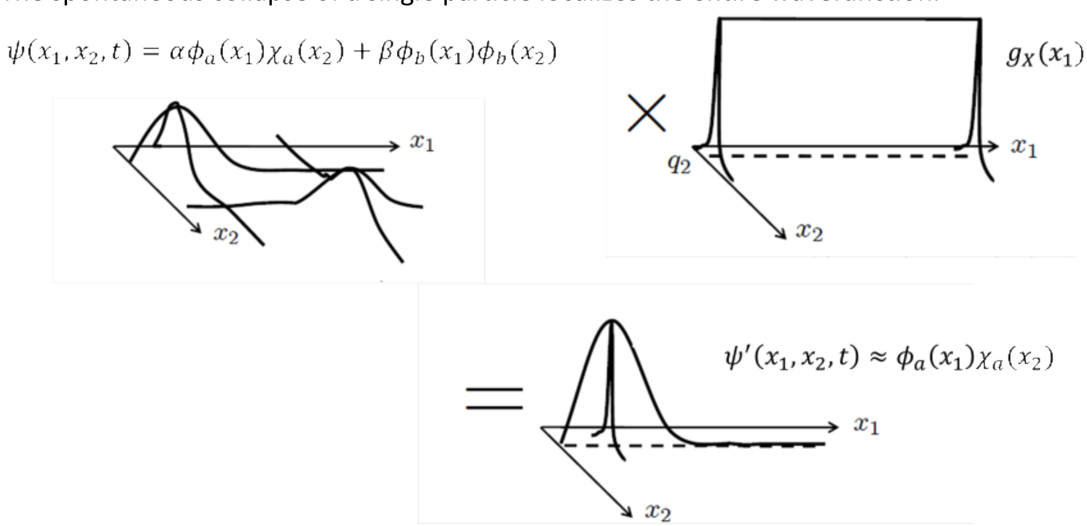
where $\phi_a(x)$ and $\phi_b(x)$ are localized around $x = a$ and $x = b$ with small width compared to σ and $|a - b| \gg \sigma$.

Then $P(X \approx a) \approx |\alpha|^2$, $P(X \approx b) \approx \left|\beta\right|^2$. For $X \approx a$, the state will collapse to

$$\psi'^{(x_1, x_2, \cdots, x_N, t)} \approx \phi_a(x_1)\phi_a(x_2) \cdots \phi_a(x_N)$$

and similarly for $X \approx b$.

The spontaneous collapse of a single particle localizes the entire wavefunction.



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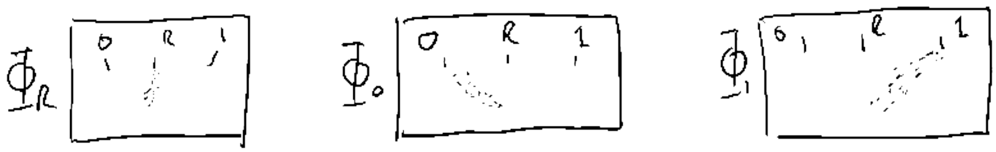
5.iii. Measurements in GRW

The pointer of a measuring device is made of a macroscopic number $N \approx 10^{23}$ of particles.



In a measurement interaction

$$[\alpha\psi_0(\boldsymbol{q}_S) + \beta\psi_1(\boldsymbol{q}_S)]\Phi_R(\boldsymbol{q}_E) \quad \rightarrow \quad \alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

but

$$\Phi_j(\boldsymbol{q}_E) = \phi_j(\vec{q}_1)\phi_j(\vec{q}_2)\cdots\phi_j(\vec{q}_N)$$

so the pointer and system will collapse extremely rapidly to either

$$\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) \quad \text{or} \quad \psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

5.iv Ontology and the Tails Problem

GRW gives us wavefunctions that are approximately localized in configuration space. But they are still functions on a $3N$ dimensional space. How is this related to what we see in 3D space?

- In other words, does GRW have a primitive ontology of local beables like de Broglie-Bohm theory?

The localizations are only approximate. $g_X(x)$ is a Gaussian function with exponentially small tails that stretch to infinity. So there are still tiny components of the wavefunction that remain in superposition. Why don't we see these?

- Note: We have to use a smooth $g_X(x)$ to avoid dynamics that causes the wavefunction to spread extremely rapidly.

GRWw: The wavefunction ontology

On this view, the wavefunction itself is the only ontology.

We have to use ideas similar to Everett/many-worlds to understand what a wavefunction means for everyday experience.

The tails problem is serious here because we have no reason to believe that components of the wavefunction with small amplitude are less important.

GRWm: The mass density ontology

We can define a mass density for particle j as

$$\rho_j(x) = m_j \int_{-\infty}^{+\infty} |\psi(x_1, x_2, \cdots, x_N)|^2 \, \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_{j-1} \mathrm{d}x_{j+1} \cdots \mathrm{d}x_N$$

The total mass density is then

$$\rho(x) = \sum_{j=1}^{N} \rho_j(x)$$

Without spontaneous collapses, this would tend to spread out and cover all space – does not capture everyday experience.

With GRW collapses, the mass density tends to get localized in blobs that look like classical reality.

- There are still blobs with very small mass spread out everywhere (tails problem). Need to argue that you cannot experience or perceive a sufficiently small mass density.

GRWf: The Flash Ontology

The localization events themselves happen at specific points X, t in spacetime.

For macroscopic systems they happen extremely frequently.

The flash ontology proposes that the world is made of small "matter events" in spacetime, where a piece of matter appears that is localized at (X, t) for each spontaneous collapse.

What we see are these flashes. Because they happen rapidly, it looks like continuous motion of particles.

Flashes happen with very small probability where the wavefunction has small amplitude. Because you need several flashes in a row to perceive something, this arguably solves the tails problem.

5.v. Generalizations of GRW

In GRW, the localizations happen at discrete times, via a dynamics that is not unified with the Schrödinger equation.

It is possible to have a continuous time stochastic process causing the collapses, which can be unified with Schrödinger dynamics as a stochastic differential equation. This is called Continuous Spontaneous Localization (CSL).

Just as GRW is indistinguishable from decoherence, CSL is indistinguishable from the theory of quantum continuous measurements.

Some people have proposed explicit mechanisms where classical fluctuating fields cause the collapse.

- Gravity (Penrose)
- Integrated Information (McQueen, Chalmers)

5.vi. Empirical Tests of GRW

Because GRW implies that there is necessarily decoherence when the system consists of enough particles, various parameter ranges for λ and σ can be ruled out empirically if we see coherence in large systems. It can be distinguished from standard quantum theory.

We can also rule out some parameter ranges as Perceptually Unsatisfactory, e.g. if it implies that a dust particle can be in a superposition of two observably distinct positions for more than a few microseconds then we would not have a solution to the measurement problem.

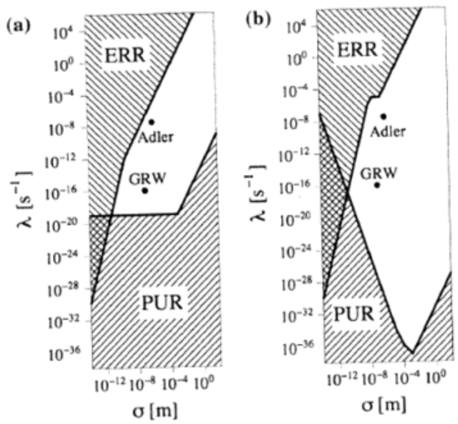


Fig. 9.10 Map of parameter space, again for both GRW and CSL theories, showing now both the "Empirically Refuted Region" (ERR) and the "Perceptually Unsatisfactory Region" (PUR) as discussed in the text. From Ref. [8]. Figure © IOP Publishing. Reproduced with permission. All rights reserved. https://doi.org/10.1088/1751-8113/45/6/065304

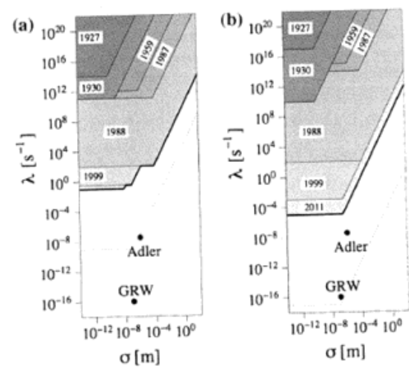


Fig. 9.11 The "Empirically Refuted Region" (ERR) of the GRW and CSL parameter spaces has steadily advanced, in recent decades, leaving an ever-narrowing window of parameter values which are compatible both with experimental and perceptual evidence. This suggests that, within perhaps a couple of decades, we will either have direct experimental evidence in support of the spontaneous collapse models, or the models will have been ruled out as either empirically or perceptually unacceptable. From Ref. [8]. Figure © IOP Publishing. Reproduced with permission. All rights reserved. https://doi.org/10.1088/1751-8113/45/6/065304

From W. Feldman, R. Tumulka, Parameter diagrams of the GRW and CSL theories of wavefunction collapse, J. Phys. A, 45:065304 (2012)
As reproduced in T. Norsen , Foundations of Quantum Mechanics, (Springer, 2017)

5.vii. Summary

Spontaneous collapse theories supplement Schrödinger dynamics with a physical collapse mechanism that localizes the state.

These theories can be ruled out empirically by generating superpositions involving large numbers of particles in different locations.

The ontology of these theories is less clear than de Broglie-Bohm. Three ontologies have been proposed, but it is not clear if they all solve the tails problem.

It is not obvious how to generalize these theories to quantum field theory. Can it be done in a Lorentz invariant way?

6. Everett/Many-Worlds

The approach described here is a mixture of:

- Oxford Everettianism: D. Wallace, The Emergent Multiverse (OUP, 2012)
- Zurek's ideas on decoherence: W. Zurek, arXiv:quant-ph/0306072 (2003). R. Blume-Kohout, W. Zurek Phys. Rev. A:062310 (2006)

The basic idea is to view the dynamical axioms as unproblematic:

- The quantum state is an ontological physical state and it evolves unitarily in time. The entire universe obeys these rules.

In orthodox interpretation this comes into conflict with the measurement axioms:

- so Everett proposed we simply discard the measurement axioms.
- They are to be derived as effective/emergent rules that an observer who is a quantum subsystem would use.

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The ontology of Everett is not localized in spacetime (no local beables), so we must regard the latter as emergent.

In a Schrödinger cat state like:

$\alpha|\uparrow\rangle \otimes |\text{alive cat}\rangle + \beta|\downarrow\rangle \otimes |\text{dead cat}\rangle$

we know that the world we experience corresponds to one of the two branches, not both. The ontology is just the entire quantum state, so there is nothing in it that picks out one branch over the other.

Our world of experience exists + ontology is just the quantum state ⇒ There are many worlds.

The parallel worlds are derived from the ontology, not posited.

Everett's idea was to define **relative states**:

- Relative to $|\uparrow\rangle$, the cat is alive.
- Relative to $|\downarrow\rangle$, the cat is dead.

But both are equally real.

6.i. The Basis Problem

We can write any quantum state in any orthonormal basis.

$\alpha|\uparrow\rangle \otimes |\text{alive cat}\rangle + \beta|\downarrow\rangle \otimes |\text{dead cat}\rangle$

$= \frac{1}{2}\left([|\uparrow\rangle + |\downarrow\rangle] \otimes \left[\alpha|\text{alive cat}\rangle + \beta|\text{dead cat}\rangle\right]\right) + \frac{1}{2}\left([|\uparrow\rangle - |\downarrow\rangle] \otimes \left[\alpha|\text{alive cat}\rangle - \beta|\text{dead cat}\rangle\right]\right)$

Why shouldn't we interpret the second decomposition as representing two worlds in which the cat is in a horribly nonclassical state?

Answer: We also have the dynamics. Identify worlds as structures in the quantum state that persist in time ⇒ **decoherence theory.**

In a typical decoherence interaction:

$|\uparrow\rangle \otimes |\text{alive cat}\rangle \ \rightarrow\ |\uparrow\rangle \otimes |\text{alive cat}\rangle \otimes |\chi_1\rangle^{\otimes N}$ and $|\downarrow\rangle \otimes |\text{dead cat}\rangle \ \rightarrow\ |\downarrow\rangle \otimes |\text{alive cat}\rangle \otimes |\chi_2\rangle^{\otimes N}$

These states do not get entangled with the environment, so they can be identified as worlds that persist in time.

The states in the other decomposition will quickly get entangled with the environment, so they are not persistent structures.

Note: In realistic models, decoherence is not exact and is relative to a level of coarse-graining. Therefore, there is no precise decomposition into worlds. Even the number of worlds is slightly sensitive to this.

This is to be expected in a model based on emergence, c.f. a finite number of particles does not have an precisely defined temperature.

There is fairly widespread agreement that decoherence theory solves the basis problem.

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6.ii. The Probability Problem

Now that we have defined worlds, we need to explain why, in a typical measurement interaction:

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\text{you}\rangle \otimes |E\rangle \rightarrow \alpha|\uparrow\rangle \otimes |\text{you}_\uparrow\rangle \otimes |E_\uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you}_\downarrow\rangle \otimes |E_\downarrow\rangle$$

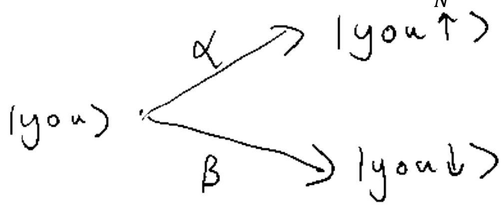
you should, before the measurement, assign probabilities $|\alpha|^2$ and $|\beta|^2$ to the two worlds that will be created.

Even the meaning of the probabilities is nontrivial:

- It is not as if you will become either $\text{you}_\uparrow$"or $\text{you}_\downarrow$ and you don't know which. Both have equal claim to be your successor.
- It is as if you are going to be cloned twice, the original you killed, and you have to assign probabilities to your two successors.

World Counting

An intuitively appealing rule is "world counting": If there are N worlds after a branching event, then the probability of each world should be $\frac{1}{N}$.



$\text{Prob}(\uparrow) = \text{Prob}(\downarrow) = \frac{1}{2}$ which is wrong.

This is so intuitively appealing to some people that they take it to obviously rule out many-worlds.

However, it is not at all obvious why you should do world counting when all worlds exist on an equal footing.

This appeals to people who only remember high school probability where

$$p = \frac{\text{number of favorable cases}}{\text{total number of possible cases}}$$

This rule is not endorsed by any interpretation of probability that is seriously entertained today.

Interpretations of Probability

There are broadly three ways of interpreting probabilities:

1. **Frequentist:** probability is the long run relative frequency of an outcome in multiple repetitions of an experiment.
2. **Bayesian:** Probabilities represent the degrees of belief of a rational agent – they constrain rational decision making.
3. **Objective chance:** Probabilities represent objective facts about the way a single experiment is performed – perhaps a disposition to produce a certain outcome, or facts about what the relative frequency would be if repeated.

All interpretations of probability are controversial. When deriving probability in many worlds we should distinguish:

- Problems that are common to classical probability, which we can't blame on Everett.
- Problems that are specific to the many-worlds interpretation.

Several frequentist derivations of probability in many-worlds have been attempted. There is fair agreement that they are failures, although increasingly sophisticated versions continue to crop up from time to time.

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Decision Theoretic Approach

Deutsch and Wallace have developed an approach based on subjective Bayesian probability and objective chance. – D. Deutsch, Proc. Roy. Soc. A 455:3129-3137 (1999). D. Wallace, The Emergent Multiverse (OUP, 2012).

The full argument requires some sophisticated decision theory. Will present a more heuristic version here.

Suppose we are uncertain about which of a finite set X of possibilities might occur, e.g. $X = \{1,2,3,4,5,6\}$ for the outcome of a dice roll.

A subset of X is called an **event.** It represents a proposition we can state about the outcome, e.g. $\text{outcome is odd} = \{1,3,5\}$.

The axioms of classical probability are:

- $0 < P(E) < 1$
- $P(\emptyset) = 0, P(X) = 1$
- If E and F are disjoint then $P(E \cup F) = P(E) + P(F)$

In subjective Bayesianism, probabilities represent the subjective degrees of belief of a decision making agent. Why should they obey these axioms?

The Dutch Book Argument

We define a way of measuring degrees of belief:

- Your probability for E is the price $\$P(E)$ at which you would be willing to buy or sell any number of the following lottery tickets

Price $\$P(E)$	Pays \$1 if E occurs Pays \$0 otherwise

Rationality criterion: You should not enter into a system of bets that causes you to make a loss for every possible outcome.

- From this, we can *derive* the axioms of probability.

Example: Suppose you set $P(E) > 1$.

Then you would be willing to buy a ticket that pays \$1 if E occurs and nothing otherwise for a price $\$P(E)$.

- If E occurs then you have lost: $\$P(E) - \$1 > \$0$.
- If E does not occur then you have lost: $\$P(E) > 0$.

Sure loss in both cases, so rationality implies $\$P(E) \leq 1$.

Similar arguments apply for the other axioms, and you can also show the converse: that any assignment satisfying the laws of probability cannot lead to a sure loss.

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Application to Many-Worlds

Adapting this idea to many-worlds allows us to assign meaning to probabilities, which is half the battle.

Keep definition of probabilities in terms of willingness to make bets on measurement outcomes.

Modified Rationality criterion: You should not enter into a system of bets that causes *all* of your successors to make a loss.

$$|\psi\rangle \otimes |\text{you}\rangle \longrightarrow \alpha_1|\psi_1\rangle \otimes |\text{you}_1\rangle + \alpha_2|\psi_2\rangle \otimes |\text{you}_2\rangle$$

This gives a meaning to the probabilities of future worlds, and the Dutch Book implies you should assign probabilities $P(j)$ to the worlds that satisfy the usual axioms:

$P(j) \geq 0, \quad \sum_j P(j) = 1.$

It still remains to argue that $P(j) = \left|\alpha_j\right|^2$

The Deutsch-Wallace Postulates

Deutsch and Wallace add the following postulates:

1. **State supervenience:** The probabilities you should assign depend only on the quantum state (here's where objective chance comes in).
2. **Microstate indifference:** You only care about the $ you win on a branch. The rest of it can be changed without affecting your betting preferences.
3. **Branching indifference:** You don't care if worlds branch into even more worlds later on, provided you have the same $ on the new branches.
4. **Continuity:** Probabilities should be a continuous function of the quantum state.

The Equal Amplitude Case

You measure a system prepared in the state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, get $\$a$ for $\uparrow$ and $\$b$ for $\downarrow$.

$\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle|\$a\rangle + |\downarrow\rangle|\$b\rangle)$

Suppose you did the same thing with opposite prizes $\$a$ for $\downarrow$ and $\$b$ for $\uparrow$.

$\rightarrow \frac{1}{\sqrt{2}}(|\downarrow\rangle|\$a\rangle + |\uparrow\rangle|\$b\rangle)$

By microstate indifference, you can flip the spin afterwards without changing your betting preferences.

$\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle|\$a\rangle + |\downarrow\rangle|\$b\rangle)$

But now both cases give the same physical state. Since this applies for all possible choices of bets and prizes $P(\uparrow) = P(\downarrow) = \frac{1}{2}$.

Clearly this generalizes to an equal superposition of N branches, which would give $P(j) = \frac{1}{N}$.

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Rational Amplitudes and General Amplitudes

For rational amplitudes, we can use branching indifference to branch into an equal superposition, e.g.

$$\sqrt{\tfrac{2}{3}}|\psi_1\rangle|\$a\rangle + \sqrt{\tfrac{1}{3}}|\psi_2\rangle|\$b\rangle \quad \rightarrow \quad \sqrt{\tfrac{1}{3}}\left(|\psi_1^1\rangle|\$a\rangle + |\psi_1^2\rangle|\$a\rangle + |\psi_2\rangle|\$b\rangle\right)$$

We know the probabilities are $\frac{1}{3}$ in the equal superposition, so we'll get:

$$P(\psi_1) = P(\psi_1^1) + P(\psi_1^2) = \tfrac{2}{3}$$

From this, we'll get $P(j) = \left|\alpha_j\right|^2$ for rational amplitudes.

We then get the general case by continuity.

Some Objections

See S. Saunders et. al. (eds.), Many Worlds?, (OUP, 2010) for many papers pro and contra.

Bayesian probability is supposed to be about reasoning in the face of uncertainty. There is no uncertainty here – all successors exist. You have skewed the meaning of decision theory:

- Reinterpret probability as degree with which you should care about your successors (Greaves)
- Apply a similar argument after you have become entangled with the measuring device, but before you are aware of the outcome – self-locating uncertainty about which branch you are on (Vaidman, Carroll & Sebens)

Kent's objection: Suppose I create two clones of you, put one in a room with α painted on the wall and the other in a room with β on the wall, and then kill the original. Why should I assign probabilities $|\alpha|^2$ and $|\beta|^2$ to the two clones? How is this different from the situation in many worlds?

- The quantum state is the only ontology available to determine objective chances. If not that then what? (Wallace)

I think that Kent's objection is a good argument against state supervenience. The world-splitting

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\mathrm{you}\rangle \quad \rightarrow \quad \alpha|\uparrow\rangle \otimes |\mathrm{you}_\uparrow\rangle + \beta|\downarrow\rangle \otimes |\mathrm{you}_\downarrow\rangle$$

Occurs in exactly the same way regardless of the values of α and β. The two resulting worlds look exactly the same.

If the decoherence is sufficiently thoroughgoing then α and β play no further role in the dynamics of the two separate branches.

The only way in which α and β could be relevant is if some external super-observer decides to perform an interference experiment between the two branches. But this is precisely the case in which it doesn't matter what the probabilities of the branches are because you's memory will be erased in the process.

- I can't imagine anything that should be *less* relevant for the objective chances than the amplitudes.

This amounts to an argument *against* the reality of the quantum state in the many worlds interpretation. If we could throw out the amplitudes of the branches from the ontology and make do with a pure subjective Bayesian interpretation of probability then this would get around Kent's objection.

I think this can be done, resulting in an interpretation that I call **ironic many worlds**.

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6.iii. Summary

Everett starts from the premise that a quantum state evolving unitarily is the entire ontology of quantum theory

From this we *derive* that there must be many-worlds.

The basis problem is solved to most people's satisfaction by decoherence.

The probability problem is much more controversial:

- To the extent that the Born rule can be derived at all in many-worlds, I believe that Deutsch-Wallace or related approaches work best.

However, state supervenience is a problematic assumption due to Kent's objection. Suggests to me that we should go for a many-worlds interpretation in which the ontology does not include all of the quantum state (cue a large number of objections from Lev Vaidman).

7. Copenhagenish Interpretations

7.i. What is a Copenhagenish Interpretation?

There Is No Copenhagen Interpretation

Copenhagen Interpretation is supposed to refer to the views of some of the founders of quantum theory, e.g. Bohr, Heisenberg, Pauli, …

- If you read their views, they are all slightly different and contradictory.

Bohr's views are most closely associated with the word "Copenhagen"

- but Bohr is notoriously difficult to read and has been interpreted in very different ways in the intervening years.
- Some issues, e.g. Bell's theorem, contextuality, ψ-ontology, were not even fully formulated in Bohr's lifetime. You won't find a clear statement on any of them in Bohr's writing.

Historical note: Don Howard (Philosophy of Science, 71:669-682 (2004)) argues that the idea of a unified "Copenhagen Interpretation" was invented by Heisenberg in the mid 1950's. Before that people spoke of ideas in the "Copenhagen spirit", but the idea of a complete and conclusive interpretation was not mentioned.

Copenhagenish Interpretations

In the intervening years, many scholars have developed more fully worked out interpretations in the Copenhagen spirit – **Copenhagenish Interpretations.**

Examples:

Objective	Perspectival
Copenhagen (Bohr)	Qbism, i.e. Quantum Bayesianism mk 2 (Fuchs, Schack)
Quantum Bayesianism mk 1 (Caves, Fuchs, Schack)	Relational Quantum Mechanics (Rovelli)
Quantum Pragmatism (Healy)	Brukner
Information Interpretation (Bub, Pitowsky)	

- **Objective:** There is an objective fact of the matter about what an observer observes.
- **Perspectival:** What is true depends on where you are sitting.

I am more interested in analyzing coherent sets of ideas than in history, so I try to formulate what is common to all Copenhagenish interpretations, without claiming to accurately represent Bohr.

7.ii. Principles of Copenhagenish Interpretations

There are 4 common principles of Copenhagenish interpretations:

1. **Observers Observe (No solipsism)**
2. **Universaility**
3. **Anti-ψ-ontology**
4. **Completeness**

Observers Observe (No solipsism)

I know from my experience that I am the type of entity that experiences definite outcomes when I make a quantum measurement.

I posit that there are other similar entities in the universe (e.g. grad students) and I don't doubt that they have the same experience.

This does not mean that consciousness, human observers, etc. is *necessary* in order for a definite outcome to occur, e.g. we could accept a decoherence account of when definite outcomes occur, only that a human observation is *sufficient* for a definite outcome to occur.

Objective version: When you make a measurement and observe the result then these are objective facts.

Perspectial version: When you make a measurement and observe the result then these are facts *for you*. There is no fact of the matter for me unless I repeat the measurement myself or interact with you.

Universality

Quantum theory is a fundamental physical theory.

Anything in the universe (if not everything at once) can in principle be described by quantum theory.

There are no fundamentally "classical" or "non quantum" systems in the universe.

⇒ In principle, I can arrange a situation in which I would describe the state of a grad student as a superposition of macroscopically distinct states.

Copenhagenism is not operationalism: there is no undefined primitive of *measurement* that is put in by hand. In this regard it is similar to Everett.

You might have thought that the measurement problem immediately rules out observers observe + universaility.

- Universaility implies: $\alpha|\uparrow\rangle|M_\uparrow\rangle + \beta|\downarrow\rangle|M_\downarrow\rangle$
- Observers observe implies: either $|\uparrow\rangle$ or $|\downarrow\rangle$

But this assumes we believe that quantum states are objective ontic states, or assumes something like the eigenvalue-eigenstate link.

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Anti ψ-ontology

Quantum states are not ontic, i.e. not intrinsic properties of an individual quantum system.

- Instead they represent: our knowledge/our information/our beliefs/what we can say/advice about the quantum system, depending which Copenhagenish view we are considering.

Therefore, two different descriptions of a measurement need not be contradictory, e.g. they could represent the descriptions of two different observers who have access to different information.

Note: I would be happy to call this ψ-epistemic, but some Copenhagenists dislike that label.

Completeness

Further, there is no deeper description to be had, i.e. no ontic states assigned to systems we are describing quantum mechanically. This is either because:

1. Quantum systems have properties but they are **ineffable**: it is literally impossible to talk about them. The moon is there when nobody is looking, but it is fundamentally *impossible* to describe its properties in language, pictures, mathematics, computer code, or anything else.
2. Quantum systems have no properties. The moon is not there when nobody is looking.

Option 1 is necessary for an objective Copenhagenish interpretation.

Option 2 can be made perspectival, i.e. *for me* the moon has no properties when *I* am not looking at it. It may have properties for other observers.

In either case quantum states represent knowledge/information/beliefs/what we can say/advice about the outcomes of future measurements we might make, not about some underlying reality.

7.iii. The Heisenberg Cut

The four principles: **observers observe, universality, anti ψ-ontology**, and **completeness** are in a certain amount of tension.

If universality is true, I can describe another observer making a measurement as

$$\alpha|\uparrow\rangle|\text{you } \uparrow\rangle + \beta|\downarrow\rangle|\text{you } \downarrow\rangle$$

But if completeness is true then I cannot ascribe you any properties when I describe you as a quantum system in this way.

In particular, if I want to account for *my own* observations, then that is ascribing a property *to me* so I cannot include myself in my quantum descriptions.

Therefore, I necessarily have to split the world into two parts:

- The part I am going to describe quantum mechanically.
- The part I am going to exclude from that description so that I can ascribe it properties (the “classical” part).

The split between these two parts is called the **Heisenberg cut.**

If universality is true, then there cannot be a fundamental place where I have to put the Heisenberg cut. It is moveable.

- This was called the **shifty split** by John Bell.

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In Bohr's view, the location of the cut should be decided pragmatically:

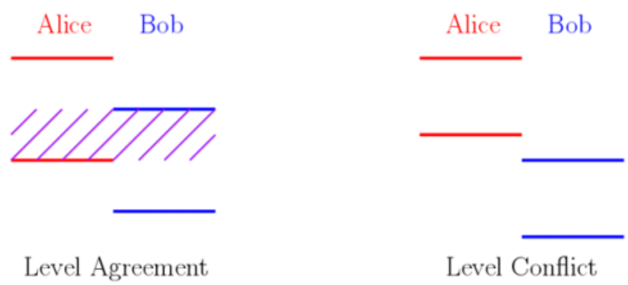
- There will be a **lowest level** I can place the split: If I coherently interfere degrees of freedom I put in the "classical" part I will get the predictions wrong.
- There is also a **highest level**: I must always put the split before myself in order to account for my own observations.

Today, we might use decoherence theory to decide where the lowest level is.

There will be a range of possible levels of description between the highest and lowest levels. The fact that different quantum states are assigned at different levels does not matter because we are anti-ψ-ontologists. So long as the levels agree on the predictions for the experiments *actually performed*, everything is fine.

Each observer has a different range of levels between their highest and lowest.

You might have thought that, for any two observers, it is always possible to find a range of levels that they can agree upon:



The Wigner's friend experiment shows that level conflicts can happen.

7.iv. Wigner's Friend

The Wigner's friend experiment is just like Schrödinger's cat, except that Wigner puts his friend inside a box to make a measurement instead of a cat.

- The difference is that the friend is unambiguously an observer.
- We can place a large enough environment inside the box, or whatever you think is necessary for an observation to occur inside.

After the measurement, but before he opens the box, Wigner can place the cut above his friend and use the state:

$\alpha|\uparrow\rangle|\text{friend}\uparrow\rangle + \beta|\downarrow\rangle|\text{friend}\downarrow\rangle$

The friend's highest level is below herself, so she necessarily uses:

$|\uparrow\rangle$ or $|\downarrow\rangle$

Friend: Come on Wigner, put your cut lower so we can reach level agreement!

Wigner: Sorry, I am contemplating doing an interference experiment on you, so this is my lowest possible level.

There is a level conflict.

Level conflicts happen (admittedly in rather impractical experiments).

However, you might have thought that in the long run, after the whole experiment is over, level agreement will always be possible.

Reason: As soon as the friend tells Wigner her measurement outcome, they will both be able to place the cut below the friend, and both be able to agree upon: $|\uparrow\rangle$ or $|\downarrow\rangle$

7.v. Wigner's Enemy

But this doesn't always happen. If Wigner actually does a coherent experiment on his friend then disagreement persists.

I call this the **Wigner's Enemy** experiment, because it involves Wigner erasing his friend's memory, which is a pretty nasty thing to do.

- Friends don't recohere friends.

Let's cheat a bit and assume that the friend's "ready" state is $|\text{friend} \uparrow\rangle$, so we can treat her as a qubit.

Let's give a specific unitary interaction for the measurement:

$|\uparrow\rangle|\text{friend} \uparrow\rangle \rightarrow |\uparrow\rangle|\text{friend} \uparrow\rangle$ $|\uparrow\rangle|\text{friend} \downarrow\rangle \rightarrow |\uparrow\rangle|\text{friend} \downarrow\rangle$

$|\downarrow\rangle|\text{friend} \uparrow\rangle \rightarrow |\downarrow\rangle|\text{friend} \downarrow\rangle$ $|\downarrow\rangle|\text{friend} \downarrow\rangle \rightarrow |\downarrow\rangle|\text{friend} \uparrow\rangle$

This unitary is its own inverse, so Wigner can undo the measurement and recohere his friend by applying the unitary a second time.

According to Wigner, undoing the measurement yields:

$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|\text{friend} \uparrow\rangle$

According to the friend, before the undoing, she was either in state:

$|\uparrow\rangle|\text{friend} \uparrow\rangle$ or $|\downarrow\rangle|\text{friend} \downarrow\rangle$

Consider the case $|\uparrow\rangle|\text{friend} \uparrow\rangle$. Then, the undoing leaves her state unchanged.

After the undoing, both agree that friend is uncorrelated with the system, so there is no level conflict, but they disagree on the state of the spin.

Possible Resolutions

For the friend to compute what happens in the undoing, she needed to apply quantum mechanics to herself – conflicts with the idea that her highest level is before herself.

So we could say that the friend's application of quantum theory is illegitimate. Only Wigner's description is reliable.

- Problematic as it says there are some possible experiments for which some observers necessarily have no physical description.

Or we could say that the friend's application of quantum theory is OK in this case. We need not insist that different observers assign the same pure state to a system even when there is level agreement. We are anti-ψ-ontologists after all.

In any case differences of quantum state assignment do not prove that objective observations do not exist. For that we need a Bell-Wigner mashup.

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7.vi. The Bell-Wigner Mashup

Timeline

2015: Caslav Brukner gives a no-go theorem for “observer independent facts” using a CHSH-Wigner mashup

In R. Bertlmann and A. Zeilinger (eds.) *Quantum [Un]Speakables II*, (Springer 2017) arXiv:1507.05255

2016: Frauchiger and Renner give a no go theorem for “self consistency of single world interpretations” later changed to “Quantum theory cannot consistently describe the use of itself” using a Hardy-Wigner mashup.

- Nature Communications 9, 3711 (2018) arXiv:1604.07422
- Jonathan Oppenheim mentions a discussion where Lluis Masanes described a CHSH-Wigner mashup on scirate https://scirate.com/arxiv/1604.07422

Matt Pusey discusses the Masanes version in the context of QBism in a talk https://youtu.be/_9Rs61l8MyY

Should I call this the Brukner-Frauchiger-Renner-Masanes-Pusey theorem?

The CHSH-Wigner Mashup

Consider two systems prepared in the maximally entangled state

$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB}+|11\rangle_{AB})$.

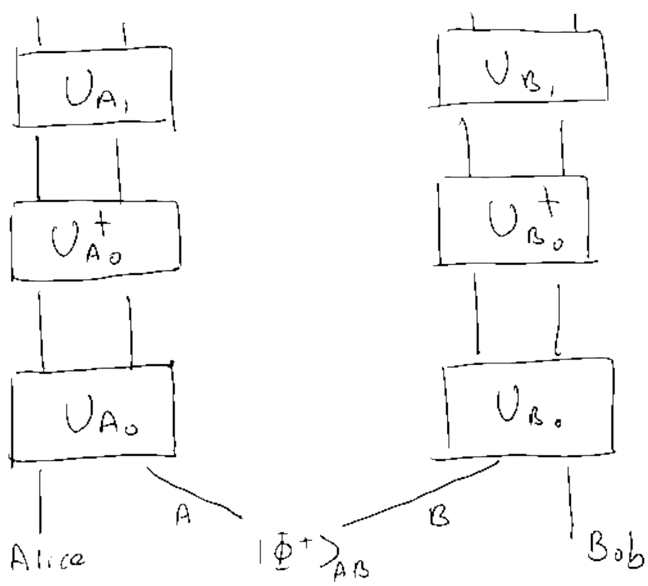
Alice has two observables A_0, A_1 she can measure on system A and Bob has two observables B_0, B_1 he can measure on system B. These are all two-outcome observables, as in Bell-CHSH.

We are going to consider the experiment from the point of view of a third observer, Wigner, who describes the whole thing in terms of coherent unitary interactions.

For any observable that Alice or Bob measures, Wigner can undo the measurement by reversing the coherent unitary interaction.

Instead of the usual Bell setup, we’ll have Alice and Bob measure *both* of their observables, one after the other, with a reversal from Wigner in between.

Wigner's Perspective



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According to the objective version of observers observe, there is a fact of the matter about what the outcome of A_0, A_1, B_0 and B_1 is on every run of the experiment. Denote these outcomes a_0, a_1, b_0 and b_1.

If we repeat this experiment multiple times, then relative frequencies exist, so a joint probability distribution

$P(a_0, a_1, b_0, b_1)$

exists.

Fine's Theorem

Theorem: The existence of a locally causal model for a Bell experiment is equivalent to the existence of a joint probability distribution over all the observables, the marginals of which give the correct operational predictions.
A. Fine, Phys. Rev. Lett. 48:291 (1982).

We only need the converse part here:

Proof:

Simply let $\lambda = (a_0, a_1, b_0, b_1)$ and $\Pr(\lambda) = P(a_0, a_1, b_0, b_1)$.

Let $\Pr(a|x,\lambda) = \delta_{a,a_x}$ and $\Pr\left(b\middle|y,\lambda\right) = \delta_{b,b_y}$.

Then,

$$\Pr\left(a,b\middle|x,y\right) = \sum_\lambda \Pr(a|x,\lambda)\Pr(b|y,\lambda)\Pr(\lambda) = P(a_x, b_y)$$

Which Probabilities have to be Quantum?

Since there is a local model, the outcomes in the CHSH-Wigner experiment has to satisfy the CHSH inequality, but we know that the quantum predictions do not.

Conclusion: At least one of the marginals $P(a_0,b_0), P(a_1,b_0), P(a_0,b_1), P(a_1,b_1)$ must fail to agree with the quantum predictions.

Which marginals absolutely *have* to obey the quantum predictions?

- Depends on how Wigner performs the experiment.
- If a pair of outcomes persists for a very long time, such that Alice and Bob can discuss them, write Nature papers about them, etc. then their marginals have to obey the quantum predictions, otherwise quantum theory would be falsified.
- If a pair of outcomes does not persist for long enough for Alice and Bob to discuss them then their marginal does not strictly have to obey quantum theory.

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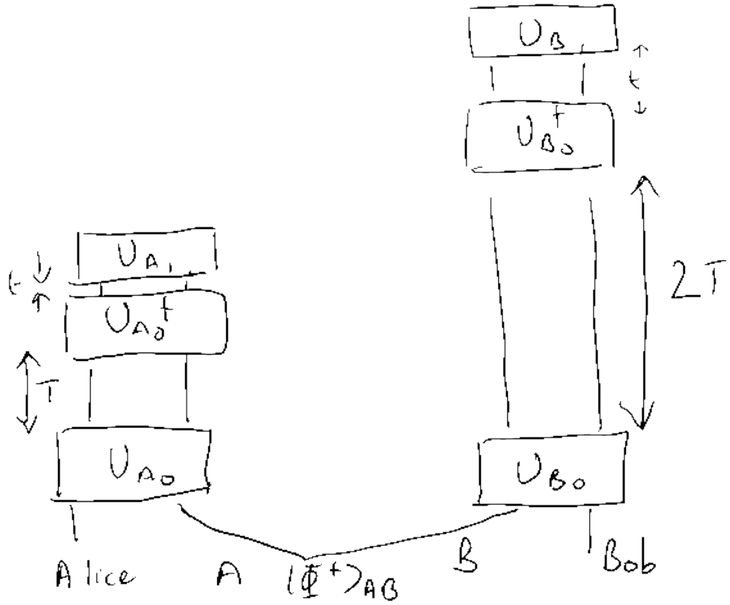
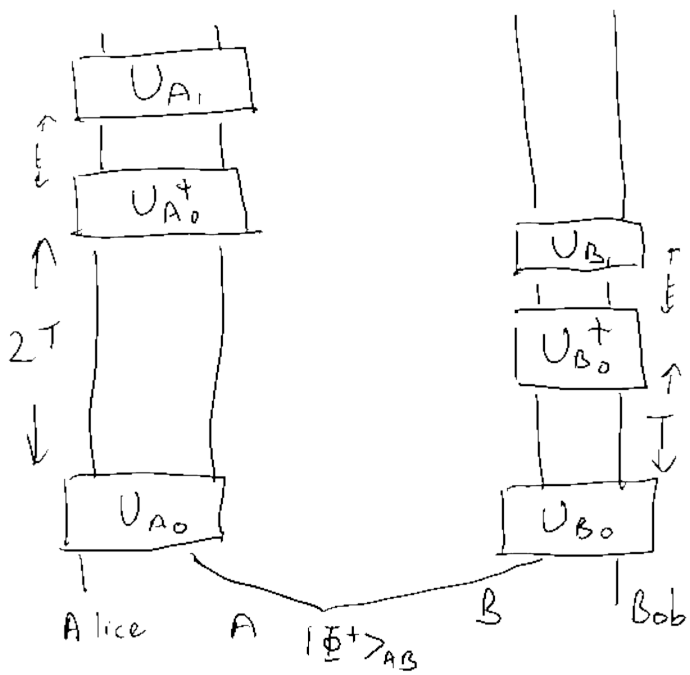


Let $T = 10$ years $t = 0.5$ ns.

Then $P(a_0, b_0), P(a_1, b_0)$ and $P(a_1, b_1)$ have to be quantum.

But not $P(a_0, b_1)$



Let $T = 10$ years $t = 0.5$ ns.

Then $P(a_0, b_0), P(a_0, b_1)$ and $P(a_1, b_1)$ have to be quantum.

But not $P(a_1, b_0)$

We can't get a contradiction, because there is always at least one marginal that doesn't have to be quantum.

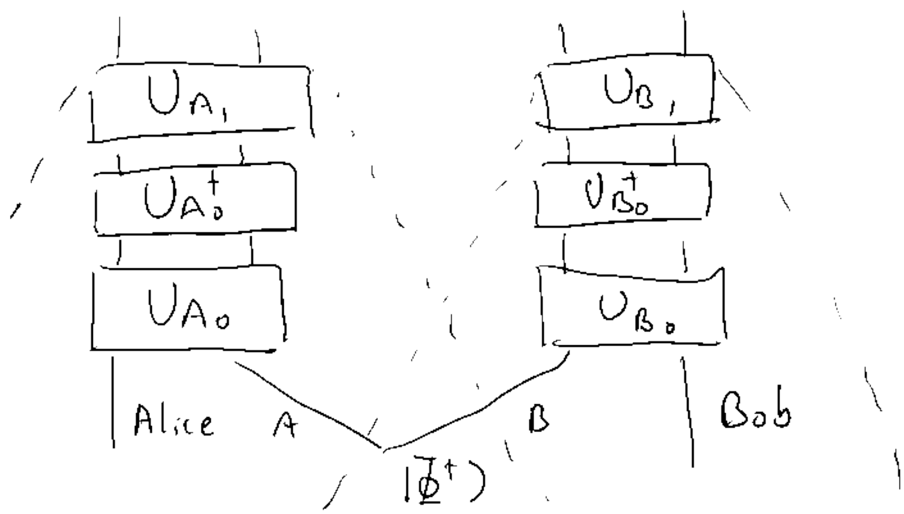
Note, however, that there is nothing in the formalism of quantum theory that would explain why we get different non-quantum marginals in these two experiments.

We would have to imagine some mechanism that communicates to Alice's system whether or not Bob's second measurement has happened yet and vice versa.

In a hidden variable model, we may be prepared to posit such a mechanism, but Copenhagenish quantum theory is just supposed to be raw quantum mechanics, interpreted anti-realistically.

Since Copenhagenists only have the quantum formalism to rely on, it is reasonable that sequences of observations that are described the same way in the quantum formalism ought to make the same predictions.

⇒ The same marginals have to be quantum in both experiments, so all of them do, and we get a contradiction.



We can even perform the experiment in such a way that all four pairs of observations:

$(A_0, B_0), (A_1, B_0), (A_0, B_1), (A_1, B_1)$

are spacelike separated, so for every pair, there is a frame in which they coexist.

Strictly speaking, only $P(a_1, b_1)$ has to be quantum in this case, as for all the others there is not enough time for Alice and Bob to compare results before the erasure.

But there seems no good reason for arbitrarily choosing a non-quantum marginal in this case.

Just performing the experiment a bit faster should not affect what is quantum, and this is a faster version of both variants.

7.vii. Perspectival Copenhagenish Interpretations

For a Copenhagenist, the obvious thing to give up is the objectivity of outcomes.

If Alice's outcomes only exist for Alice and Bob's outcomes only exist for Bob and neither exist for Wigner, then there is no global (Wigner) perspective on which all outcomes can be said to exist, and hence no joint probability distribution.

QBism

QBism is an interpretation of this type. It is the combination of:

- Pure subjective Bayesianism: All probabilities are subjective Bayesian, so quantum states always represent the degrees of belief of a decision-making agent. Different agents have different states. There is never a requirement for two agents to assign the same state (even with no level conflict).
- Perspectival Copenhagenism: An agent's quantum state describes beliefs about their own personal reality. If another agent's observation is not reflected in that state then it does not exist for them.

Thursday, June 20, 2019

Relational Quantum Mechanics

Rovelli's Relational Quantum Mechanics (should be called Perspectival Quantum Mechanics):

- Physical systems only have properties from the perspective of other systems, but these perspectival properties are objective.
- E.g. The measurement has an outcome from the perspective of (the physical system called) the friend, but not from the perspective of (the physical system called) Wigner.
- These properties are determined by the eigenvalue-eigenstate link, but only applied perspectivally. There is no conflict in the measurement problem because the two descriptions are from the perspective of different physical systems.
- Rovelli thinks there is nothing special about measurements. I can equally talk about the properties of a single electron from the perspective of another electron.
- I think this runs into a basis problem like Everett, but we can solve it by only assigning properties from the perspective of systems that are decohered.

The Technological Interpretation

Both QBism and Relational Quantum Mechanics are *more perspectival* than required by the Bell-Wigner mashup. I propose the following minimally perspectival Copenhagenish interpretation.

We note that it is always possible *in principle* for an all powerful super-observer to come along and recohere every observation made in the history of the universe.

Therefore, there is never a time when it is safe to say that an observation has objectively occurred *for everyone*.

However, most of the time it is technologically impractical to recohere an observation, e.g. would require placing the Earth in a Dyson sphere in order to collect all photons scattered off the experimental apparatus.

For Alice, it is safe to say that an observation has occurred if Alice lacks the technological ability to recohere the observation.

What counts an an observation is relative to the technological ability of the observer.

- A bit bizarre to have ontology depend on technology, but it implies that we all agree on objective reality most of the time, and avoids having different realities from the perspective of every electron in the universe.

7.viii. Summary

Unless we are willing to permit contortions about which marginals are allowed to be non-quantum, the Bell-Wigner mashup rules out objective Copenhagenish interpretations, i.e. most of them except QBism and Relational QM.

It is remarkable that we can constrain Copenhagenish interpretations at all.

The perspectival move will not appeal to many Copenhagenists, as Copenhagen is usually thought to be built on level-headed empiricism, i.e. the things we see in the lab do straightforwardly happen.

It is this straightforward empirical attitude that drives many to Copenhagen instead of Everett, which says that there is a long path from the ontology to understanding what we see in the lab as an emergent phenomenon.

Bell-Wigner may drive you towards realism, but we have plenty of results, e.g. Bell, that make realism problematic too.

We should probably investigate more exotic types of ontology that might get around all of these no-go results.