Network-Locality

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UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

QSIT Quantum Science and Technology
National Centre of Competence in Research

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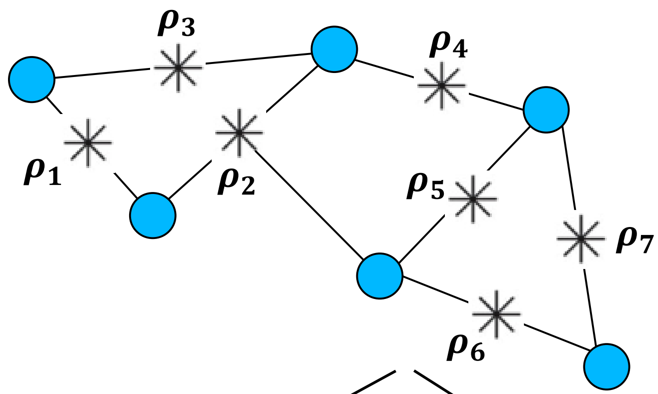
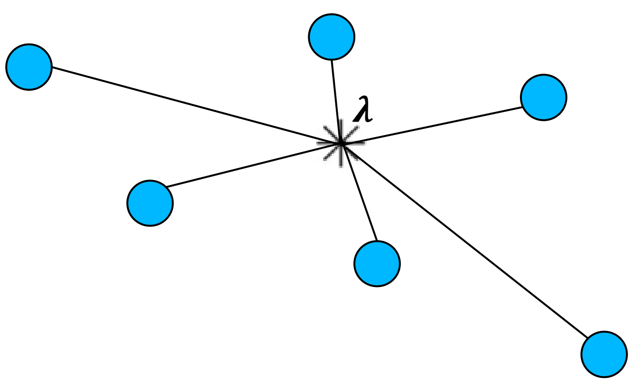
A Quantum Network:

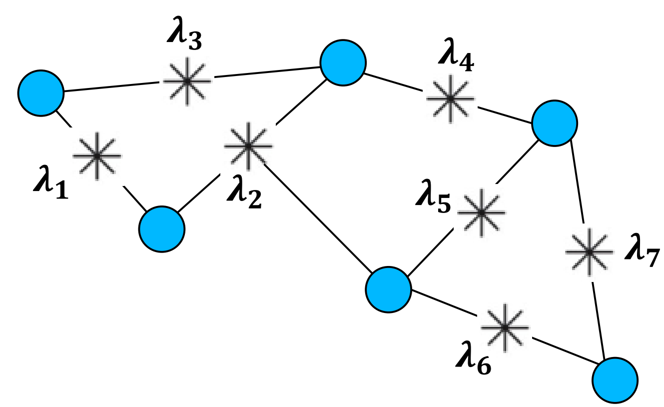


Fine Genuine Quantum Properties?

Bell NonLocality approach:



Network-Locality approach:



See also research at Perimeter (Waterloo) , ICFO (Barcelona), UFRN (Natal), Neel (Grenoble) …

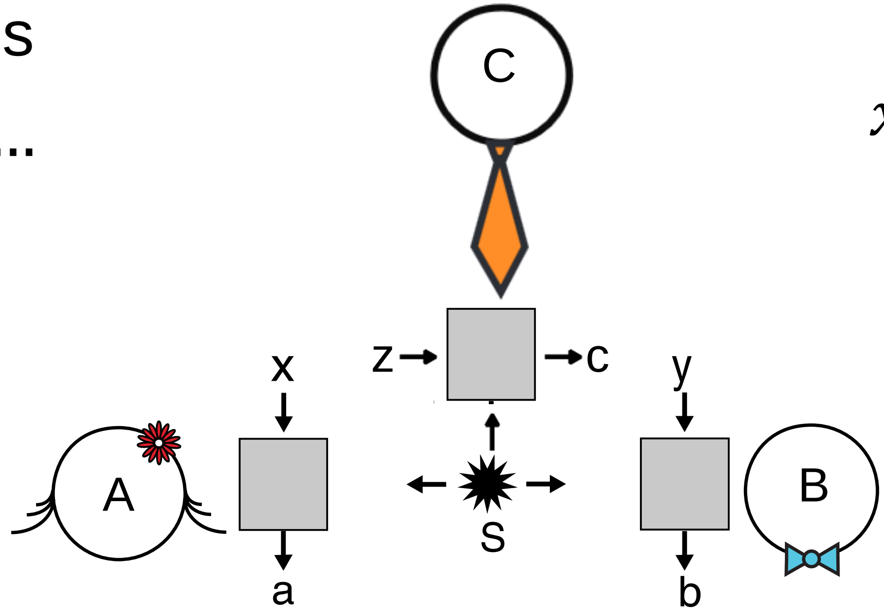
Bell nonlocality

p parties

$A, B, C, \ldots$

q inputs

$x, y, z \in \{1, \ldots, q\}$



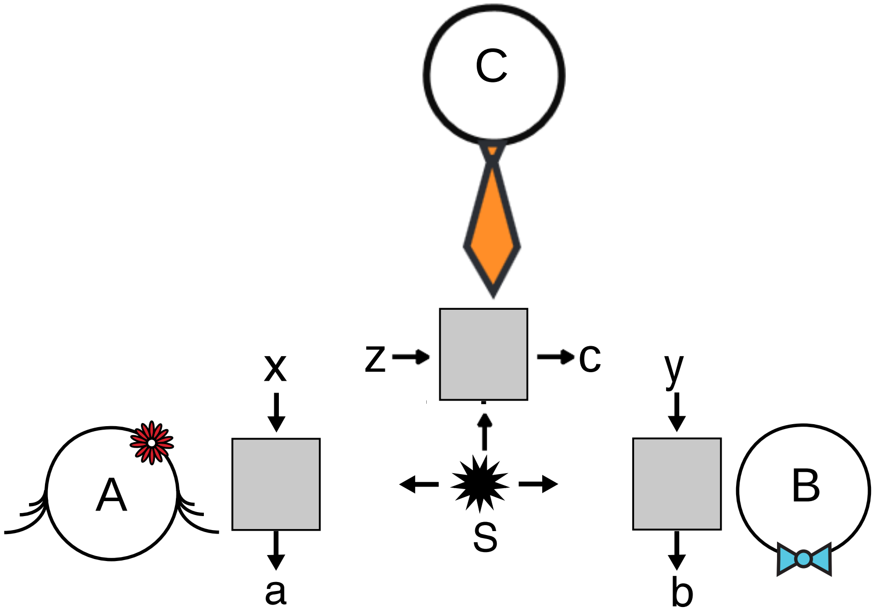
r outputs

$a, b, c \in \{1, \ldots, r\}$

Behavior $\vec{P} = \{p(abc|xyz)\}$

1 common source:
Classical/Quantum/Other

Bell nonlocality

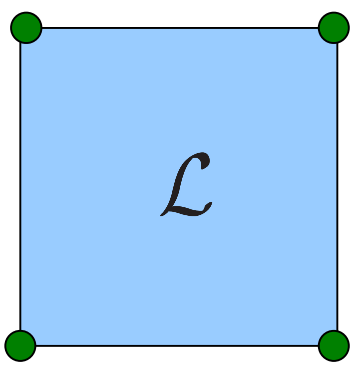


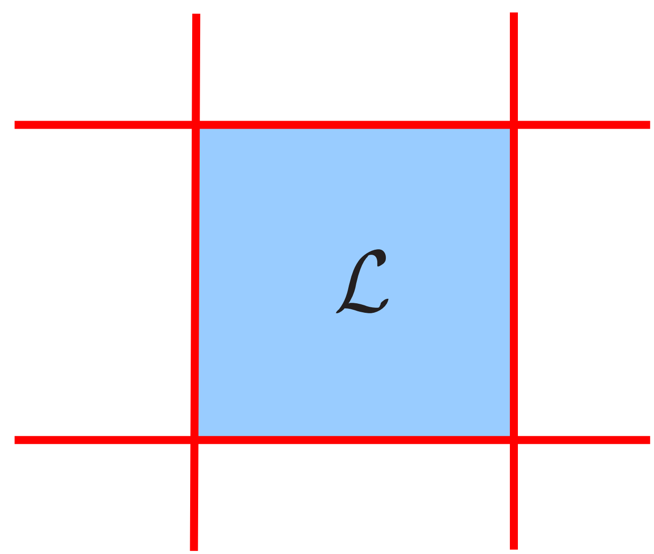
For a local hidden variable:

Characterize $\mathcal{L} = \{\vec{P}\} \subset \mathbb{R}^{q^p r^q}$?

Bell nonlocality

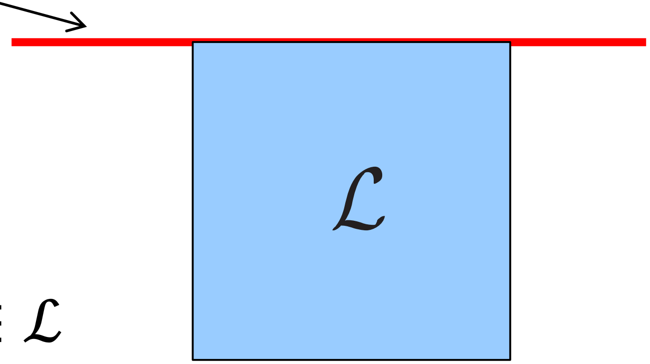
- $\mathcal{L}$ is a polytope
- Vertices of $\mathcal{L}$ are deterministic distributions
- 2 characterization of the polytope $\mathcal{L}$:
  - Enumerate all deterministic distributions
  - Enumerate all facets





Bell nonlocality

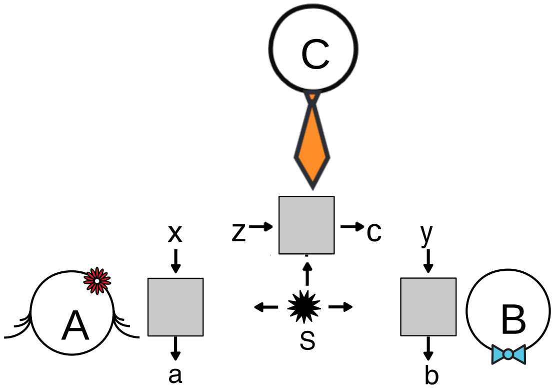
Hyperplane $\vec{S} \in \mathbb{R}^{q^p r^q}, S_k \in \mathbb{R}$
represents a Bell inequality

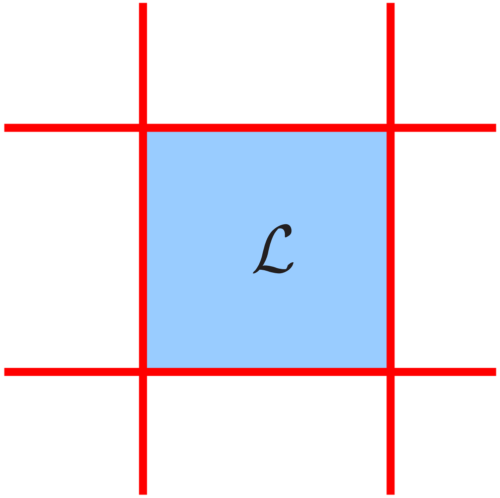


Facets:

$$\vec{S} \cdot \vec{P} = \sum_{abcxyz} s_{xyz}^{abc} p(abc|xyz) \leq S_k \,, \forall \vec{P} \in \mathcal{L}$$

Bell nonlocality

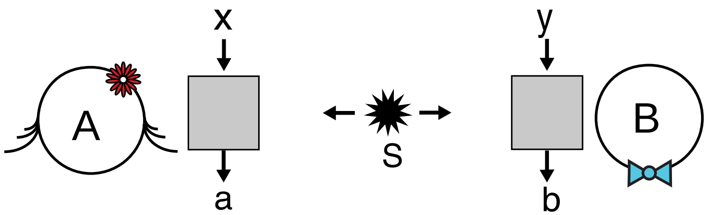


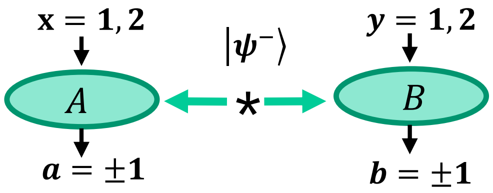


- $\mathcal{L}$ is characterized by Bell inequalities, which are « easily » enumerable.

- Starting point to find quantum violations

Bell nonlocality



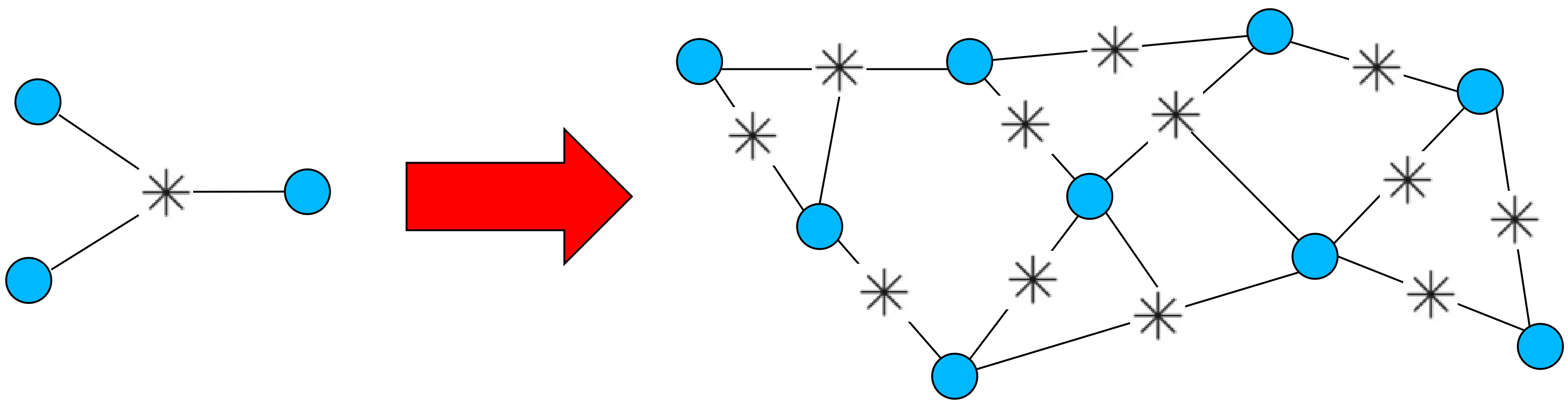


A quantum Non-Local experiment

$\vec{P} \notin \mathcal{L}, \vec{P} \in \mathcal{Q}$:

The CHSH experiment

Generalisation to Network-Locality



$\vec{P} \in \mathcal{L}$

$\vec{P} \in \mathcal{Q}$

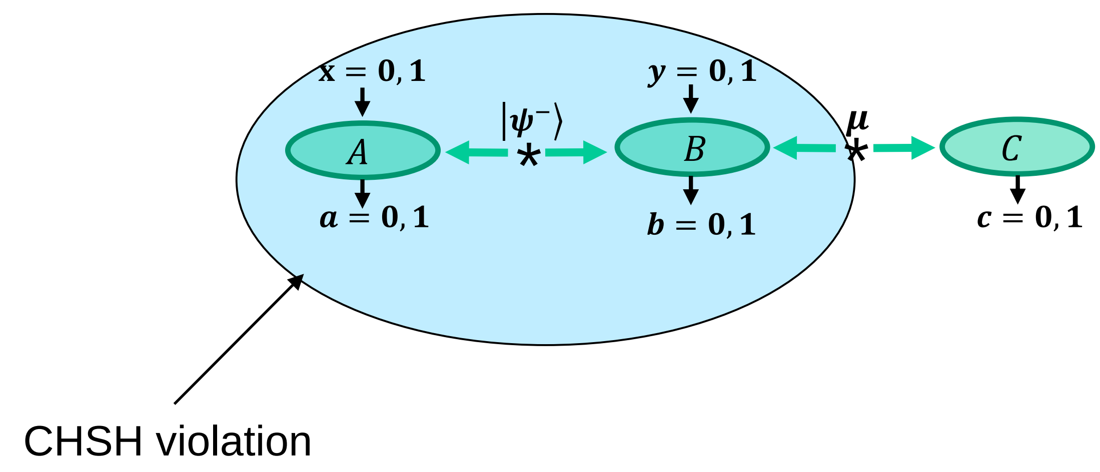
$\vec{P} \in \mathcal{N}\mathcal{L}$

$\vec{P} \in \mathcal{N}\mathcal{Q}$

Independent sources → non convex problem

Network-Locality

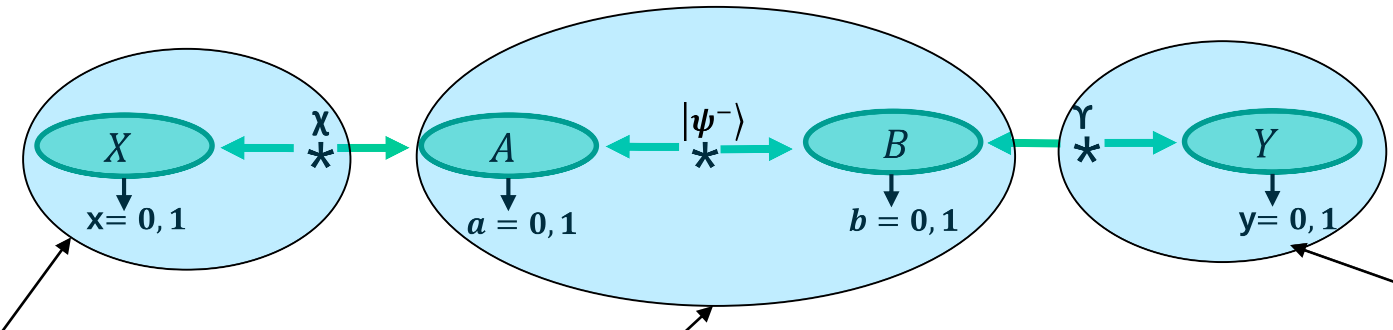
Some violations from Bell Nonlocality:



Not genuine Network-Locality violation!

Network-Locality

Some violations from Bell Nonlocality:



Answer x=χ

CHSH violation with input χ , ϒ

Answer y=χ

Not genuine Network-Locality violation!

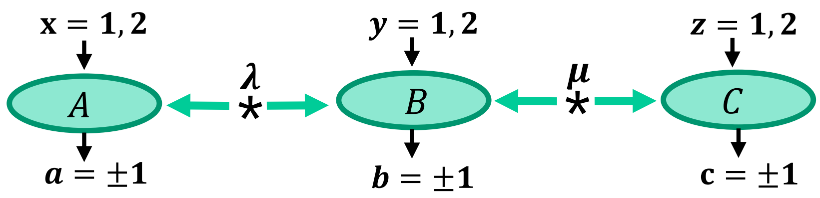
- Bell-Nonlocality VS Network-Locality
- Bilocality & Extensions
- Triangle scenario

- Bell-Nonlocality VS Network-Locality

- Bilocality & Extensions

- Triangle scenario

1^{st} bilocality inequality



Correlators:

$$\langle A_x B_y C_z \rangle$$

Mean value of abc for input xyz

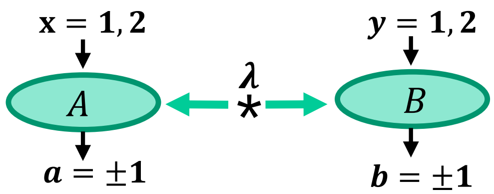
$$\mathrm{I}_1 = \frac{1}{4}\langle (A_1 + A_2) B_1 (C_1 + C_2) \rangle_{\lambda,\mu}$$

$$\mathrm{I}_2 = \frac{1}{4}\langle (A_1 - A_2) B_2 (C_1 - C_2) \rangle_{\lambda,\mu}$$

$$\sqrt{|\boldsymbol{I_1}|} + \sqrt{|\boldsymbol{I_2}|} \leq \mathbf{1}$$

C.Branciard, N.Gisin, and S.Pironio,
Characterizing the Nonlocal Correlations Created via Entanglement Swapping, Phys. Rev. Lett.104,170401 (2010)

CHSH



$$S = |\langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle - \langle A_2 B_2 \rangle|$$

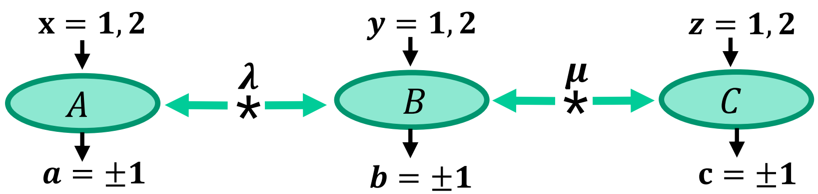
Let:

$$\left.\begin{aligned} \hat{A}_1 &= A_1 + A_2 \\ \hat{A}_2 &= A_1 - A_2 \end{aligned}\right\} |\hat{A}_1| + |\hat{A}_2| \leq 2$$

Then:

$$\begin{aligned} S &= \left|\left\langle \hat{A}_1 B_1 + \hat{A}_2 B_2 \right\rangle\right| \\ &\leq \int \left|\hat{A}_1 \cancel{B_1}\right| + \left|\hat{A}_2 \cancel{B_2}\right| \qquad \text{(Triang. Ineq.)} \\ &\leq \left\langle \left|\hat{A}_1\right| + \left|\hat{A}_2\right| \right\rangle \quad \leq \quad 2 \end{aligned}$$

1st bilocality inequality



- Let $I_i = \frac{1}{4}\langle \hat{A}_i B_i \, \hat{C}_i \rangle$:

$$|I_i| \le \frac{1}{4}\langle |\hat{A}_i||\hat{C}_i| \rangle = \frac{1}{4}\langle |\hat{A}_i| \rangle \langle |\hat{C}_i| \rangle$$ (Triang. Ineq.)

- Then:

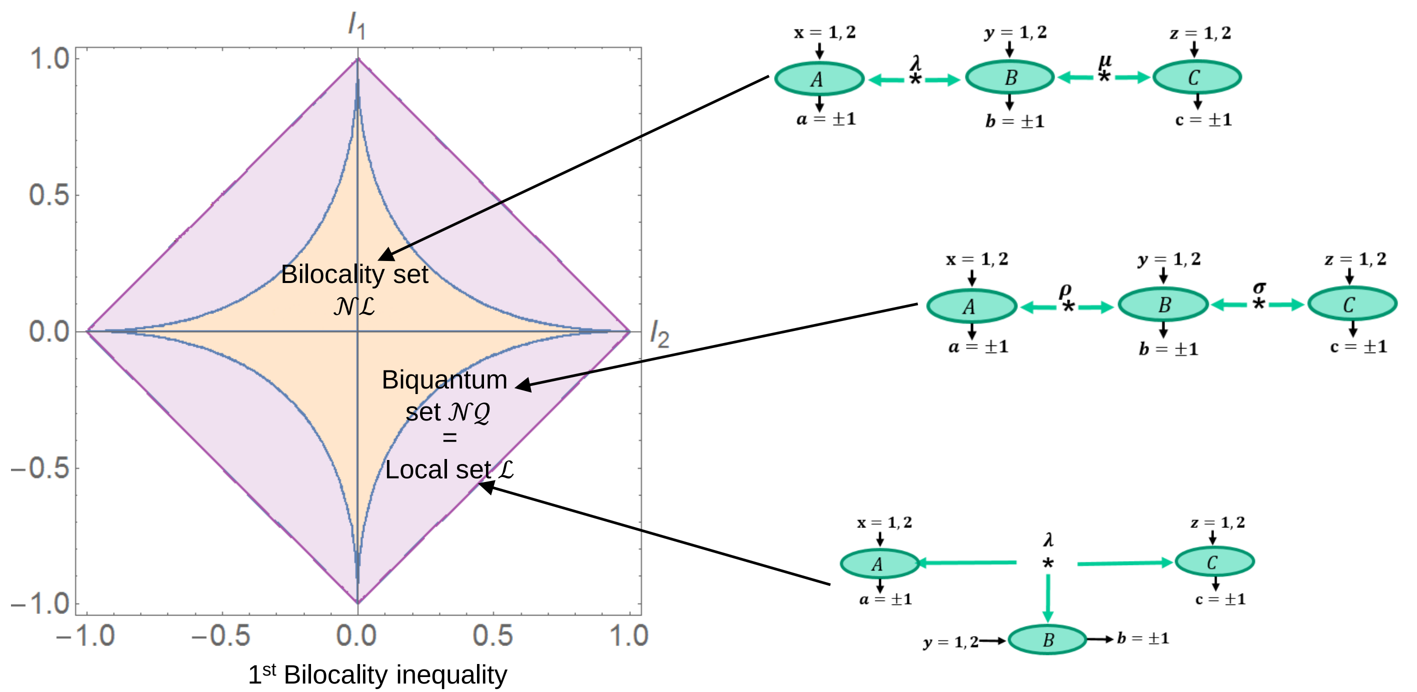
$$S = \sqrt{|I_1|} + \sqrt{|I_2|} \le \frac{1}{4}\left(\sqrt{\langle |\hat{A}_1| \rangle}\sqrt{\langle |\hat{C}_1| \rangle} + \sqrt{\langle |\hat{A}_2| \rangle}\sqrt{\langle |\hat{C}_2| \rangle}\right)$$

$$\le \frac{1}{2}\sqrt{\langle \left|\hat{A}_1\right| + |\hat{A}_2| \rangle}\sqrt{\langle \left|\hat{C}_1\right| + |\hat{C}_2| \rangle}$$ (C.S. Ineq.)

$$\le 1$$

1st bilocality inequalities

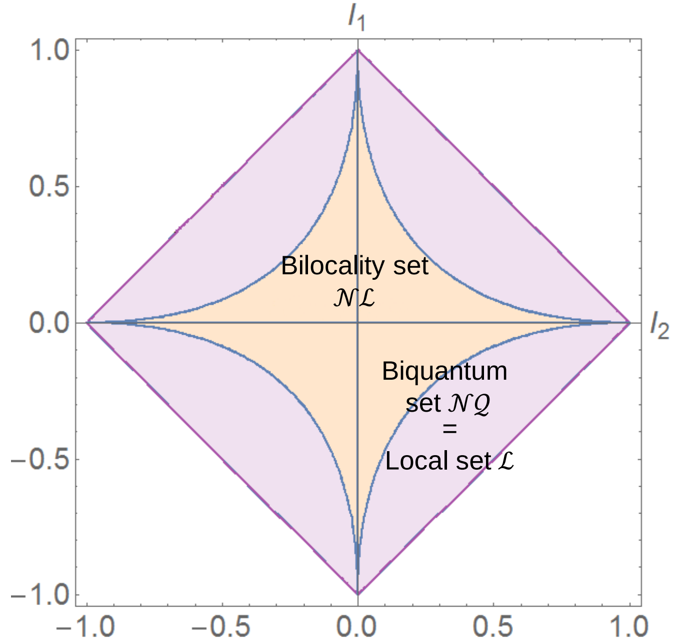
$$\sqrt{|I_1|} + \sqrt{|I_2|} \leq 1$$



C.Branciard, D.Rosset, N.Gisin, S.Pironio (2012), *Bilocal versus non-bilocal correlations in entanglement swapping experiments,* Phys. Rev. A 85, 032119

1st bilocality inequalities

$$\sqrt{|I_1|} + \sqrt{|I_2|} \leq 1$$

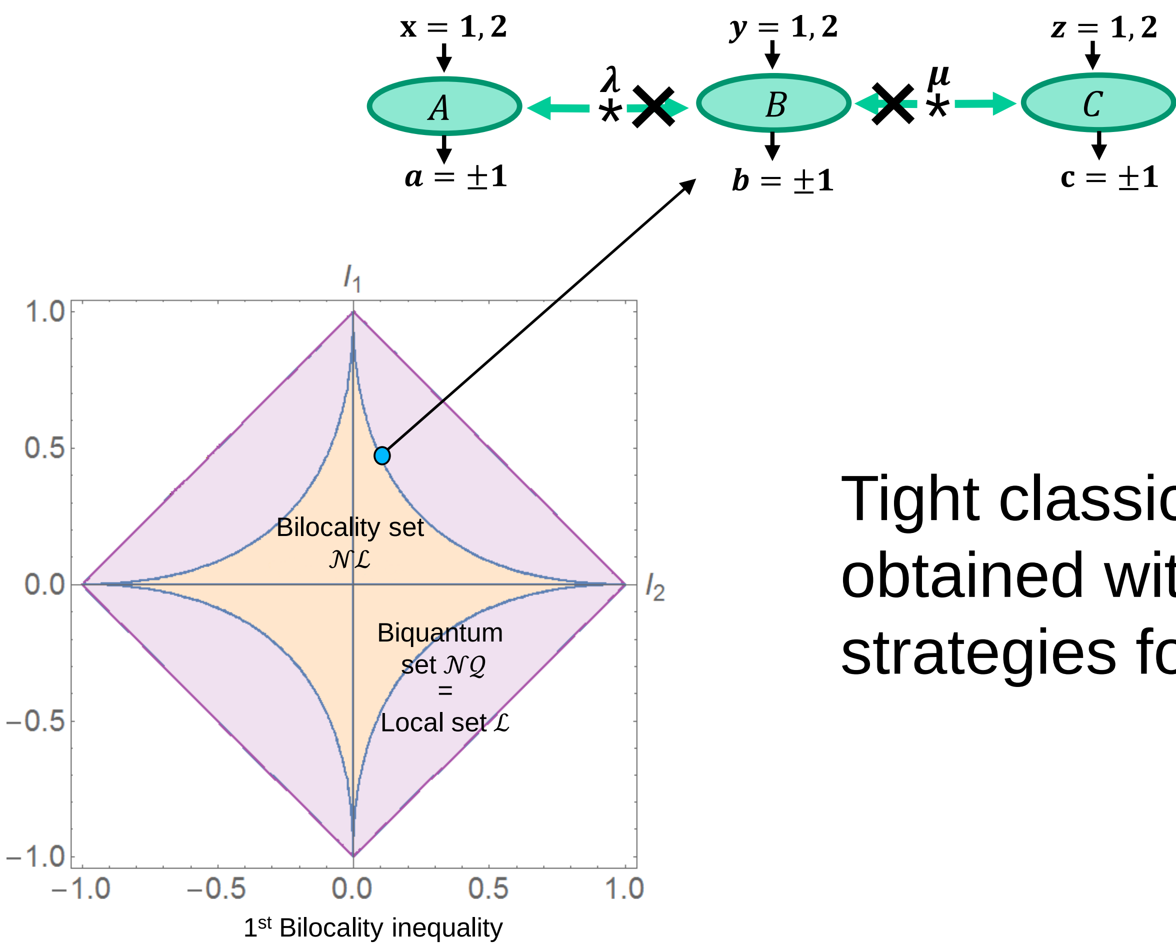


1st Bilocality inequality

- Partial characterization of $\mathcal{NL}$
- Tight
- Quantum violations
- Bilocal set: nonconvex

C.Branciard, D.Rosset, N.Gisin, S.Pironio (2012),
Bilocal versus non-bilocal correlations in entanglement swapping experiments, Phys. Rev. A 85, 032119

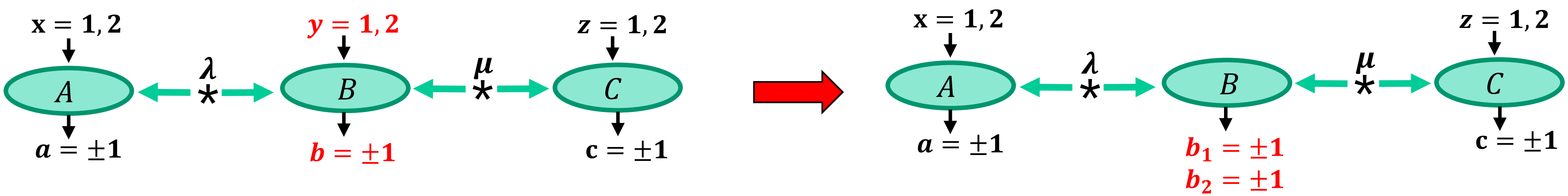
1st bilocality inequality



Tight classical bound obtained with deterministic strategies for Bob

A.Tavakoli, MO.Renou, N.Gisin, N.Brunner *(2017)*
Correlations in star networks: from Bell inequalities to network inequalities arXiv:1702.0386

2^{nd} bilocality inequality



Correlators:

$$\langle A_x B_y C_z \rangle$$

Mean value of $ab_y c$ for input xz

$$I_1 = \frac{1}{4}\langle (A_1 + A_2) B_1 (C_1 + C_2) \rangle_{\lambda,\mu}$$

$$I_2 = \frac{1}{4}\langle (A_1 - A_2) B_2 (C_1 - C_2) \rangle_{\lambda,\mu}$$

$$\sqrt{|\boldsymbol{I_1}|} + \sqrt{|\boldsymbol{I_2}|} \leq \mathbf{1}$$

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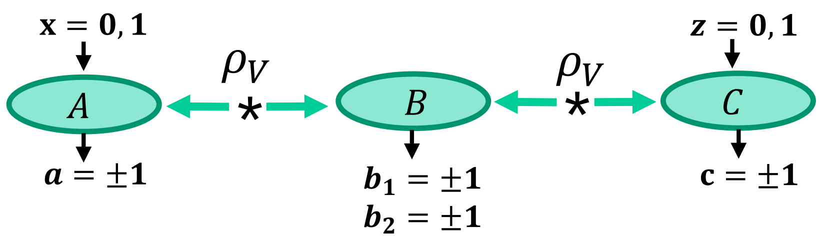
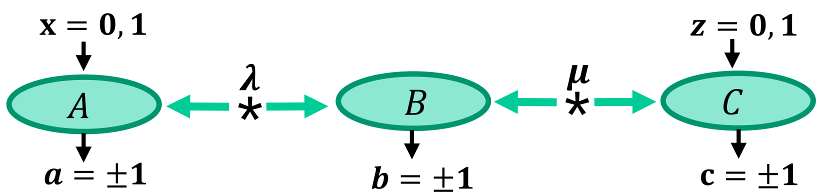
Characterizing the Nonlocal Correlations Created via Entanglement Swapping, Phys. Rev. Lett.104,170401

2nd bilocality inequality



- $\rho_V = V|\psi^-\rangle\langle\psi^-| + (1 - V)\frac{Id}{4}$
- ρ_V violates CHSH iff $V > \frac{1}{\sqrt{2}}$
- If B does a Bell State Measurement : AC share ρ_{V^2}
- Bilocality inequality is violated iff $V^2 > \frac{1}{2}$

2^{nd} bilocality inequality



Strong connections with CHSH violation:

- Violated by all pairs of pure entangled states

- With a BSM for Bob:

$$S^{max} \leq \sqrt{S_{AB}^{max}} \sqrt{S_{BC}^{max}}$$

Bilocality

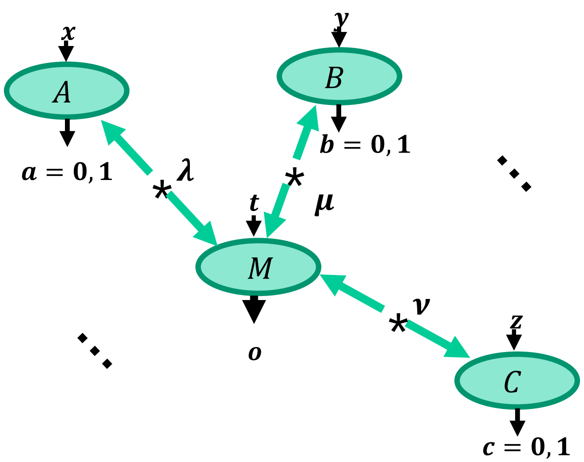
CHSH

With equality if
$\rho_{AB} = \rho_{BC}$

ρ violates CHSH $\leftrightarrow$ $\rho \otimes \rho$ violates biloc. ineq.

F.Andreoli, G.Carvacho, L.Santodonato, R.Chaves, F.Sciarrino
Maximal violation of n-locality inequalities in a star-shaped quantum network, arXiv:1702.08316
N.Gisin, Q.Mei, A.Tavakoli, MO.Renou, N.Brunner (2017)
All entangled pure quantum states violate the bilocality inequality arXiv:1702.00333

Extension: Star Network



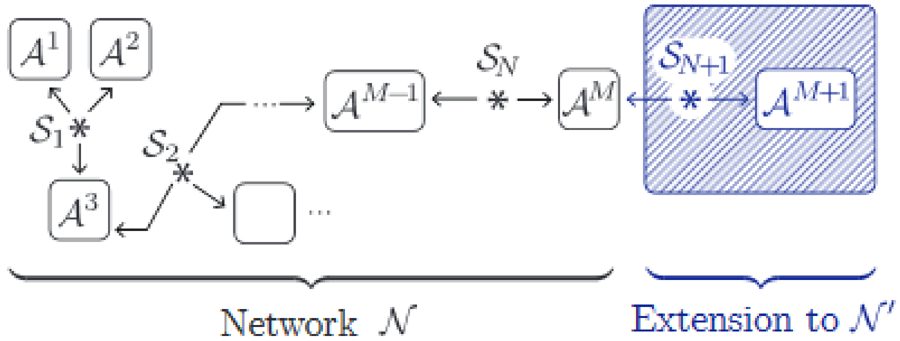
Generalizable to any star network with binary output:

- With $A_1, A_2, \ldots \to \hat{A}_1, \hat{A}_2, \ldots$; $B_1, B_2, \ldots \to \hat{B}_1, \hat{B}_2, \ldots$; ...
- And $I_i = \langle \hat{A}_i \hat{B}_i \hat{C}_i \ldots M_i \rangle$

$$\sum_{\boldsymbol{i}} |\boldsymbol{I_i}|^{\mathbf{1}/\boldsymbol{N}} \leq \boldsymbol{C}$$

A.Tavakoli, MO.Renou, N.Gisin, N.Brunner *(2017)*
Correlations in star networks: from Bell inequalities to network inequalities arXiv:1702.0386

Extension

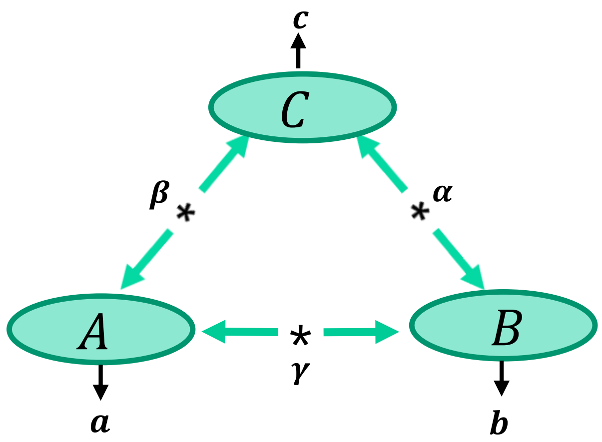


- For any correlator Bell inequality about network $\mathcal{N}$ in terms of the $\langle A^1_{x_1} \dots A^M_{x_M} \rangle$
- Using the transformation $\begin{matrix} A_1^{M+1} \\ A_2^{M+1} \end{matrix} \rightarrow \begin{matrix} \hat{A}_1^{M+1} = A_1^{M+1} + A_1^{M+1} \\ \hat{A}_2^{M+1} = A_1^{M+1} - A_1^{M+1} \end{matrix}$
- Construct a new Bell inequality with terms $\langle A^1_{x_1} \dots A^M_{x_M}\ \hat{A}_i^{M+1} \rangle$
- Proof of CHSH, go from CHSH to Bilocality Inequality, …

D.Rosset, C.Branciard, T.Barnea, G.Pütz, N.Brunner, N.Gisin
Nonlinear Bell Inequalities Tailored for Quantum Networks Phys. Rev. Lett. 116, 010403 (2016)

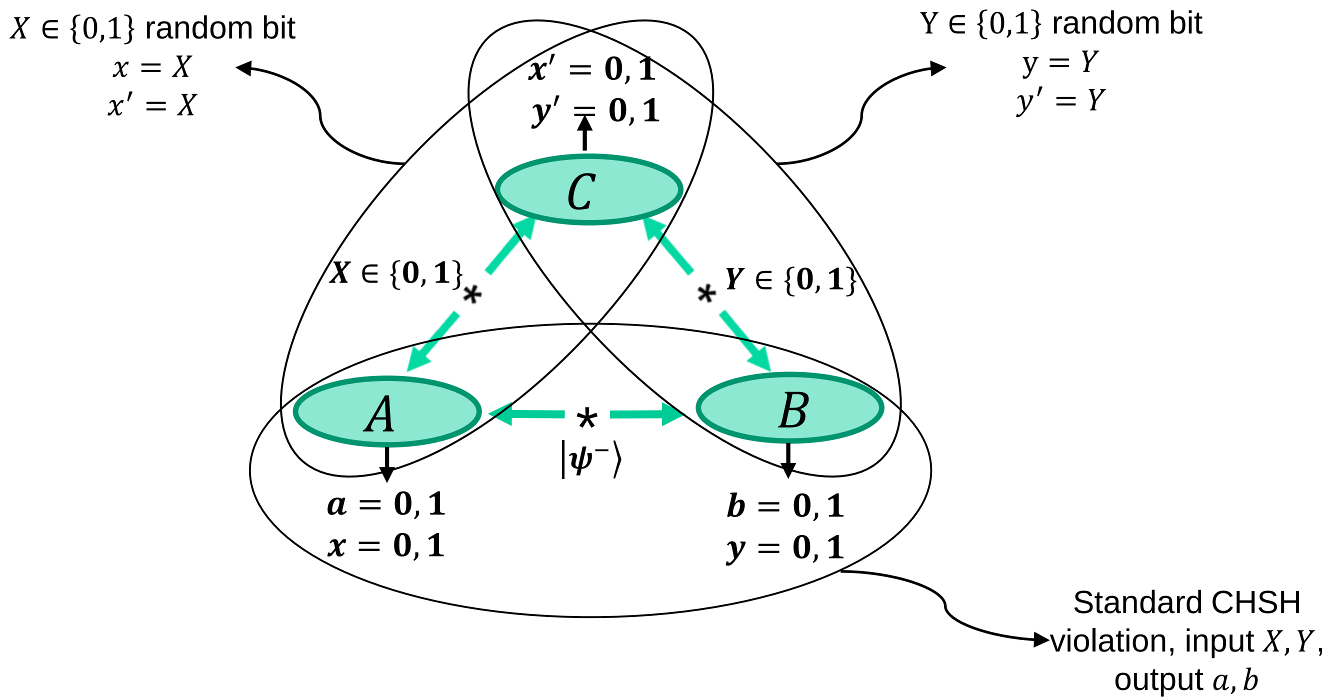
- Bell-Nonlocality VS Network-Locality
- Bilocality & Extensions
- Triangle scenario

Triangle scenario



- Triangle network
- No input

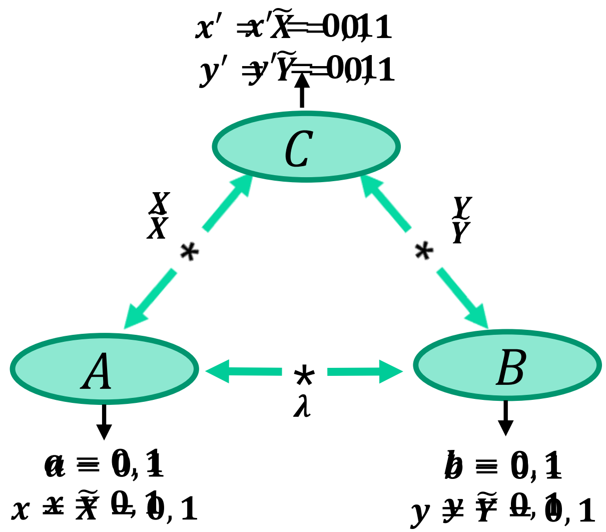
Triangle scenario, $\mathcal{NQ}$



Can it be in $\mathcal{NL}$?

T.Fritz, (2012)
Beyond Bell's Theorem: Correlation Scenarios, New J. Phys. 14 103001

Triangle scenario, $\mathcal{NL}$



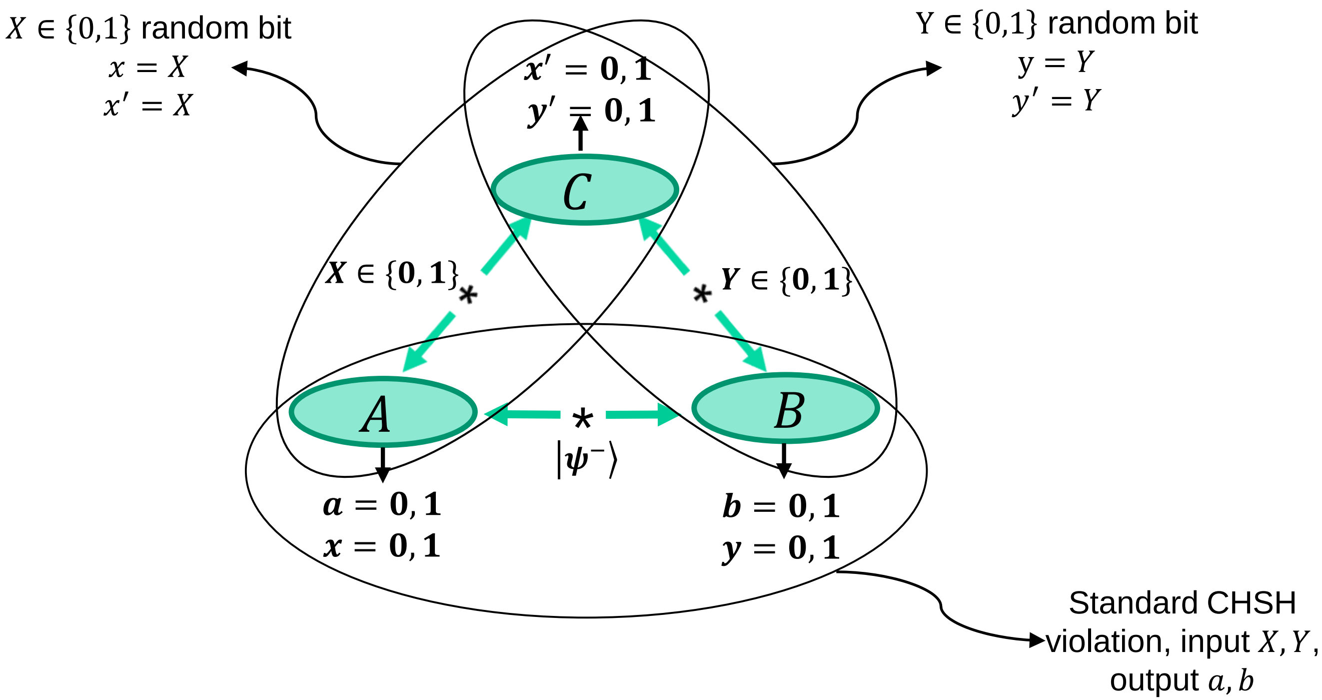
If $\vec{P} \in \mathcal{NL}$:

- $\forall \text{run}, x = x'$: $x = x' = f(X) \equiv \tilde{X}$
- $\forall \text{run}, y = y'$: $\text{y} = y' = g(Y) \equiv \tilde{Y}$
- $p(abxy)$ violates CHSH

Cannot be explained classically!

T.Fritz, (2012)
Beyond Bell's Theorem: Correlation Scenarios, New J. Phys. 14 103001

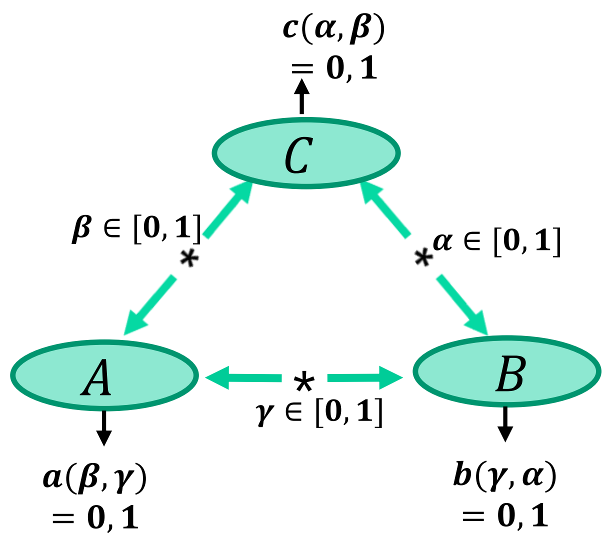
Triangle scenario, $\mathcal{NQ}$



- Need 4 outputs (to check that nobody lies about his input)
- Not genuine example of Network-Locality violation, uses Bell Nonlocality

T.Fritz, (2012)
Beyond Bell's Theorem: Correlation Scenarios, New J. Phys. 14 103001

2 output triangle scenario



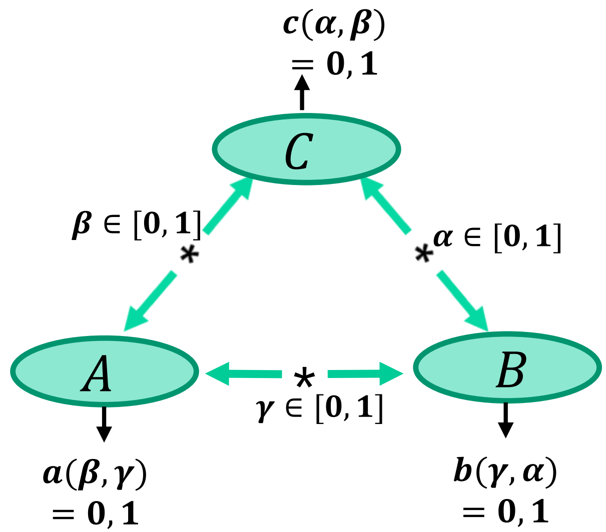
w.l.o.g. :
α, β, γ *uniformly random in* $[0,1]$

2 questions:

Characterization of $\mathcal{NL}$?

$\mathcal{NL} \neq \mathcal{NQ}$?

Strategies $\boldsymbol{S}$

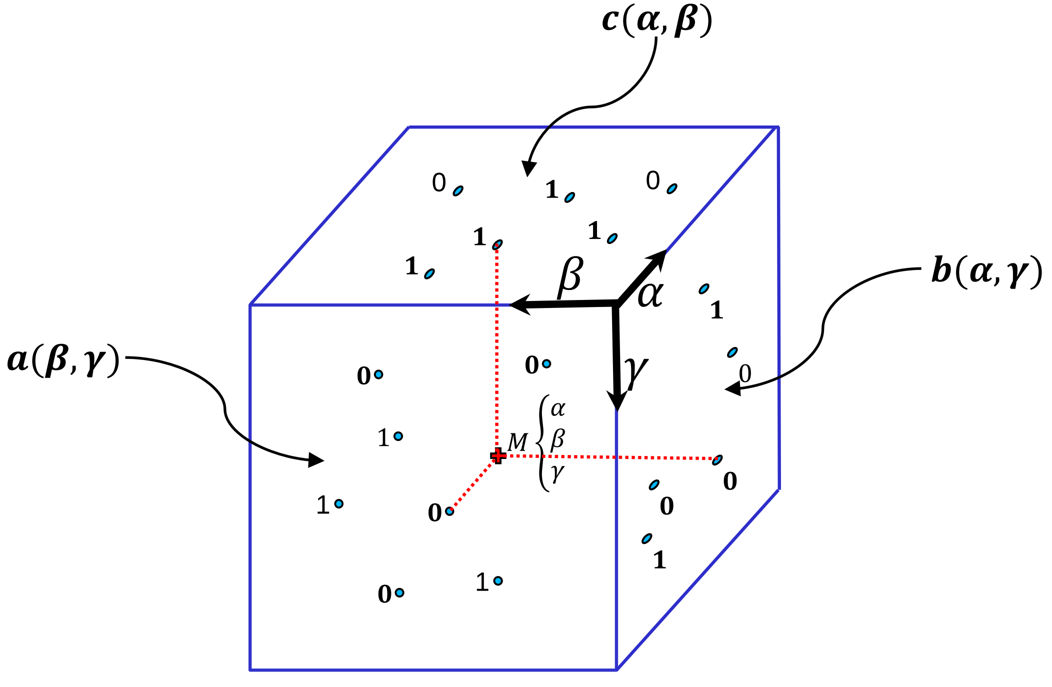


w.l.o.g. :

$\boldsymbol{S}$ ≡ *deterministic* $a(\beta,\gamma)$, $b(\gamma,\alpha)$, $c(\alpha,\beta)$

$$p(\tilde{a}\tilde{b}\tilde{c}) = \int d\alpha\, d\beta\, d\gamma\, \delta^{\tilde{a}}_{a(\beta,\gamma)} \delta^{\tilde{b}}_{b(\gamma,\alpha)} \delta^{\tilde{c}}_{c(\alpha,\beta)}$$

Strategies $\boldsymbol{S}$



Any $\boldsymbol{S}$: Given by a cube with a 0 or a 1 for each point of 3 faces

Applications

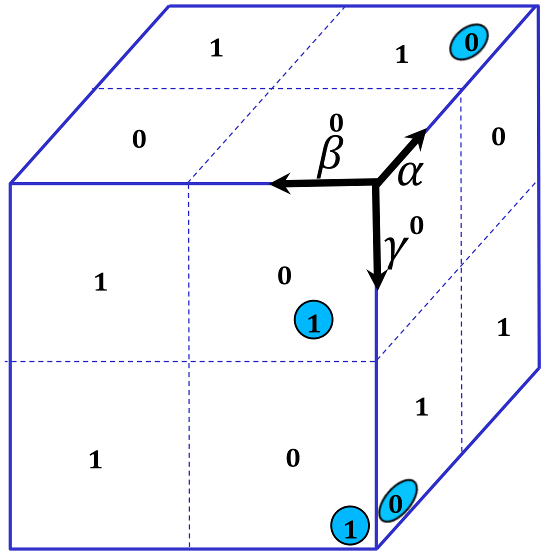
No dimensional advantage in Quantum theory

- $\mathcal{NL} \subset \mathcal{NQ} \subset \mathbb{R}^8$
- P_0: $\left\{p(abc) = \frac{1}{8}\right\}$ is "central"
- Is there some direction
  $P_0 + \delta P \in \mathcal{NQ}, \notin \mathcal{NL}$?

➢ Not possible in standard Bell Nonlocality
➢ Not obvious in Network-Locality

Applications

No dimensional advantage in Quantum theory

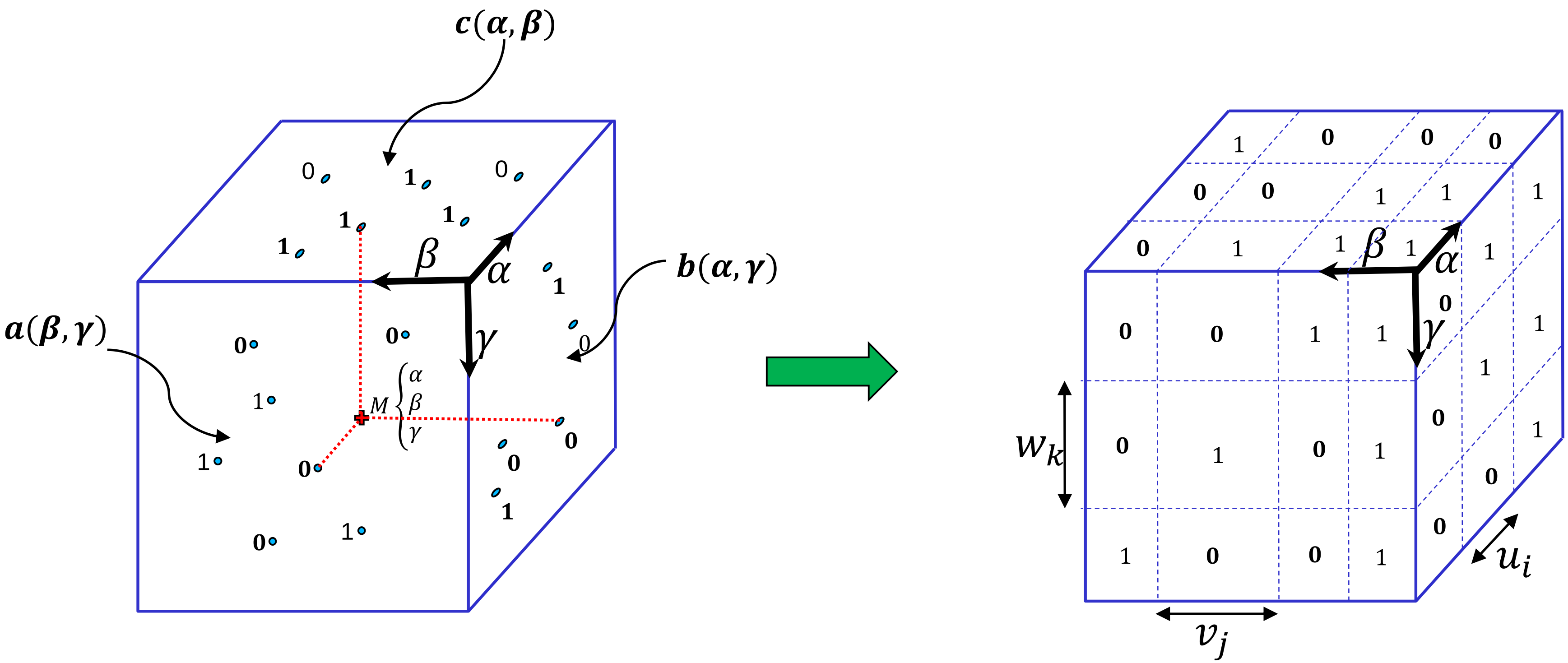


- Take $\boldsymbol{S_0}$ strategy for P_0
- Vary it: obtain $P_0 + \delta P$
- Show that any $P_0 + \delta P$ can be obtained

➢ This can be generalized to any network

MO.Renou *et al.*, in preparation

Finite rank model



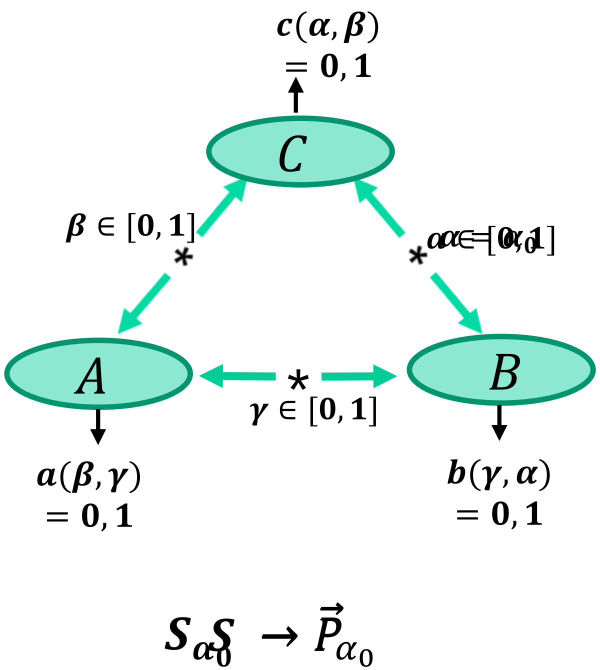
$$\vec{P} = \int d\alpha d\beta d\gamma \vec{P}_{\alpha,\beta,\gamma} \,,$$

$\vec{P}_{\alpha,\beta,\gamma}$ deterministic

$$\vec{P} = \sum_{i,j,k=1}^{8} u_i v_j w_k \, \vec{P}_{i,j,k},$$

$\vec{P}_{i,j,k}$ deterministic

Finite rank model



- Consider $\boldsymbol{S} \rightarrow \vec{P}$
- Fix source $\alpha = \alpha_0$:

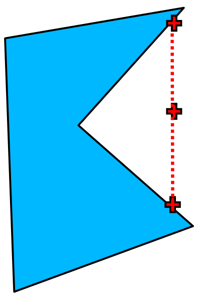
We obtain a new strategy

$$\boldsymbol{S}_{\alpha_0} \rightarrow \vec{P}_{\alpha_0}$$

- $\vec{P} = \int d\alpha_0 \vec{P}_{\alpha_0}$

Finite rank model

$$\vec{P} = \int d\alpha_0 \vec{P}_{\alpha_0}$$



Carathéodory theorem:
If $\vec{P} \in \mathbb{R}^d$ in convex hull of A, A convex, then $\vec{P}$ is the convex combination of d points of A

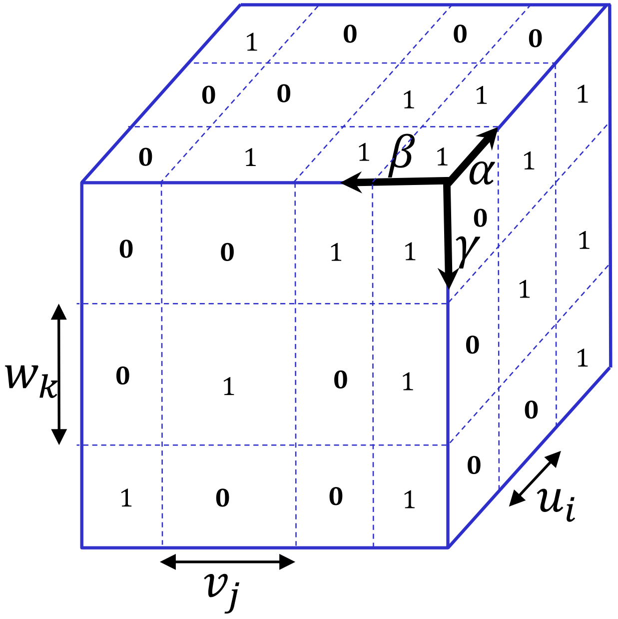
$$\vec{P} = \sum_{i=1}^{8} u_i \vec{P}_i,$$

$\vec{P}_i$ has a strategy independent from α

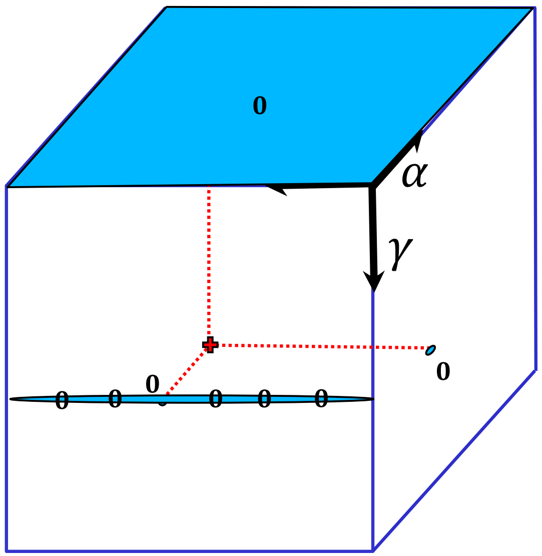
D. Rosset, N. Gisin, and E. Wolfe,
All n-local set are semialgebraic, in preparation

Finite rank model

$$\vec{P} = \sum_{i,j,k=1}^{8} u_i v_j w_k \vec{P}_{i,j,k},\ \vec{P}_{i,j,k} \text{ deterministic}$$

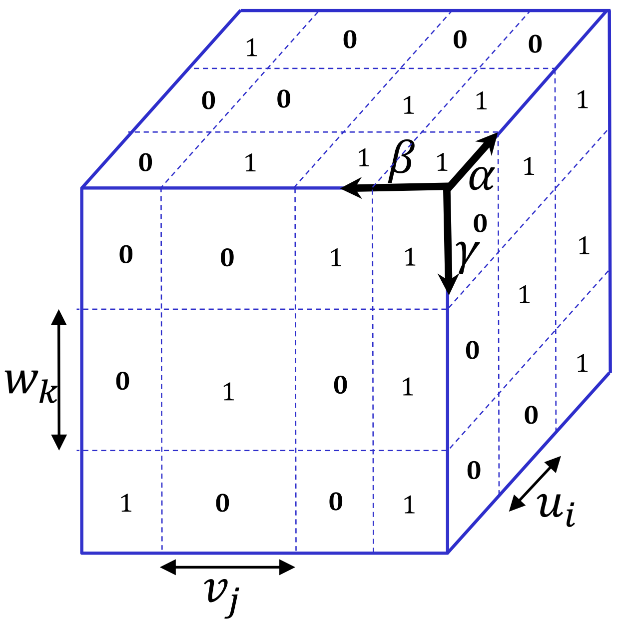


Applications



- GHZ distribution $[p_{000} = p_{111} = \frac{1}{2}]$
  $\notin \mathcal{NL}$
- ω distribution $[p_{001} = p_{010} = p_{100} = \frac{1}{3}]$
  $\notin \mathcal{NL}$

Applications

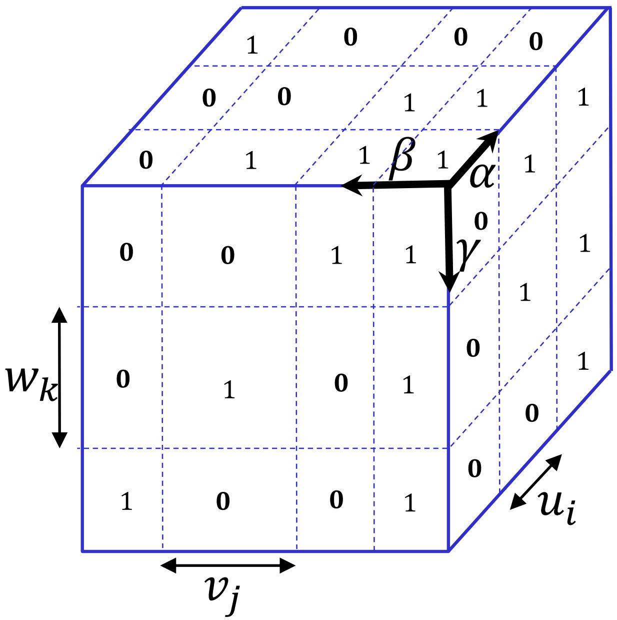


Characterization of $\mathcal{NL}$:

Given $\vec{P}$, answer to "$\vec{P} \in \mathcal{NL}$"?

- Enumerate all $0/1$ repartition, find a u_i, v_j, w_k distributions: too hard!

Applications



- Restricted problems:
  - Given p_{000}, Max(p_{111})?
  - Given p_{001}, p_{010}, Max(p_{100})?

<u>Carathéodory theorem</u>

$\Phi(\vec{P}) = \{p_{000}, p_{111}\}$ has a rank 2 strategy.

$\Psi(\vec{P}) = \{p_{001}, p_{010}, p_{100}\}$ has a rank 3 strategy.

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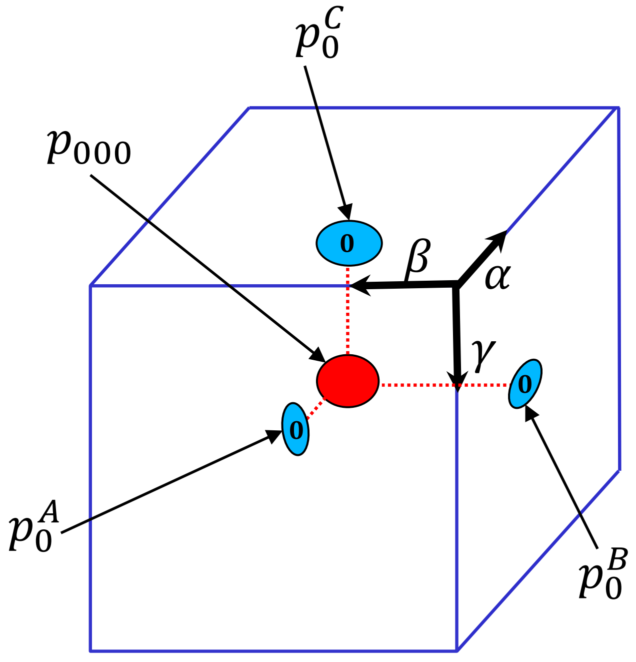
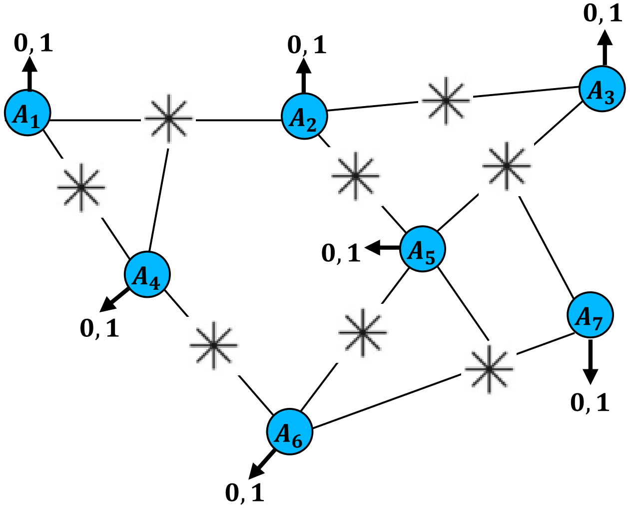
Applications



- LoomisWhitney inequality:
  $$p_{000}^2 \leq p_0^A p_0^B p_0^C$$
- Could not find violation (with maximally entangled qubit)
- Only one joint probability

Link to computer science



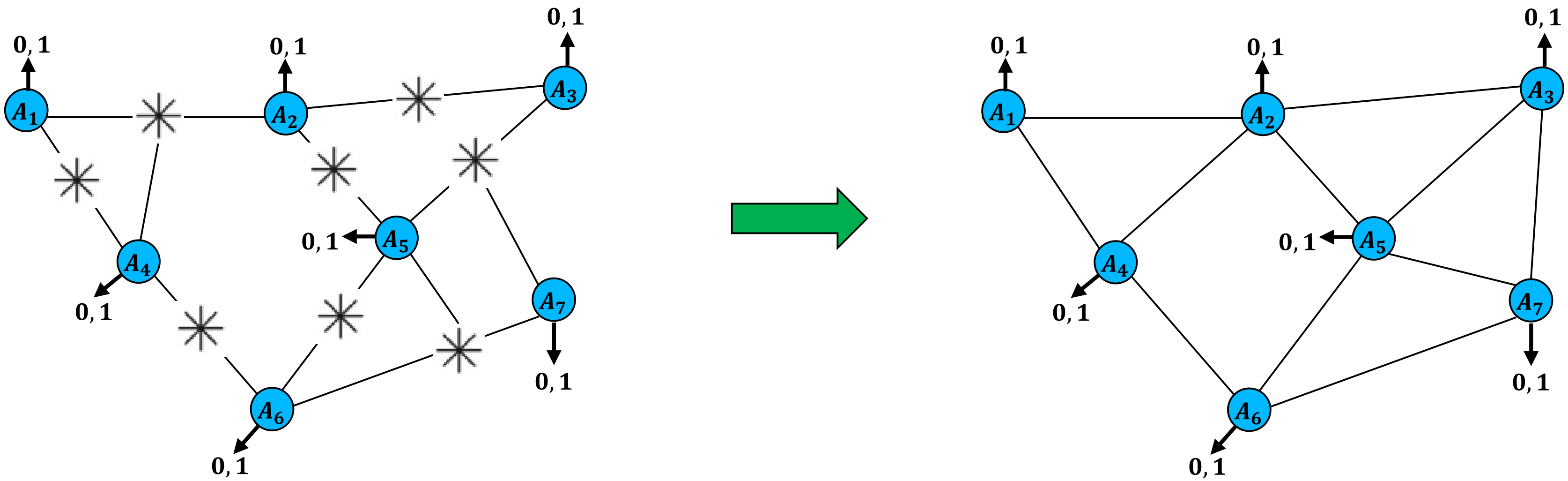
Finner inequalities:

$$p_{00\ldots0} \leq \prod_i \left(p_0^{A_i}\right)^{\nu_i},$$

The ν_i depends on the network structure

Link to computer science

Variable Graph (locality assumption) | Dependency Graph (causality assumption)



Finner inequalities:

$$p_{00\ldots0} \leq \prod_i \left(p_0^{A_i}\right)^{\nu_i}$$

Analogue inequalities:

$$p_{00\ldots0} \leq \prod_i \left(p_0^{A_i}\right)^{\mu_i}$$

$\mu_i \leq \nu_i$: Possible quantum violation only with a gap

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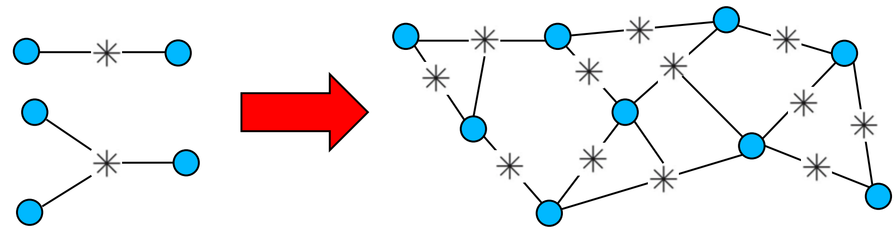
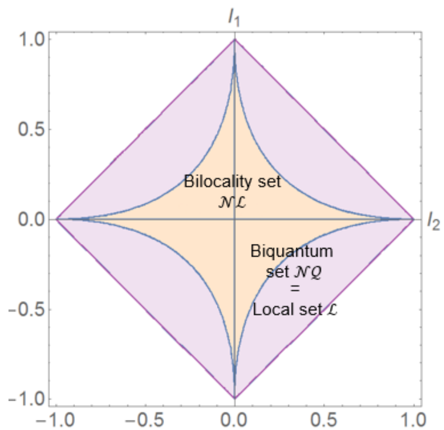
To conclude

- Bell Nonlocality / Network-Locality





- Find $\mathcal{NL}$: hard non-convex problem.
- Some $\mathcal{Q}$ violation for no-loop network

- Partial characterization of $\mathcal{NL}$.
  Other technics (Inflation).
  No “genuine” example of violation for the triangle.