quantum gravity

spacetime

responsibility

experiment

indefinite causal order

computational advantage

inference

superposition of
causal relations

Causality
in
Quantum Theory

statistical analysis

epistemology

purpose

paradox

determinism

knowledge vs reality

relativity

signalling

free will

hidden variables

evidence-based policy

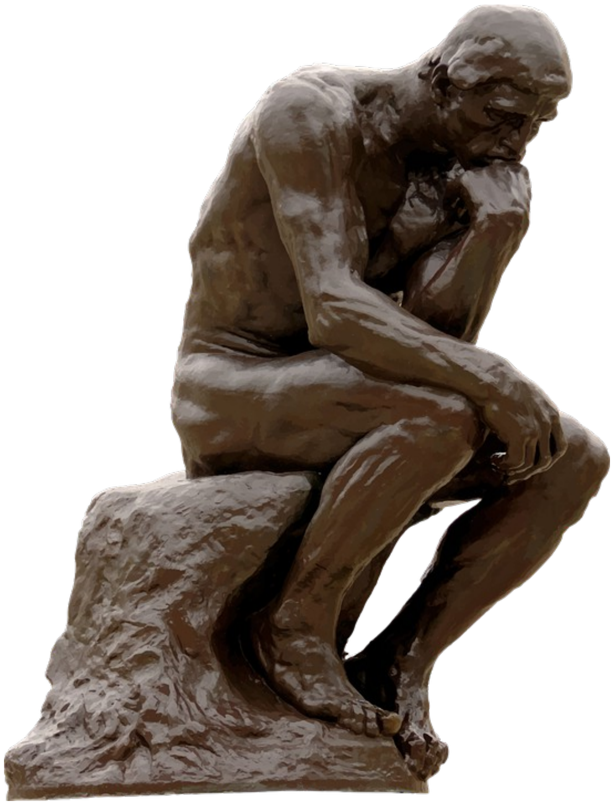
Katja Ried, Univ. Innsbruck

Solstice of Foundations — ETH Zurich, June 2017

Itinerary

1. Causality (classical version)
   - concepts and formalism
   - features and phenomena
2. Quantum indeterminism
3. Retrocausality and causal loops
4. Nonlocality
   - in quantum field theory
   - Bell inequality violations
5. Quantum causal models
   - concepts and formalism
6. Causal structure (quantum version)
   - non-classical causal relations
   - experiments

Causality
(classical version)



On what grounds can one say that A causes B?

Some more specific questions:

Are the answers different talking about events or variables?

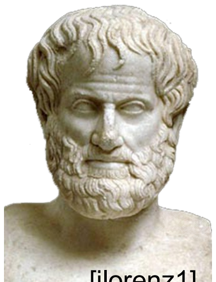
Can we always identify causes, and are they unique?

Under which circumstances can A not causally influence B?

Meta-question: What does this definition of causality accomplish?

(philosophical insights, operational predictions...)

Some possible answers



[jlorenz1]

Aristotle's four causes:
(1) material cause: constituent matter, (2) formal cause: shape, arrangement, (3) efficient or moving cause: agents (4) final cause: purpose. [Falcon]

„We may define a cause to be an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second.“ [Hume]

„If an improbable coincidence has occurred, there must exist a common cause.“ [Reichenbach]

Hans Reichenbach

In quantum field theory, „for our theory to be causal, we must require that all spacelike separated operators commute“. [Tong]

An event A is statistically independent of its non-descendants given its causal parents. [Pearl]

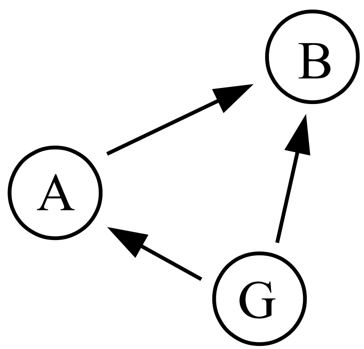
Causal Models: conceptual and mathematical framework

Origin: statistics – sociology, epidemiology, econometrics...

System modelled: relations among a set of coarse-grained random variables

Structure

pattern of influences



directed acyclic graph

+

Parameters

description of influences

$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

conditional probabilities

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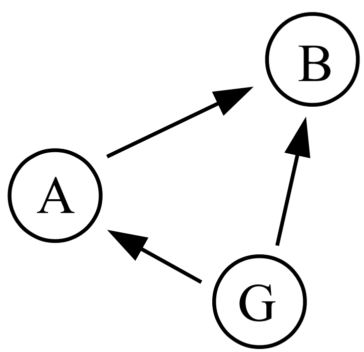


directed acyclic graph

+

Parameters

description of influences

$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

conditional probabilities

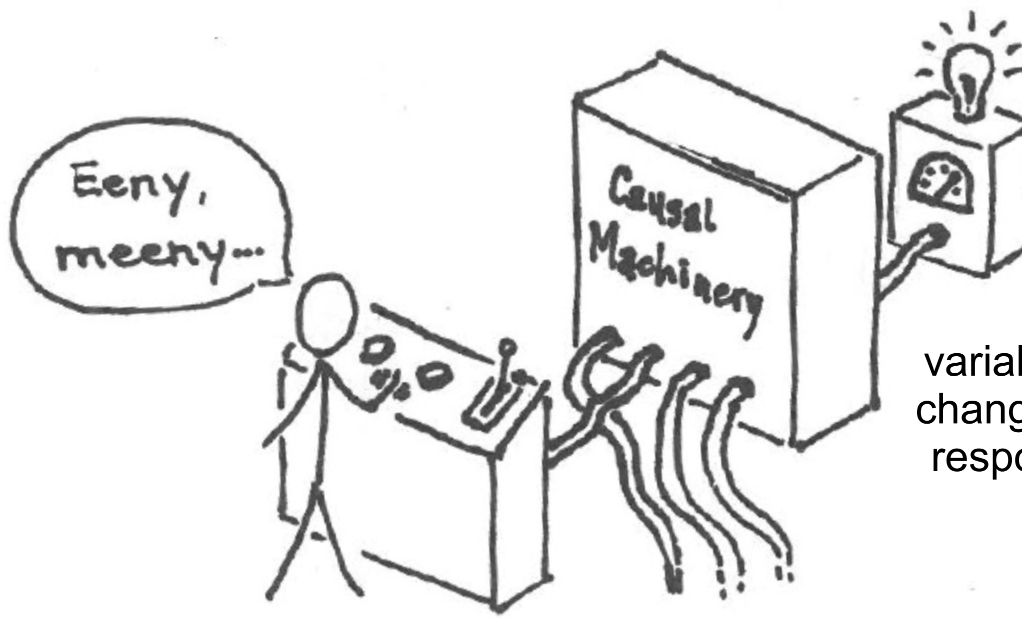
Mathematical toolbox:

- conditional probabilities: $P(Y|X) \geq 0 \qquad \sum_y P(Y|X) = 1 \; \forall \, x$
- belief propagation: $P(Y) = \Sigma_x P(Y|X=x) P(X=x)$

Causal Models: features and phenomena

Causation defined in terms of **interventions**
A has a causal effect on B if we can change the value of B by manipulating A (while leaving all else unchanged).

- practical significance: control



variable B changes in response

experimenter chooses a value for A

all other influences remain fixed

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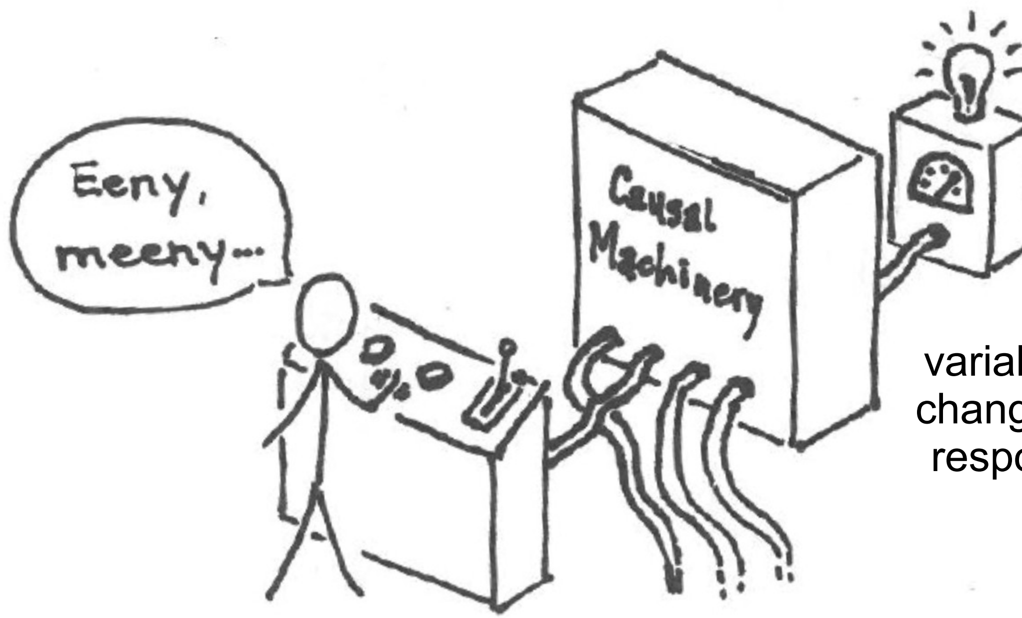

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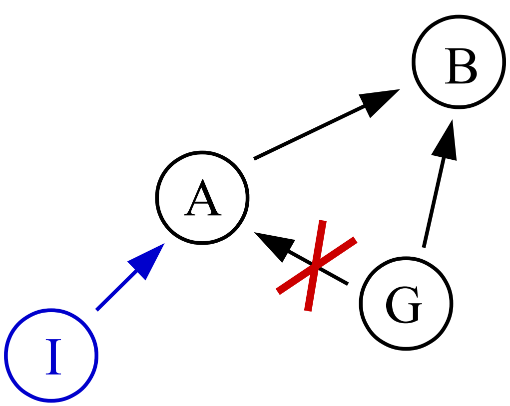

structural effect:
surgery on DAG

$$P(B|AG)$$
$$\cancel{P(A|G)}$$
$$P(G)$$

effect on conditionals: intervention overrides incoming causal influences

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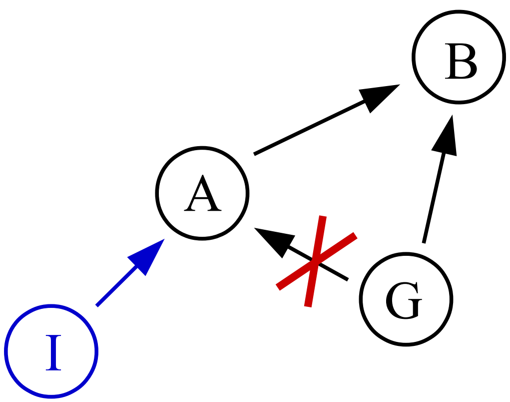

structural effect:
surgery on DAG

$$
\begin{gathered}
P(B|AG) \\
\cancel{P(A|G)} \\
P(G)
\end{gathered}
$$

effect on conditionals: intervention overrides incoming causal influences

Local intervention leaves all other causal mechanisms unaffected:
autonomy of causal mechanisms

Causal Models: features and phenomena

simple picture of interventions:
incoming causal influences
are eliminated

$$P(B|AG)$$
$$\cancel{P(A|G)}$$
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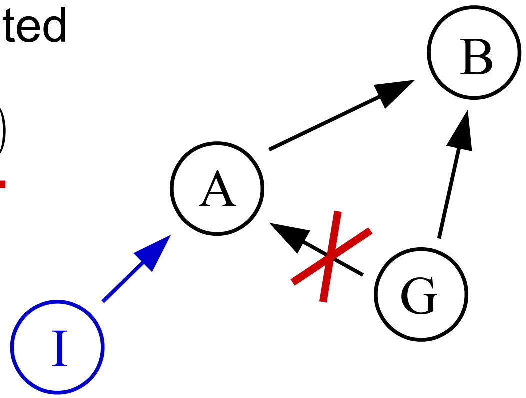

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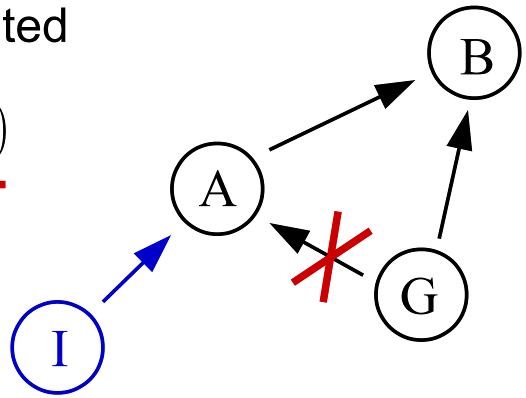
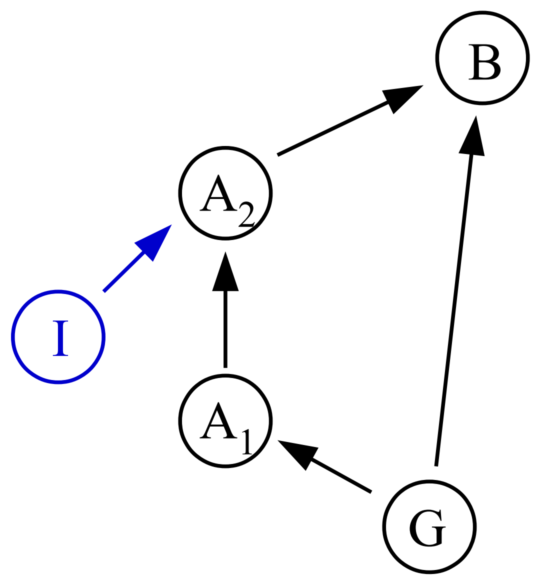


more informative perspective:
splitting of variables

randomization



$$P(B|A_2G)$$
$$P(A_2|A_1,I)$$
$$P(A_1|G)$$
$$P(G)$$

Causal Models: features and phenomena

simple picture of interventions:
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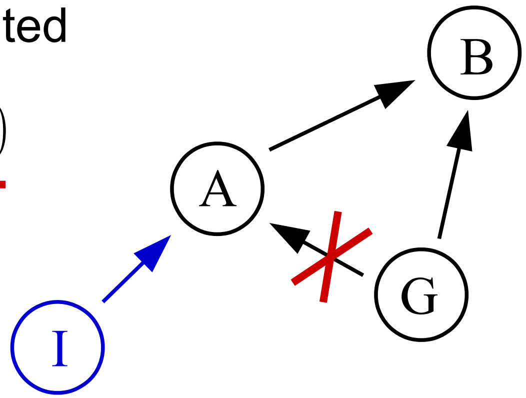

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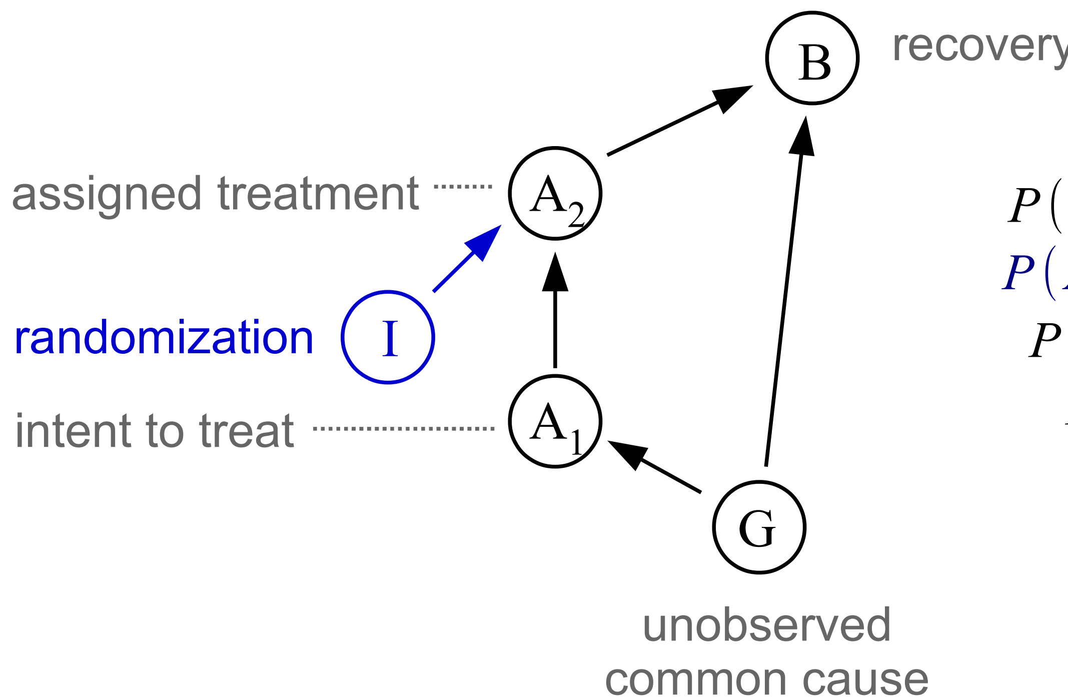

$$P(B|A_2 G)$$
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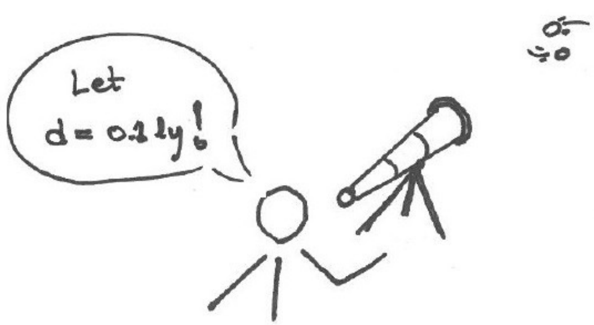
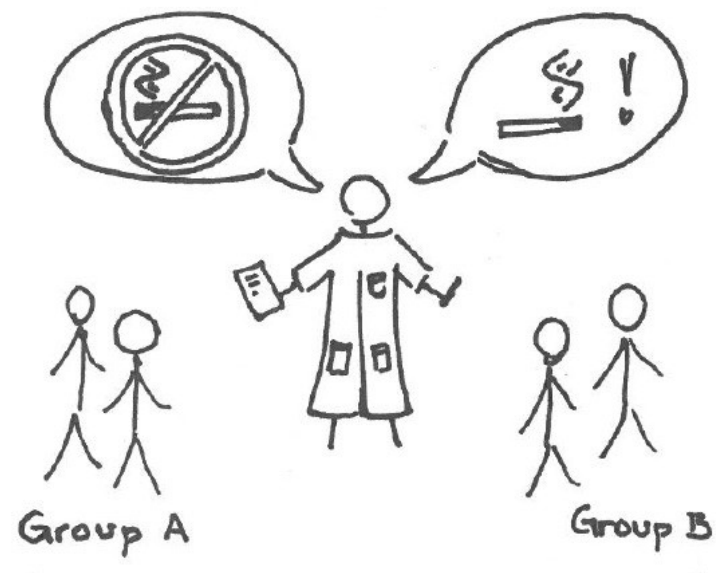
Causal Models: features and phenomena

Interventions can be problematic.



Experimental astronomy

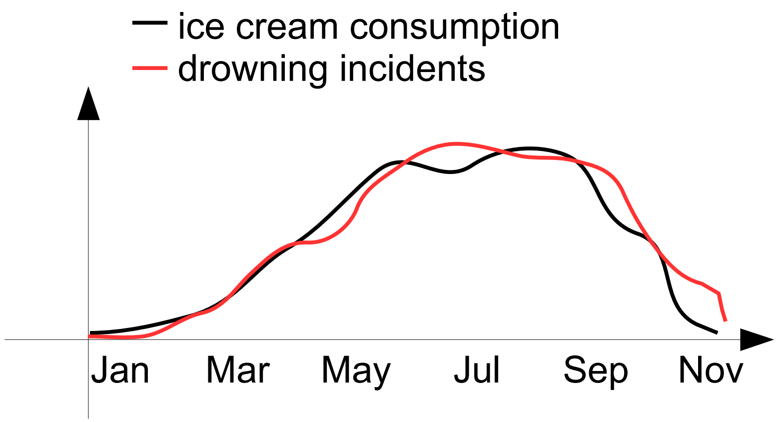


Randomized trial on the health effects of smoking

Solution: **causal inference**

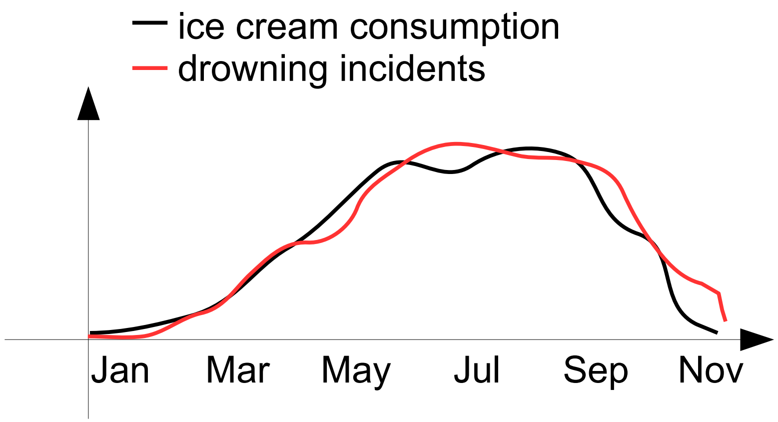
Causal Models: features and phenomena

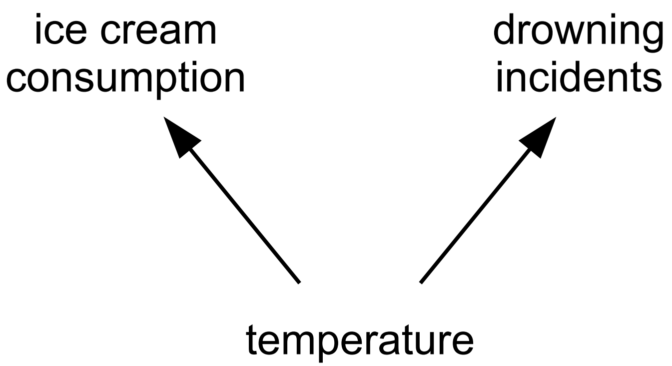
Causal inference: discovering causal relations without interventions



Causal Models: features and phenomena

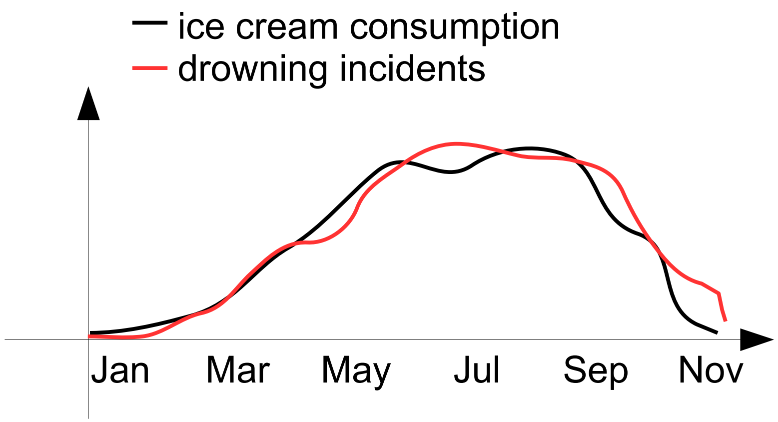
Causal inference: discovering causal relations without interventions

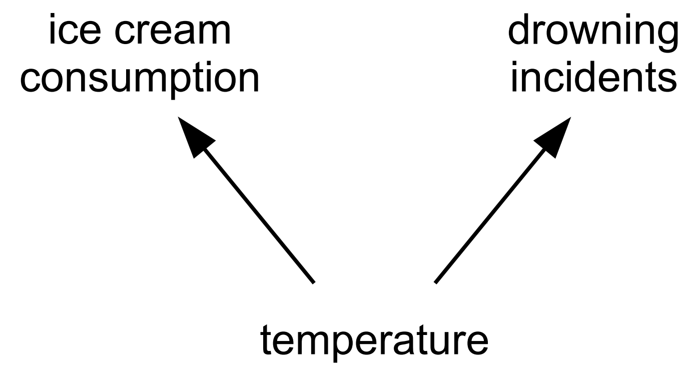


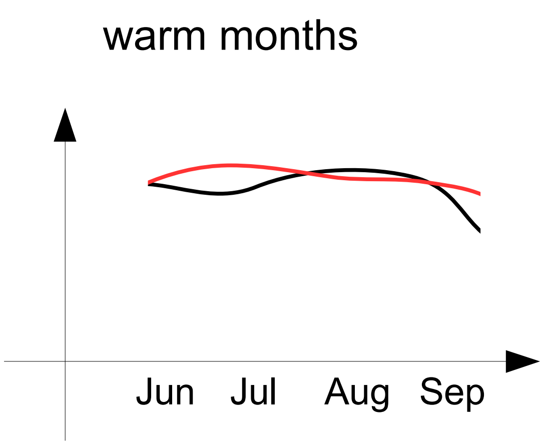


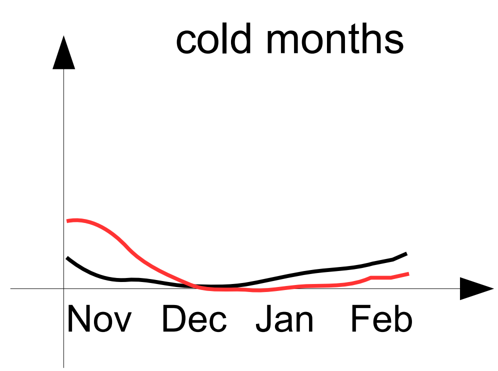
Causal Models: features and phenomena

Causal inference: discovering causal relations without interventions









Conditional independence:
two variables become statistically independent when one conditions on a third

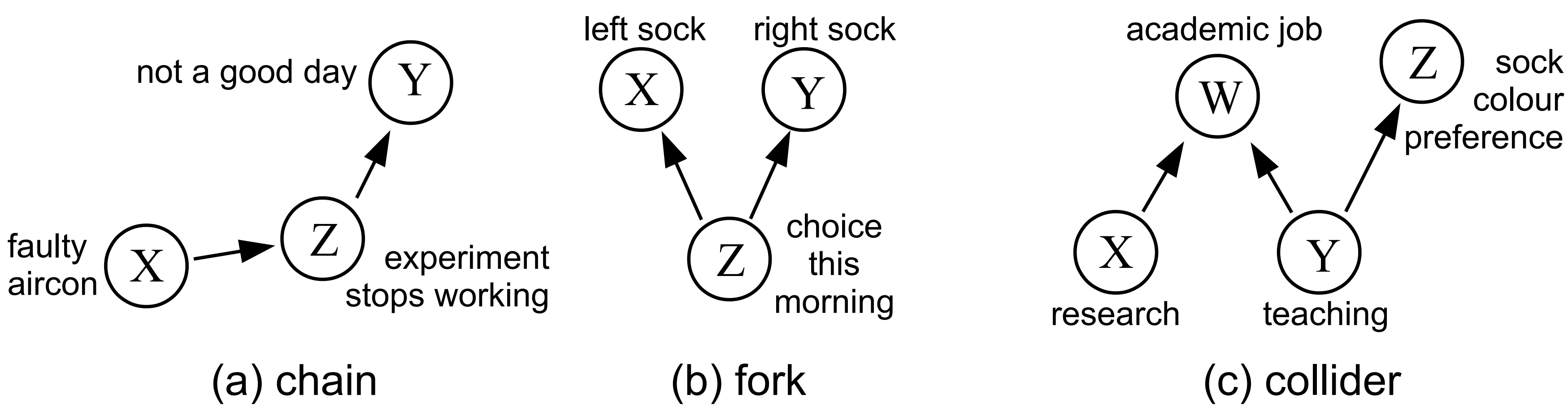
$$I \perp D \mid T$$

„If an improbable coincidence has occurred, there must exist a common cause.“ [Reichenbach]

Causal Models: features and phenomena

Conditional independences can arise from various causal structures.

Structures that lead to $X \perp Y \mid Z$:



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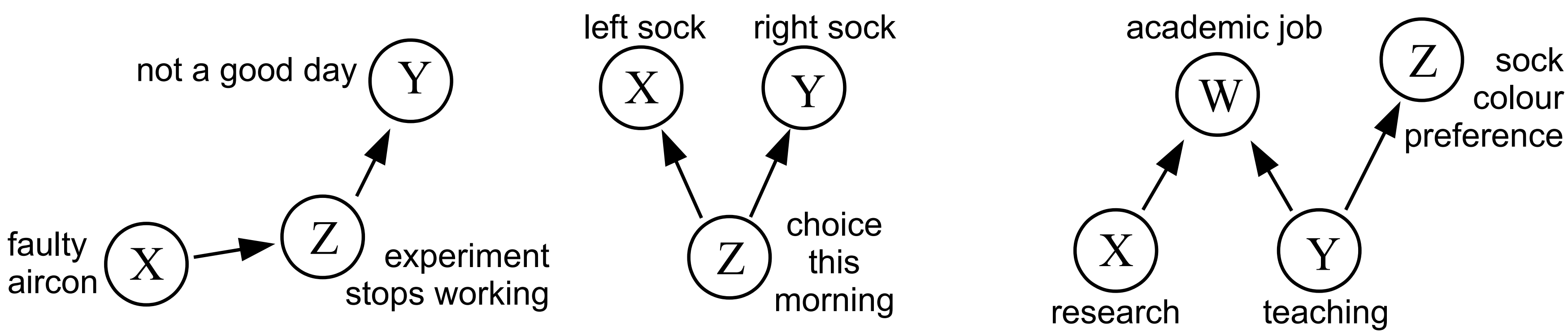
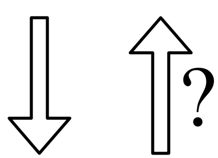


(a) chain (b) fork (c) collider

d-separation
(property of causal structure)



conditional independence
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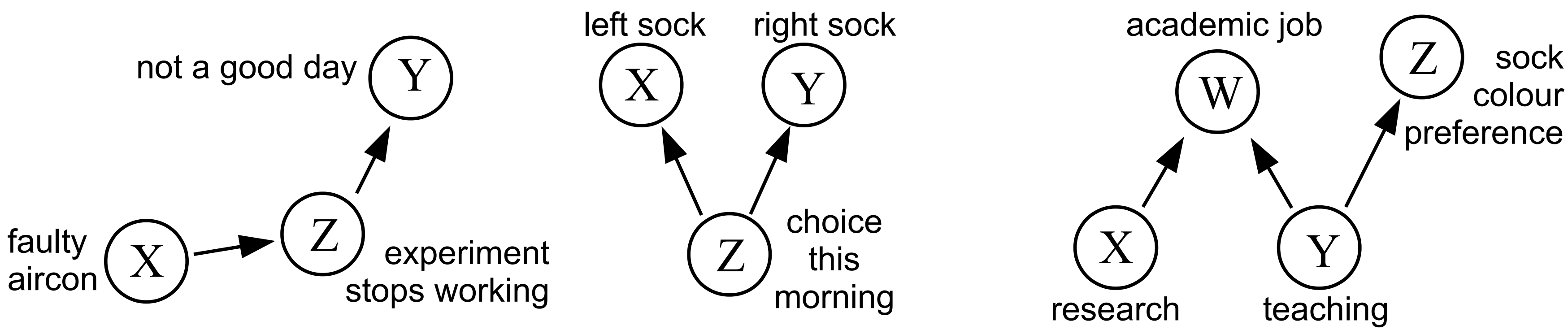

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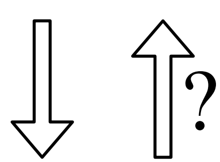
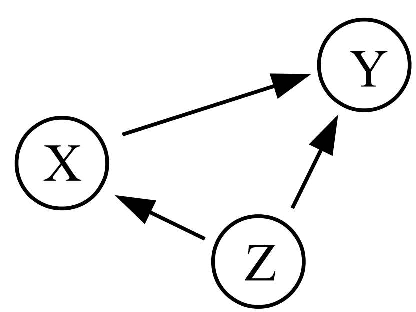


conditional independence
(property of probability distribution)

Conditional independences reveal features of causal structure if we exclude **fine-tuning**.

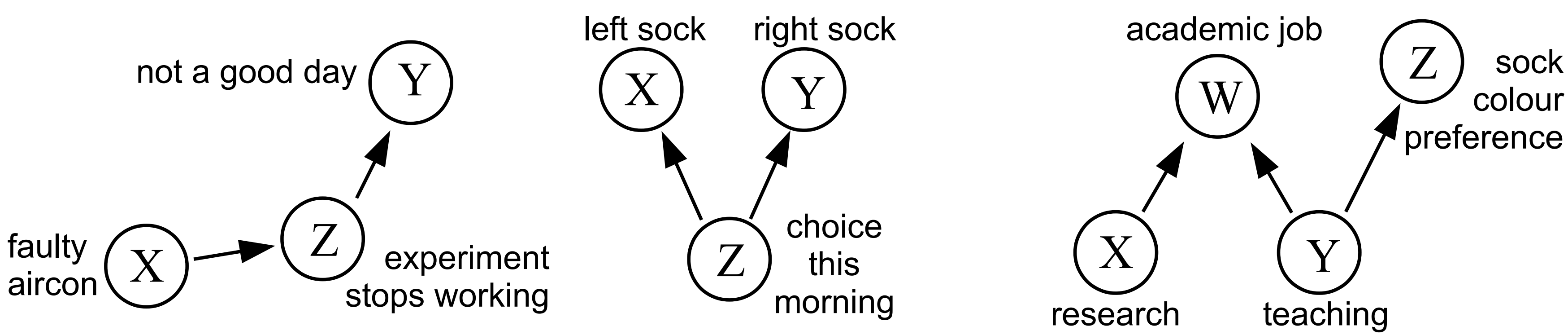


$$x = \alpha z + u_x$$
$$y = \beta z + \gamma x + u_y$$

Causal Models: features and phenomena

Conditional independences can arise from various causal structures.

Structures that lead to $X \perp Y \,|\, Z$:



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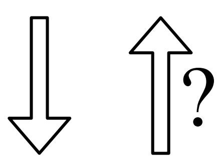

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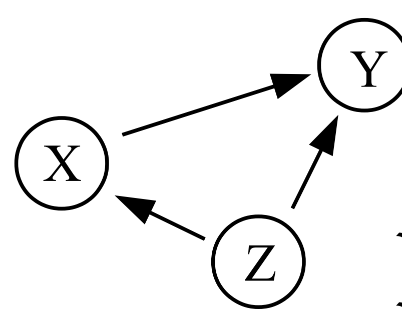


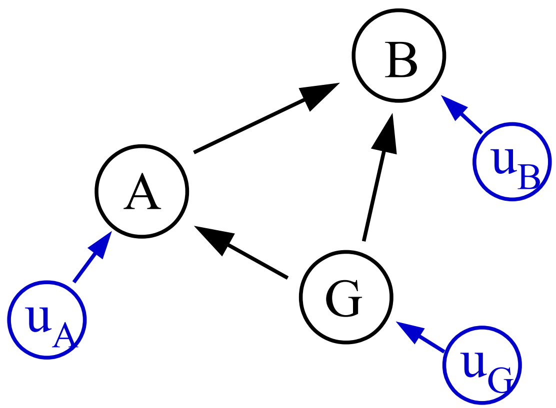
$$x = \alpha z + u_x$$
$$y = \beta z + \gamma x + u_y$$

$$\gamma = 0 \Rightarrow Y \perp X \,|\, Z$$
$$\gamma = -\beta/\alpha \Rightarrow Y \perp (X, Z)$$

Causal Models: features and phenomena

Functional causal models:

alternative description with **deterministic** relations



$$B = f_B(A, G, u_B)$$
$$A = f_A(G, u_A)$$
$$G = f_G(u_G)$$

coarse-graining →

← fine-graining

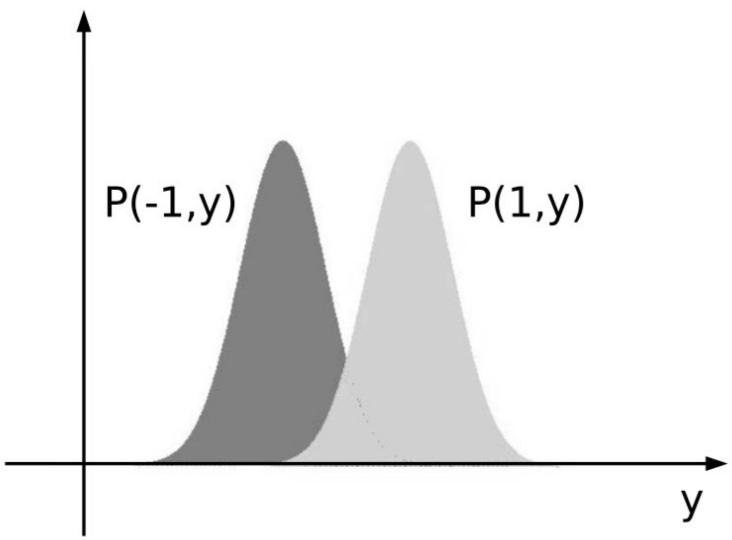
$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

autonomy ↔ independence of noise sources

Causal Models: features and phenomena

Example: using the independence of mechanisms for causal inference

Data: joint probability distribution
over two variables, x={-1,+1} and y



[Example and figure from Janzing and Schölkopf, arXiv:0804.3678]

Causal Models: features and phenomena

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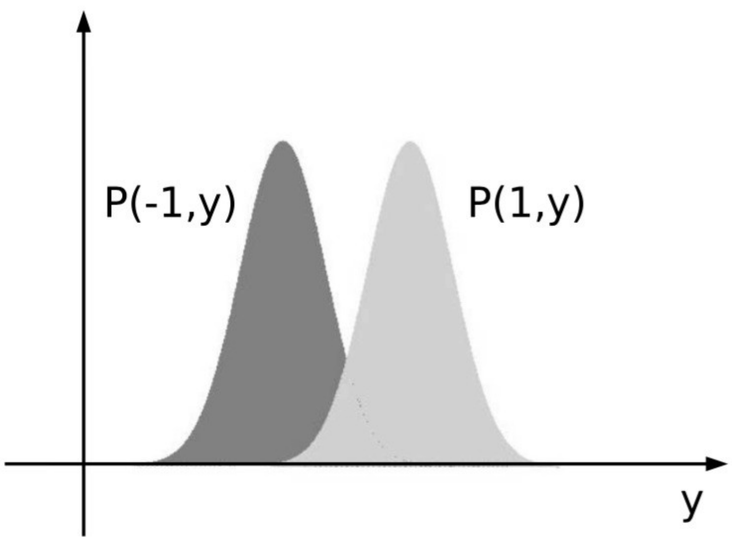


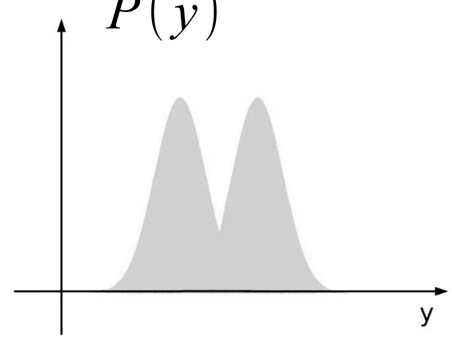
Explanation 1: x → y

$$P(x)=\frac{1}{2}\,\forall\,x$$

$$P(y|x)=\alpha\exp\left(\frac{-(y-x)^2}{\mu}\right)$$

Explanation 2: y → x

$P(y)$



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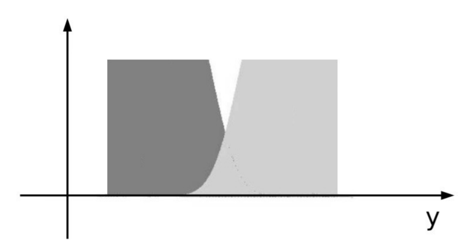

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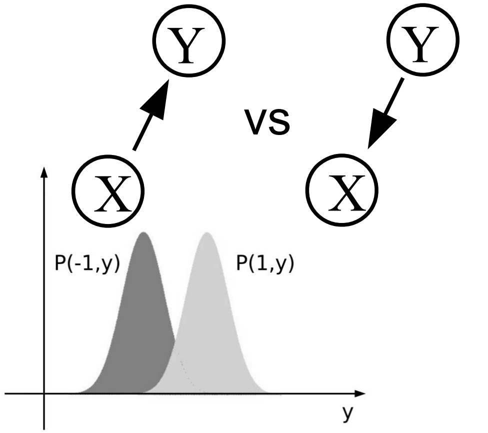
Causal Models: features and phenomena

Time in causal models



„We may define a cause to be an object, *followed by another*, and where all the objects similar to the first are followed by objects similar to the second.“ [Hume]

Causal Models: features and phenomena

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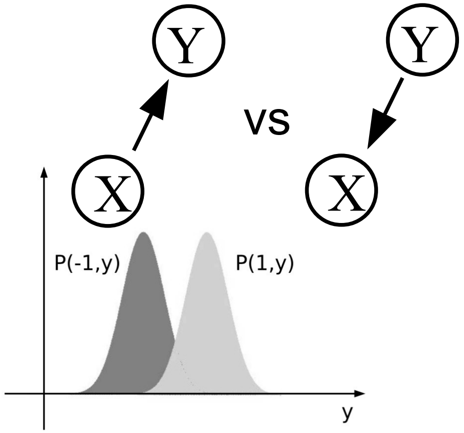

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The class of continuous timelike curves determines the topology of spacetime. [Malament, also Hawking et al]

Causal Models: features and phenomena

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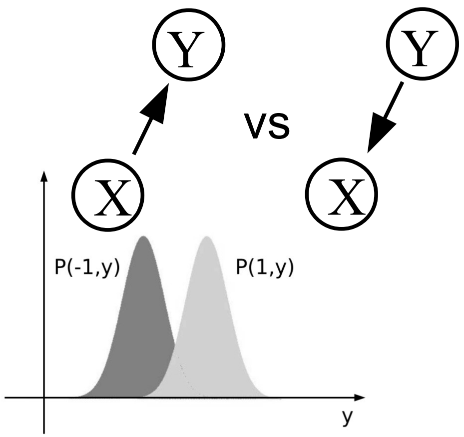

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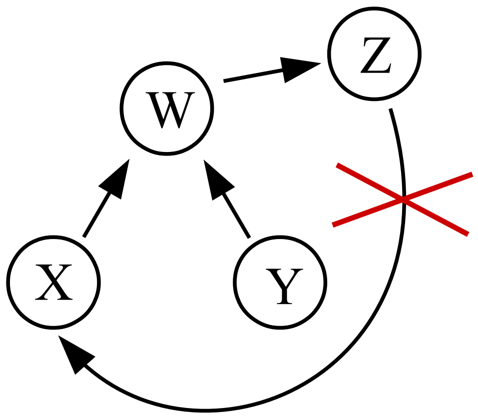
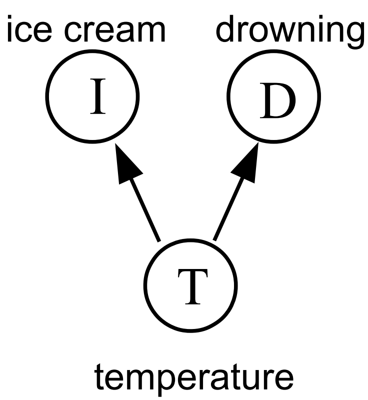


Causal structure is **acyclic**.

Causal Models: features and phenomena

Using causal information: **inference**

Example 1: Given a causal model, derive joint and marginal probability distributions.



$P(T)=\{.6,.4\}$

P(I|T)	I=0	I=1
T=0	.7	.3
T=1	.1	.9

P(D|T)	D=0	D=1
T=0	.9	.1
T=1	.4	.6

Conditional probabilities: $P(X|Y) \geq 0 \qquad \sum_x P(X|Y) = 1 \; \forall \, y$

Causal Models: features and phenomena

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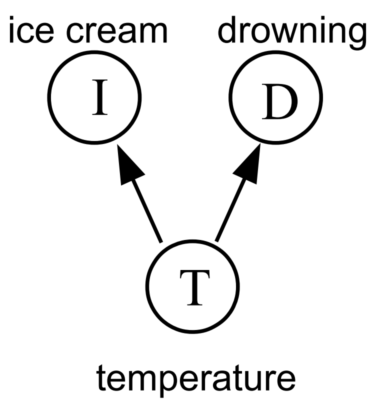

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Conditional probabilities: $P(X|Y)\geq 0$ $\sum_x P(X|Y)=1\,\forall\, y$

joint prob. distrib. P(IDT)

	I=0 D=0	I=0 D=1	I=1 D=0	I=1 D=1
T=0	.378	.042	.162	.018
T=1	.016	.024	.144	.216

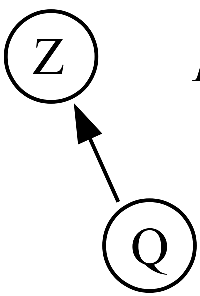
marginal prob. distrib. P(ID)

I=0 D=0	I=0 D=1	I=1 D=0	I=1 D=1
.394	.066	.306	.234

Causal Models: features and phenomena

Using causal information: **inference**

Example 2: Quantum foundations in Zurich



$$P(Z=1|Q)=\begin{cases} 10^{-4} & (Q=0) \\ 10^{-1} & (Q=1) \end{cases}$$

$$P(Q|Z=1)=?$$

Causal Models: features and phenomena

Using causal information: **inference**

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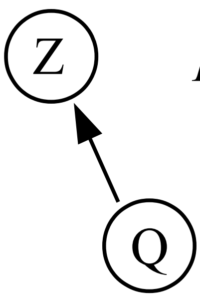

$$P(Z=1|Q)=\begin{cases}10^{-4} & (Q=0)\\ 10^{-1} & (Q=1)\end{cases}$$

$$P(Q|Z=1)=? \qquad P(Q=1)=10^{-7}$$

Causal Models: features and phenomena

Using causal information: **inference**

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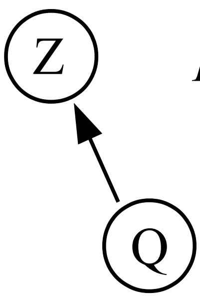


$$P(Z=1|Q)=\begin{cases}10^{-4} & (Q=0)\\ 10^{-1} & (Q=1)\end{cases}$$

$$P(Q|Z=1)=? \qquad P(Q=1)=10^{-7}$$

retrodiction: inference about the causal past

Bayesian inversion:

$$P(Z,Q)=P(Z|Q)\,P(Q)$$

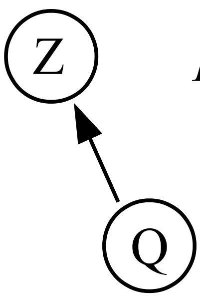
$$\rightarrow \quad P(Q|Z)=\frac{P(Z,Q)}{P(Z)}$$

P(Q|Z)	Q=0	Q=1
Z=0	1-10^{-7}	10^{-7}
Z=1	1-10^{-4}	10^{-4}

Causal Models: features and phenomena

Using causal information: **inference**

Example 2: Quantum foundations in Zurich



$$P(Z=1|Q)=\begin{cases}10^{-4} & (Q=0)\\ 10^{-1} & (Q=1)\end{cases}$$

$$P(Q|Z=1)=? \qquad P(Q=1)=10^{-7}$$

retrodiction: inference about the causal past

Bayesian inversion:

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$$\rightarrow \quad P(Q|Z)=\frac{P(Z,Q)}{P(Z)}$$

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causal vs **inferential conditionals**

- same mathematical form
- different epistemological significance

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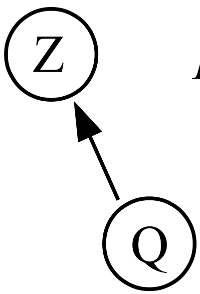

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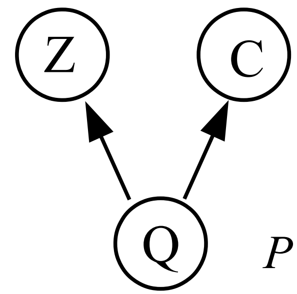
Example 3: Correlation with coffee



$$P(C=1|Q)=\begin{cases}.5 & (Q=0)\\ .8 & (Q=1)\end{cases}$$

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inference via a common cause

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Using causal information: **inference**

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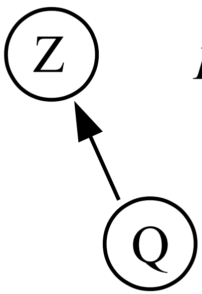

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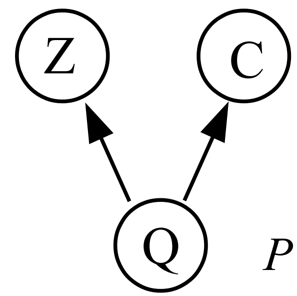

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inference via a common cause

$$\begin{aligned}P(C|Z)&=\sum_q P(C|Q=q)\,P(Q=q|Z)\\ &=\sum_q P(C,Q=q|Z)\end{aligned}$$

Note

- two-step process:

  joint $P(CQ|Z)$, then marginalize
- same mathematical form as before

Highlights: classical causal models

- definition of causation based on **interventions**
- formal consequence: **splitting** of variables
- causation vs **inference**: mathematically similar but conceptually distinct

Some essential features:

- **autonomy** of causal mechanisms

  → no **fine-tuning**: conditional independences reflect features of causal structure
- admit an account in terms of underlying **deterministic** mechanisms
- causal order: **acyclic**, aligned with temporal order

Quantum Indeterminism

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

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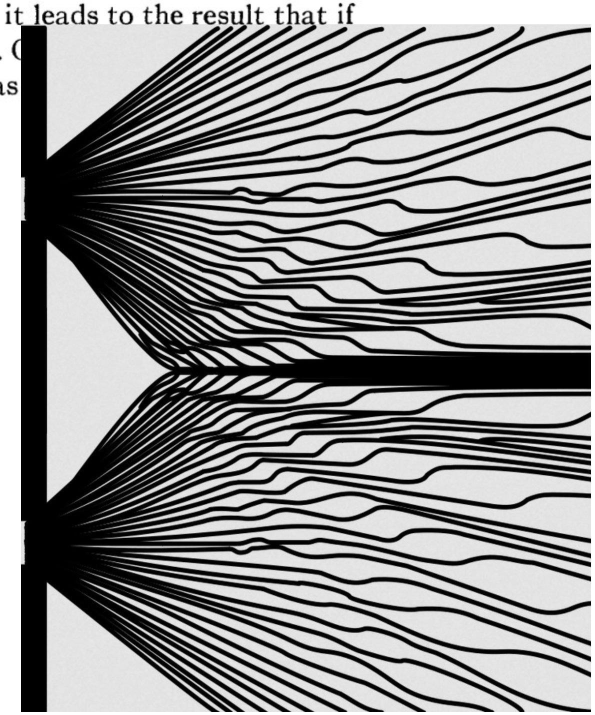

Bohmian trajectories in
a double-slit experiment

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- open question
- hidden variable models are possible
  – if one gives up other assumptions
- exploiting quantum indeterminism:

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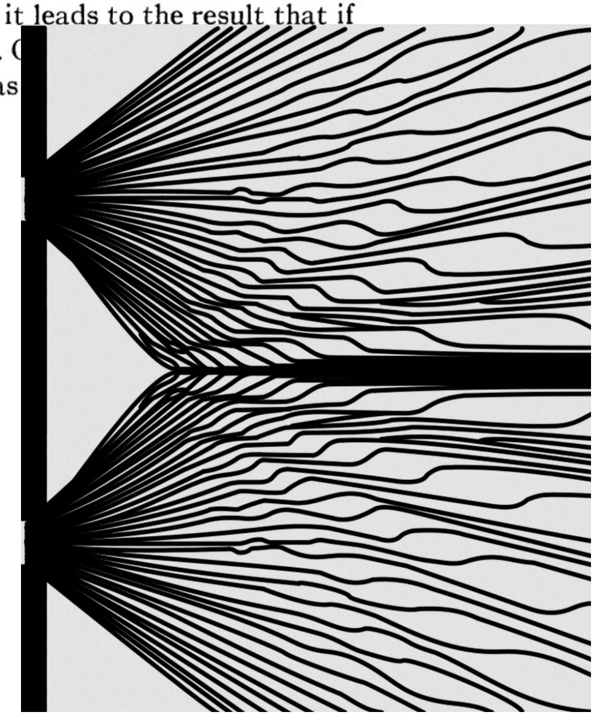
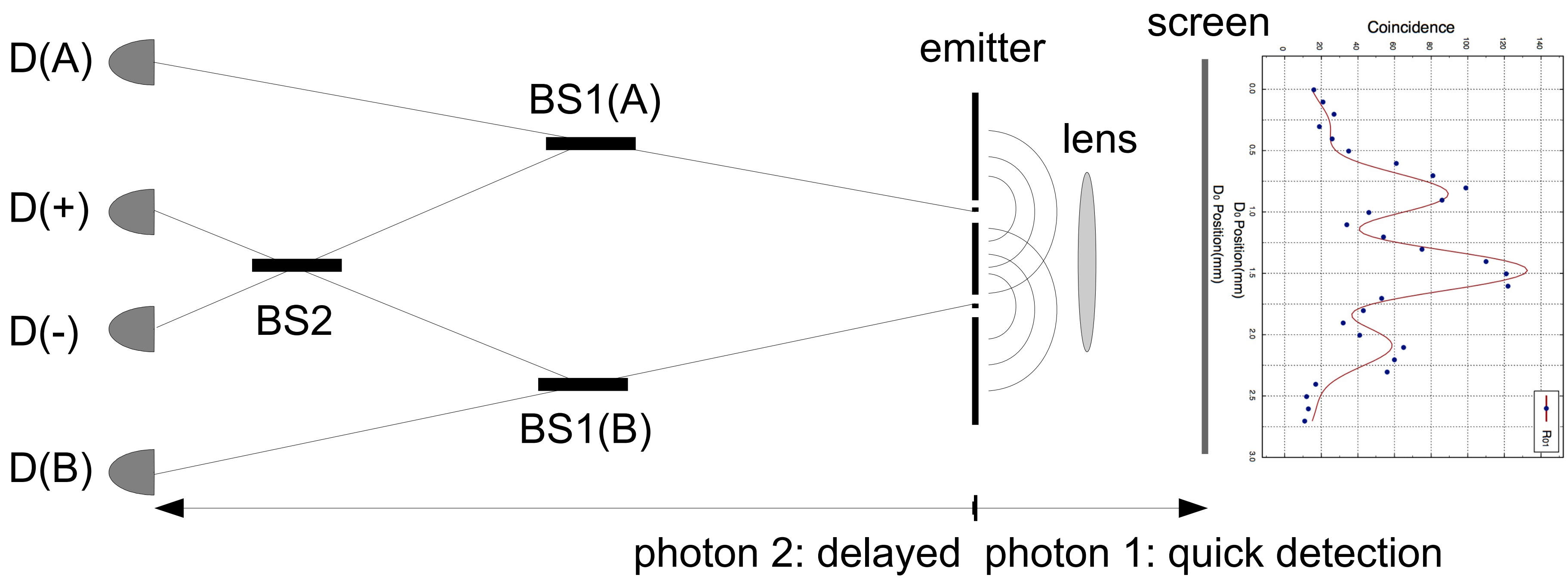
IDQ FROM VISION TO TECHNOLOGY



Bohmian trajectories in a double-slit experiment

Retrocausality and Causal Loops

The delayed-choice quantum eraser



[Kim et al, PRL 84, 1 (2000)]

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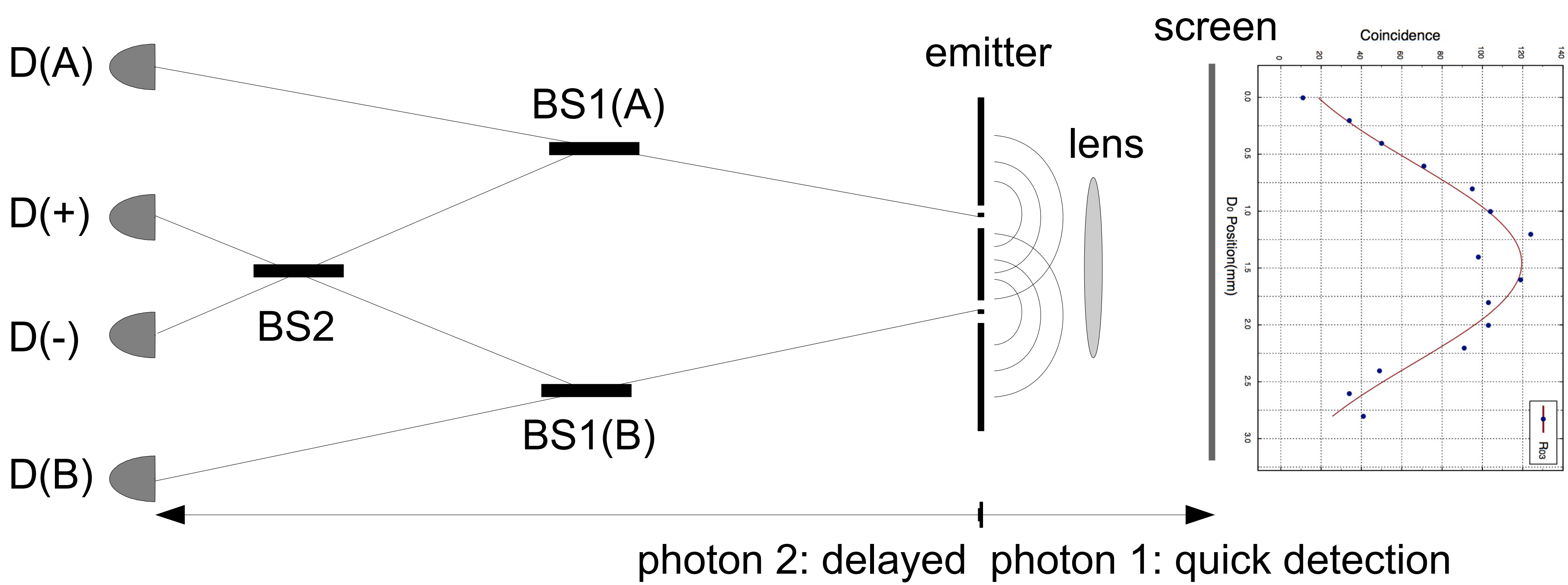

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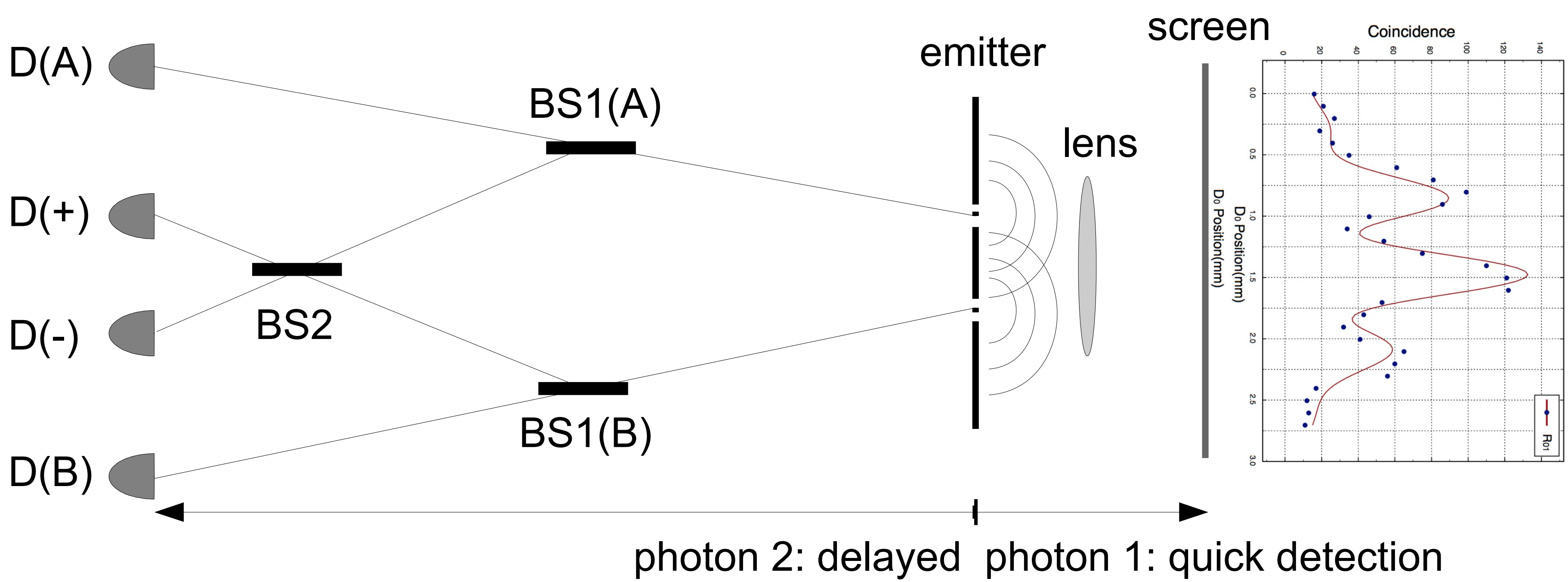


[Kim et al, PRL 84, 1 (2000)]

$$\frac{1}{\sqrt{2}}\Big[|A\rangle + e^{i\phi}|B\rangle\Big] \xrightarrow{BS1} \frac{1}{2}\Big[\big(|A\rangle+|A'\rangle\big)+e^{i\phi}\big(|B\rangle+|B'\rangle\big)\Big]$$

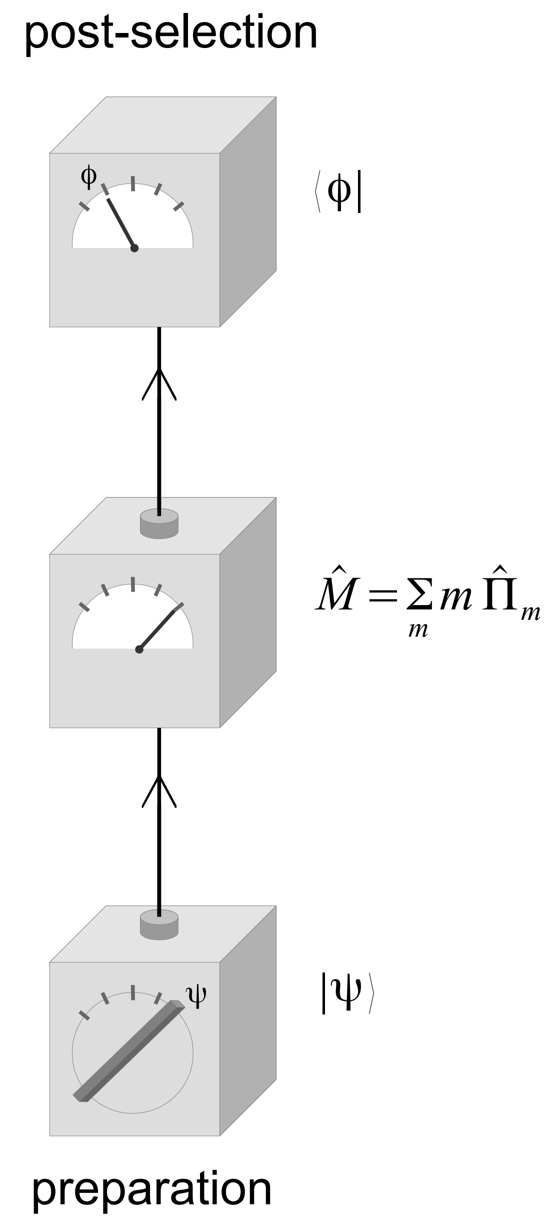
$$\xrightarrow{BS2} \frac{1}{2}\Big[|A\rangle+e^{i\phi}|B\rangle+\frac{1}{\sqrt{2}}\left(1+e^{i\phi}\right)|+\rangle+\frac{1}{\sqrt{2}}\left(1-e^{i\phi}\right)|-\rangle\Big]$$

probability of learning (i) path: $P(A)+P(B)=\frac{1}{2}$; (ii) phase: $P(+)+P(-)=\frac{1}{2}$

One cannot control which information one acquires, only post-select.

The two-state vector formalism

The two-state vector formalism (TSVF) [1] is a time-symmetric description of the standard quantum mechanics originated in Aharonov, Bergmann and Lebowitz [2]. The TSVF describes a quantum system at a particular time by two quantum states: the usual one, evolving forward in time, defined by the results of a complete measurement at the earlier time, and by the quantum state evolving backward in time, defined by the results of a complete measurement at a later time.



conventional quantum mechanics:

$$P(m|\psi) = \left|\hat{\Pi}_m|\psi\rangle\right|^2 \quad , \qquad P(\phi, m|\psi) = \left|\langle\phi|\hat{\Pi}_m|\psi\rangle\right|^2$$

$$\Rightarrow \quad P(m|\psi,\phi) = \frac{\left|\langle\phi|\hat{\Pi}_m|\psi\rangle\right|^2}{\sum_m \left|\langle\phi|\hat{\Pi}_m|\psi\rangle\right|^2}$$

TSVF: backward in time forward in time

[Vaidman, arXiv:0706.1347]

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TSVF: backward in time forward in time

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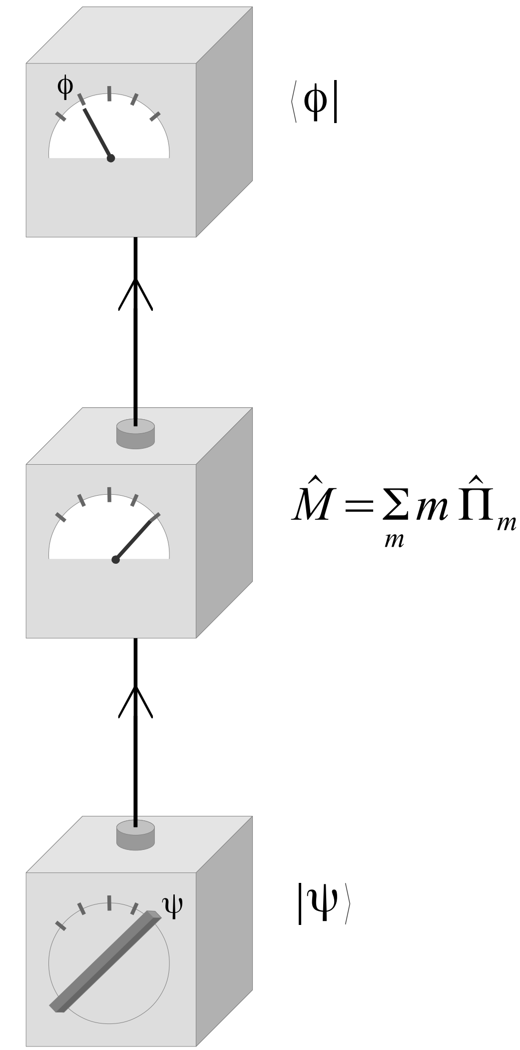
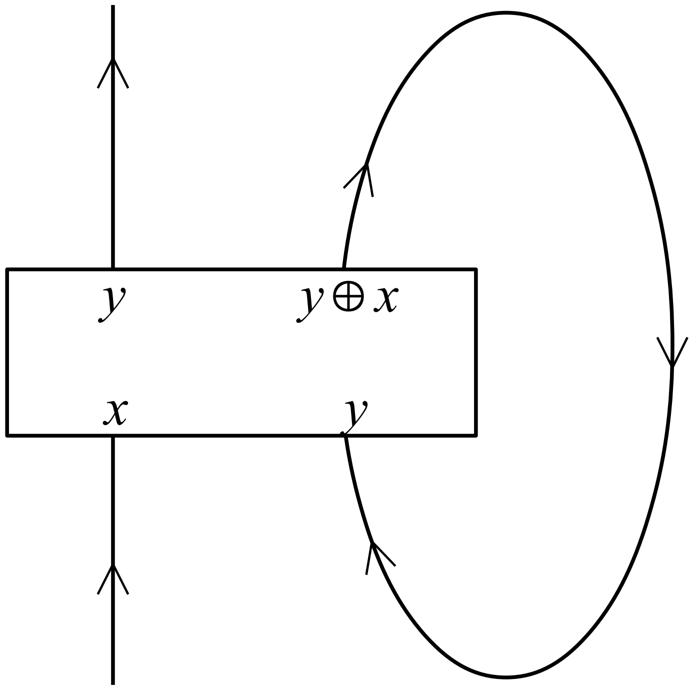


preparation

„We cannot, however, create with certainty a particular backward evolving quantum state, (…) The difference follows from the time asymmetry of the memory arrow of time.“

[Vaidman, arXiv:0706.1347]

Closed timelike curves



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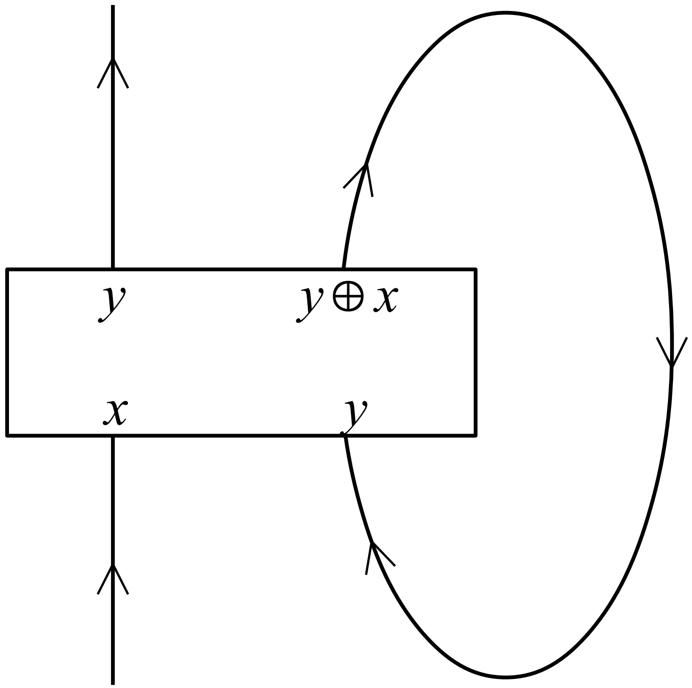
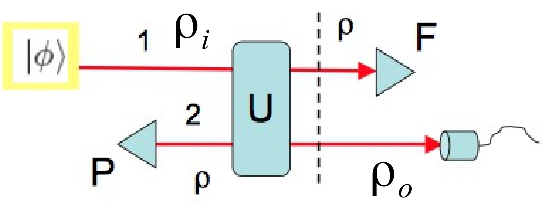


consistency condition:

$$y \oplus x = y \;\; \Rightarrow \;\; x = 0$$

Closed timelike curves

Quantum mechanics avoids „'paradoxical' constraints on the past".



[Ralph&Myers]

Consistency conditions:

$$\rho = Tr_2\left[U(\rho_i \otimes \rho)U^\dagger\right]$$

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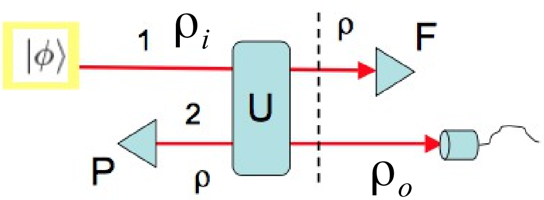

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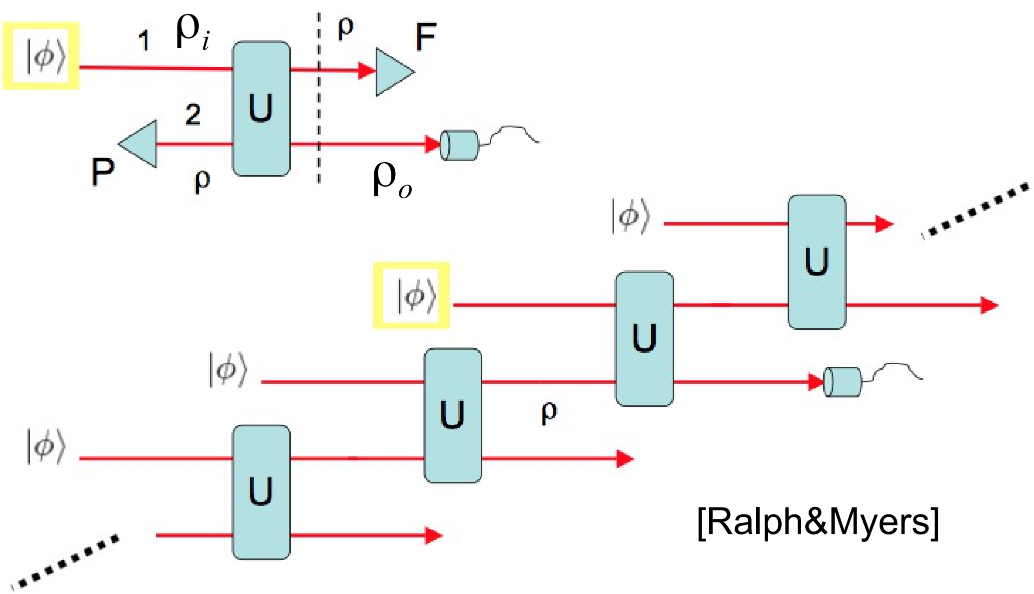

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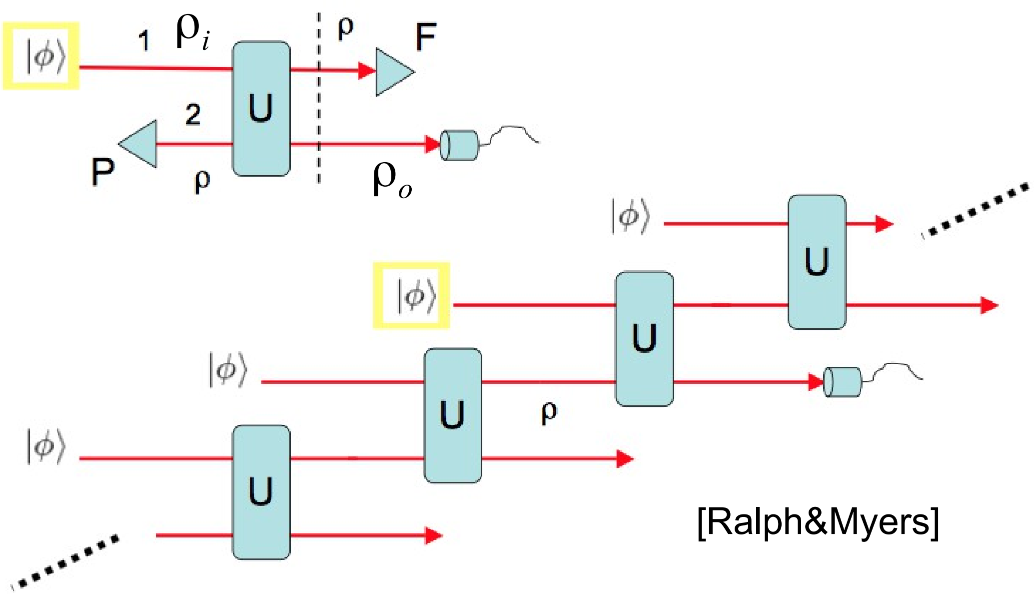

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„Qubit's view“ assumes infinitely many copies of ρ_i, leading to

- non-linear transformations
- perfect state discrimination and cloning
- inequivalence of probabilistic mixtures, breaking entanglement
- instant computation: Pspace [Aaronson&Watrous]
- ...

Nonlocality

Quantum Field Theory

Consider a classical, free, real scalar field $\phi(\vec{x}, t)$.

<u>non-relativistic case</u>: Schrödinger field

$$i\frac{\partial}{\partial t}\phi = -\frac{\nabla^2}{2m}\phi \quad \Rightarrow \quad \phi(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} a(\vec{k}) e^{i\vec{k}\cdot\vec{x}} e^{-ik_0 t} \quad , \quad k_0 = \frac{|\vec{k}|^2}{2m}$$

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$\vec{k}$ indexes independent modes (harmonic oscillators):

$$\mathscr{H} = \frac{1}{2m}\int d^3x\, \vec{\nabla}\phi^* \cdot \vec{\nabla}\phi = \int \frac{d^3k}{(2\pi)^3}\, a^*(\vec{k})\, a(\vec{k})$$

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promote $a(\vec{k})$ to annihilation operators, $\phi(\vec{x}, t)$ to field (annihilation) operator:

$$\hat{\phi}^\dagger(\vec{x},t)|0\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} e^{-ik_0 t}\, \hat{a}^\dagger(\vec{k})|0\rangle \qquad \text{single localized excitation}$$

Note: $\hat{\phi}(\vec{x}, t)$ is an observable.

Quantum Field Theory

Consider a classical, free, real scalar field $\phi(\vec{x}, t)$.

<u>relativistic case</u>: Klein-Gordon field

$$\frac{\partial^2}{\partial^2 t}\phi = (\nabla^2 - m^2)\phi \quad \Rightarrow \quad k_0^2 = |\vec{k}|^2 + m^2$$

$$\phi(\vec{x}, t) = \int \frac{d^3 k}{2\mathrm{k}_0 (2\pi)^3}\left[a(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} e^{-i|k_0|t} + a^*(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}} e^{+i|k_0|t}\right]$$

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$$\Rightarrow \begin{cases} \text{creates localized excitations: } \hat{\phi}^\dagger(\vec{x}, t)|0\rangle \\ \text{is an observable: } \hat{\phi}^\dagger(\vec{x}, t) = \hat{\phi}(\vec{x}, t) \end{cases}$$

Quantum Field Theory

propagator: probability amplitude of propagation between x and x'

$$D(x-x') = \langle 0|\hat{\phi}(x)\hat{\phi}^{\dagger}(x')|0\rangle = \int \frac{d^3k}{2\mathrm{k}_0(2\pi)^3} e^{-ik(x-x')} \langle 0|\hat{a}(\vec{k})\hat{a}^{\dagger}(\vec{k})|0\rangle$$

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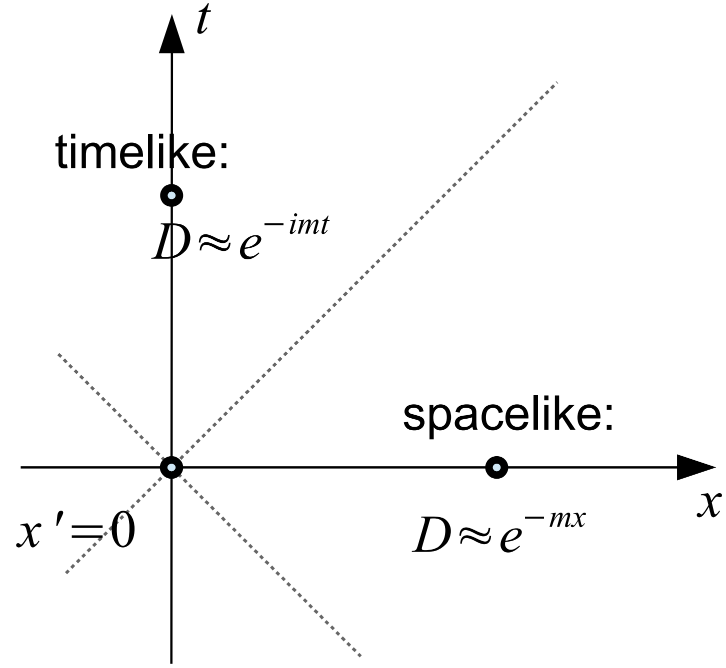

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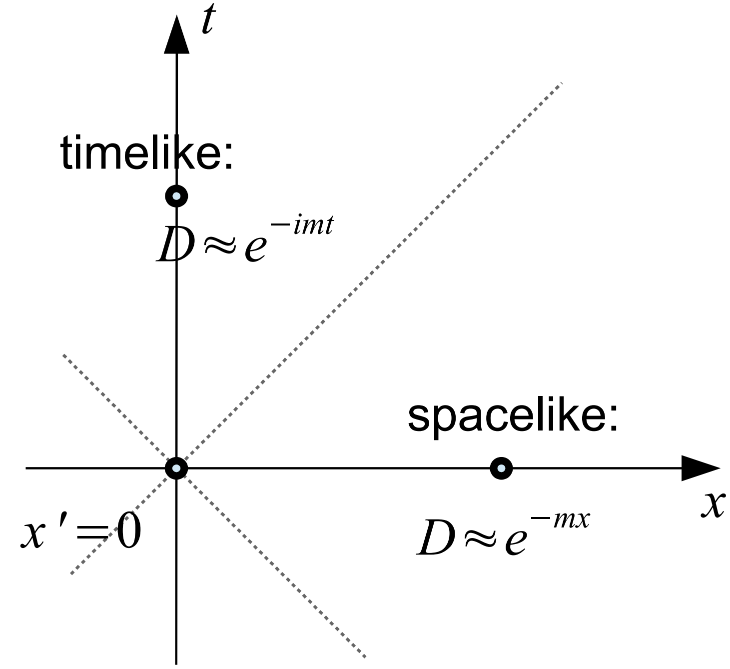
One can:

- passively observe **two-point correlations**

  between spacelike separations



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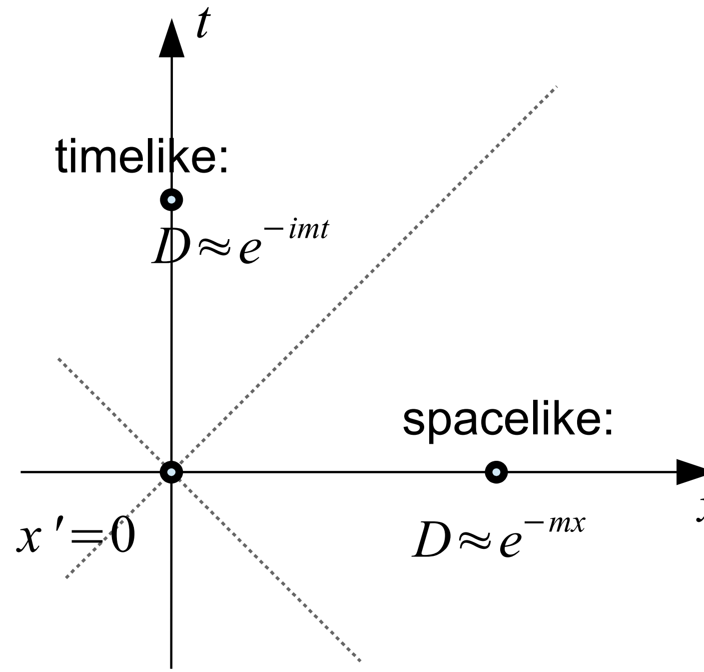


One can:

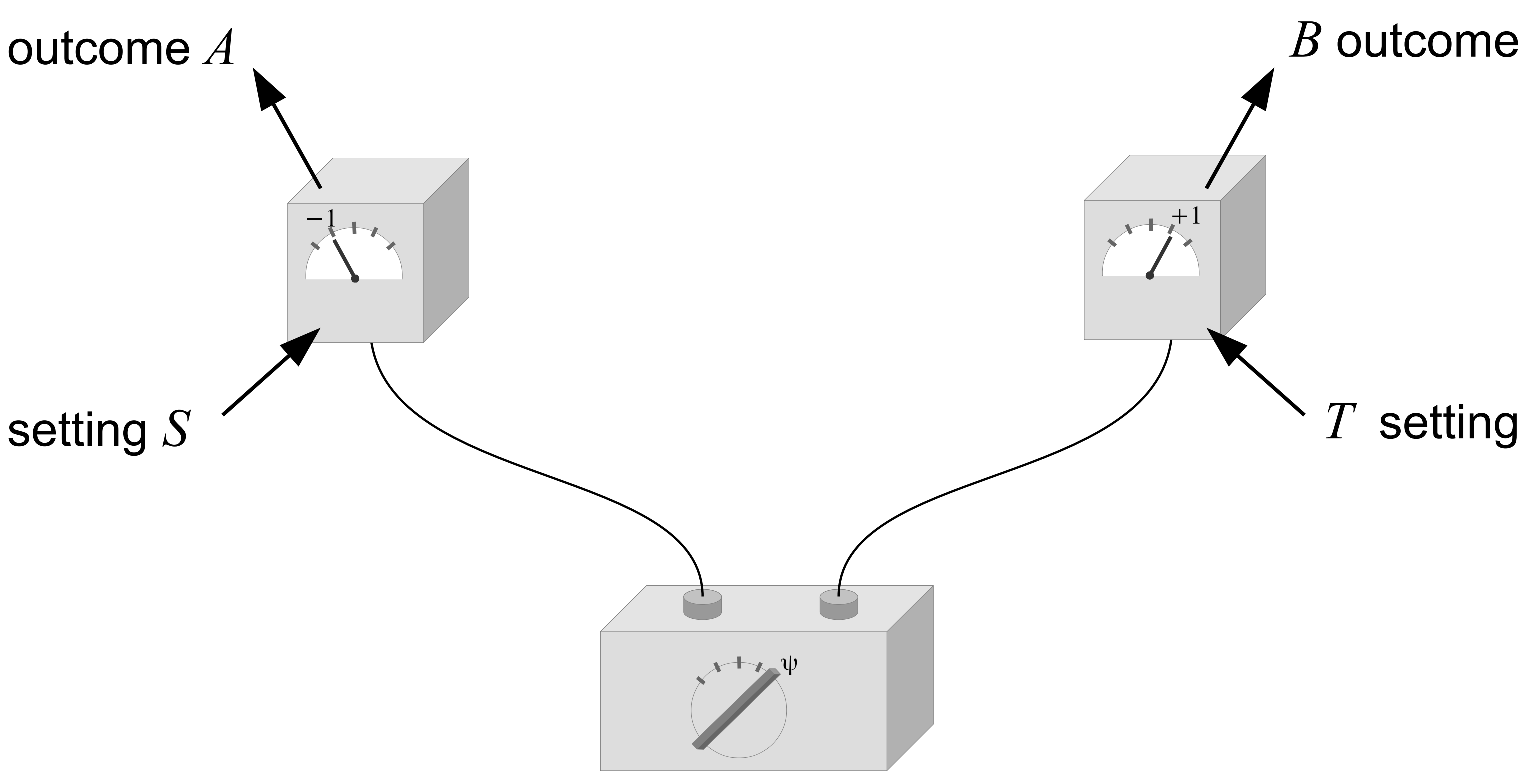
- passively observe **two-point correlations**
  between spacelike separations

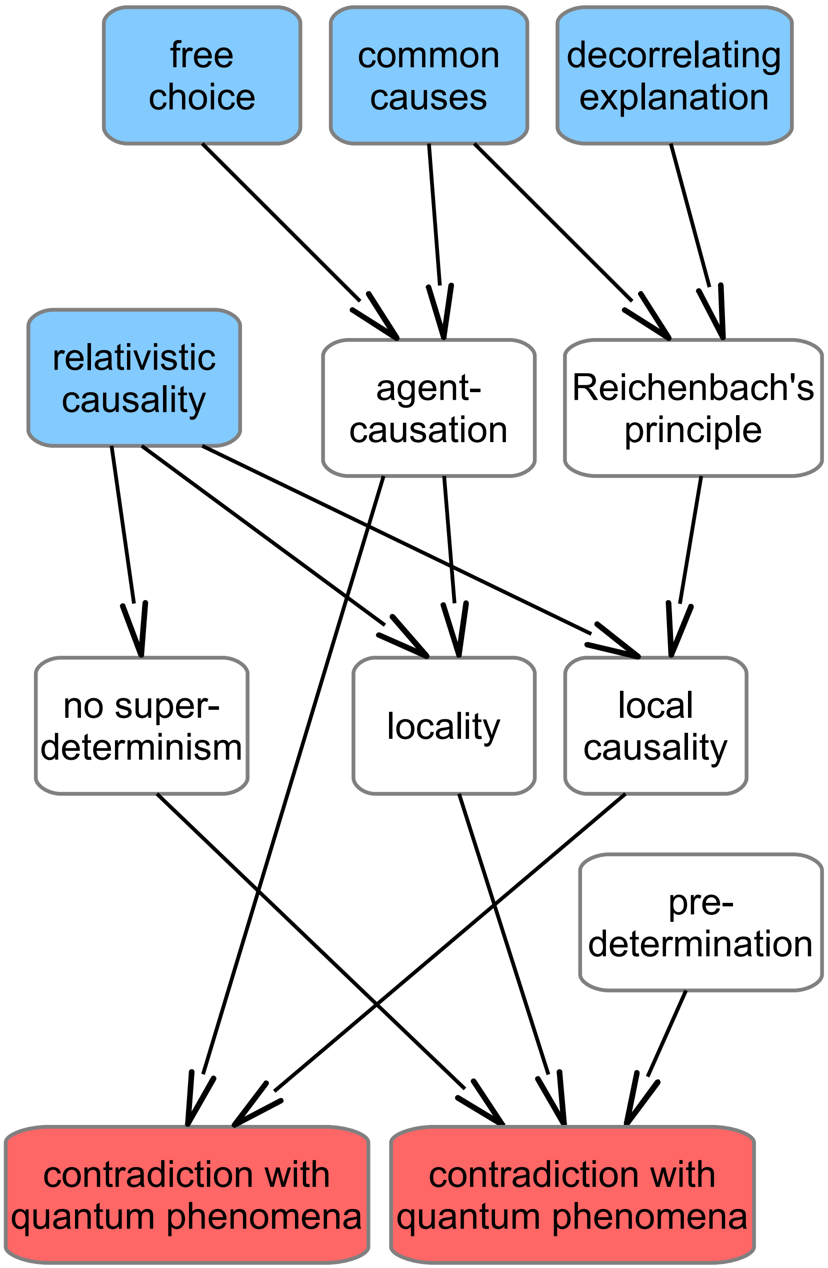
- signal by measuring the field observable:

$$\left[\langle 0|\hat{\phi}(x')\right]\hat{\phi}(x)\left[\hat{\phi}^{\dagger}(x')|0\rangle\right] - \langle 0|\hat{\phi}(x)|0\rangle$$

  depends on the **commutator** $\left[\hat{\phi}(x), \hat{\phi}^{\dagger}(x')\right]$

Bell Inequality Violations





[Wiseman&Cavalcanti]

Postulates

1. *Free choice*: A freely chosen action has no relevant *causes*.
   (Any *cause* of an event is in its *past*.)
2. *Relativistic causality*: The *past* is the past light-cone.
3. *Common causes*: If two events are correlated and neither is a *cause* of the other, then they have a common *cause* that *explains* the correlation.
4. *Decorrelating explantion*: A common *cause* C *explains* a correlation only if conditioning on C eliminates the correlation.

Consequences

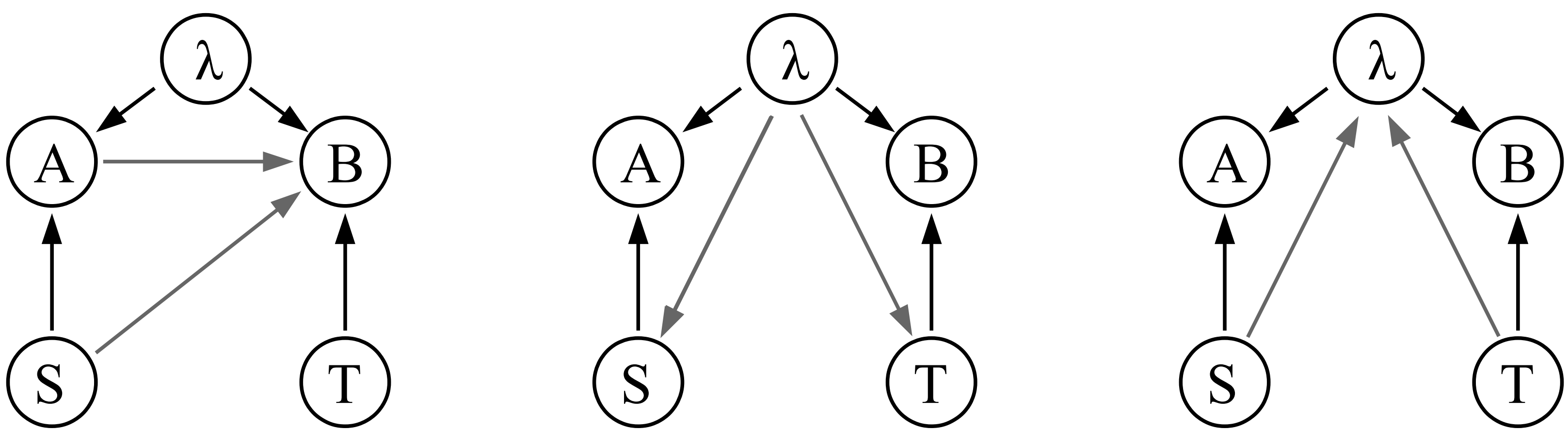
1. *Agent-causation*: If a relevant event A is correlated with a freely chosen action, then that action is a *cause* of A.
2. *Reichenbach*: If two events are correlated, and neither is a *cause* of the other, then they have a common *cause* C, such that conditioning on C eliminates the correlation.
3. *Local causality*: If two space-like separated events A and B are correlated, then there is a set of events C in their common Minkowski past such that conditioning on C eliminates the correlation.
4. *No superdeterminism*: All events on a space-like hypersurface are uncorrelated with freely chosen actions subsequent to that SLH.
5. *Locality*: The probability of an observable event A is unchanged by conditioning on a space-like-separated free choice b, even if it is already conditioned on other events not in the future light-cone of b.
6. *Local causality*: If two space-like separated events are correlated, then there is a set of events C in their common Minkowski past such that conditioning on C eliminates the correlation.

What can causal inference tell us about Bell experiments?

Inputs: conditional independences

- between settings: $S \perp T$
- no signalling: $A \perp T | S$, $B \perp S | T$

Some proposed causal structures:



superluminal influences superdeterminism retrocausality

[Wood&Spekkens]

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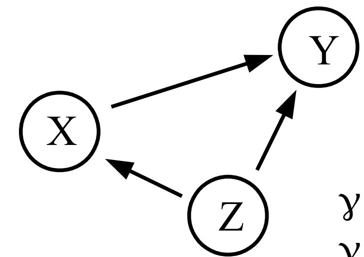
- no signalling: $A \perp T | S \;,\;\; B \perp S | T$

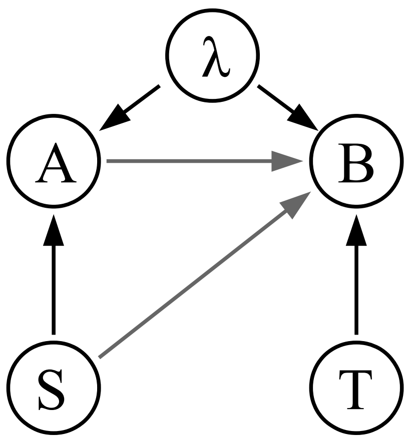
Conditional independences reveal features of causal structure if we exclude **fine-tuning**.



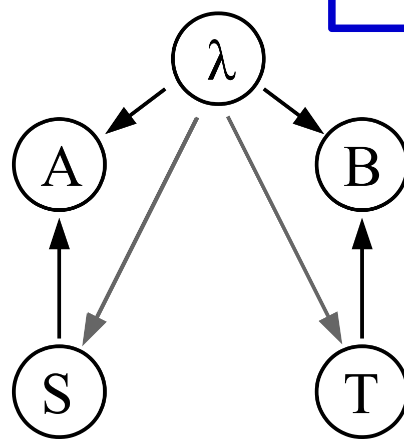
$$x = \alpha z + u_x$$
$$y = \beta z + \gamma x + u_y$$

$$\gamma = 0 \Rightarrow Y \perp X \,|Z$$
$$\gamma = -\beta / \alpha \Rightarrow Y \perp (X, Z)$$

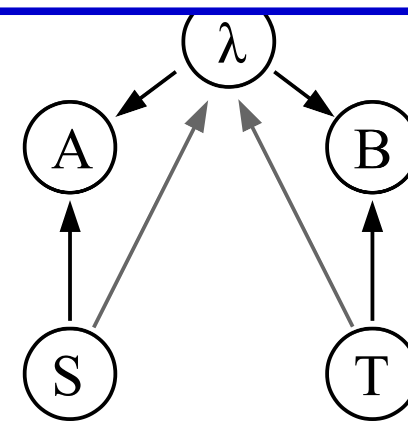
Some proposed causal structures:



superluminal influences



superdeterminism



retrocausality

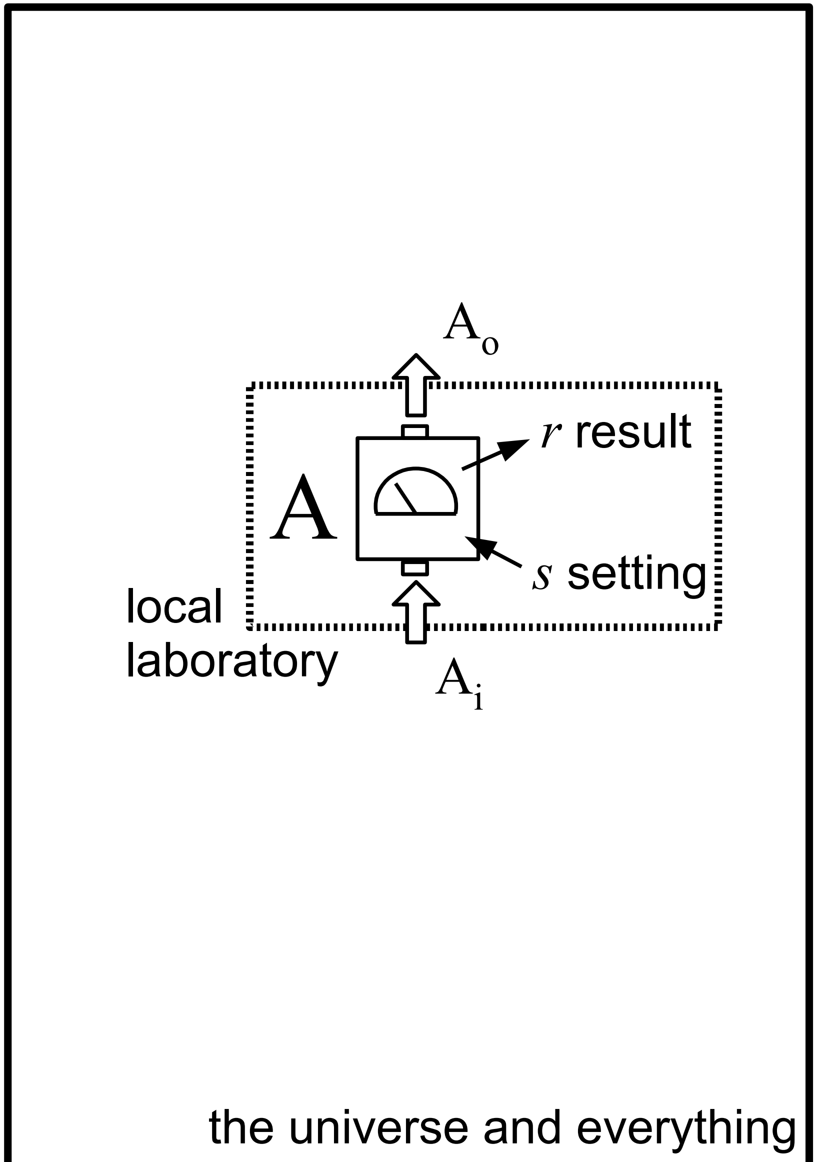
⇒ Classical causal models cannot explain Bell inequality violations because this would require fine-tuning.

[Wood&Spekkens]

Quantum Causal Models

How to describe causal relations between quantum systems?

Ansatz I: via mathematical formalism



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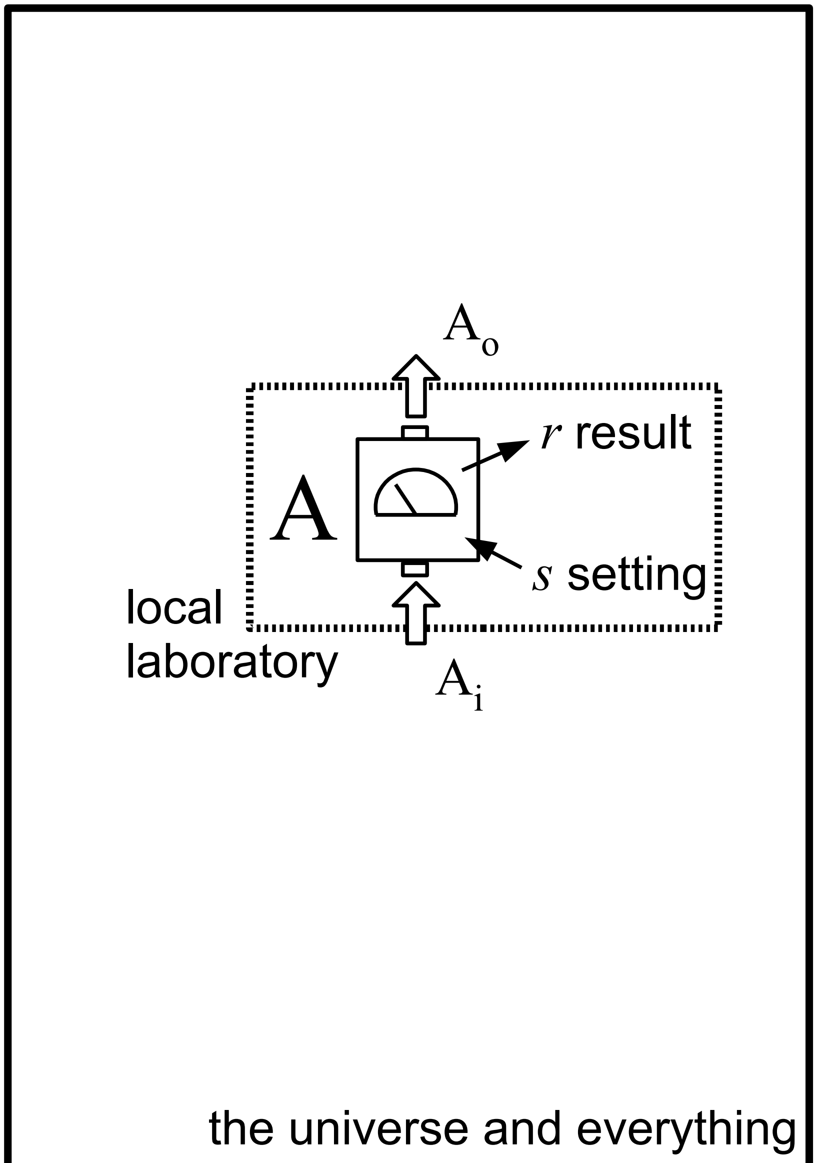


Probing is described by a *quantum instrument*:

- map from input to output states

$$\mathscr{M}^{sr} : \mathscr{L}(\mathscr{H}_i) \rightarrow \mathscr{L}(\mathscr{H}_o)$$

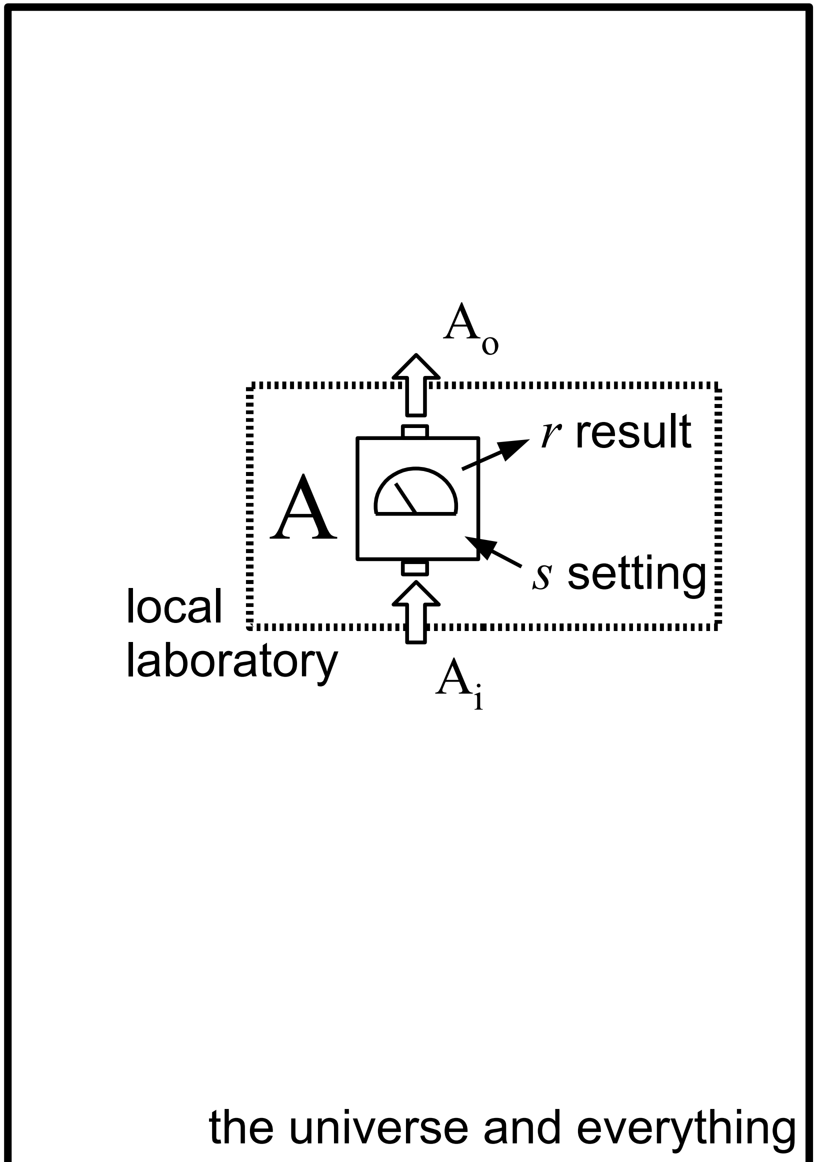
- completely positive

$$(\mathscr{M}^{sr} \otimes \mathscr{I}_B)(\rho_{A_i B}) \geq 0 \quad \forall \rho, s, r$$

- sum over results trace-preserving

$$Tr\left[\sum_r \mathscr{M}^{sr}(\rho)\right] = 1 \quad \forall \rho, s$$

Ansatz I: via mathematical formalism



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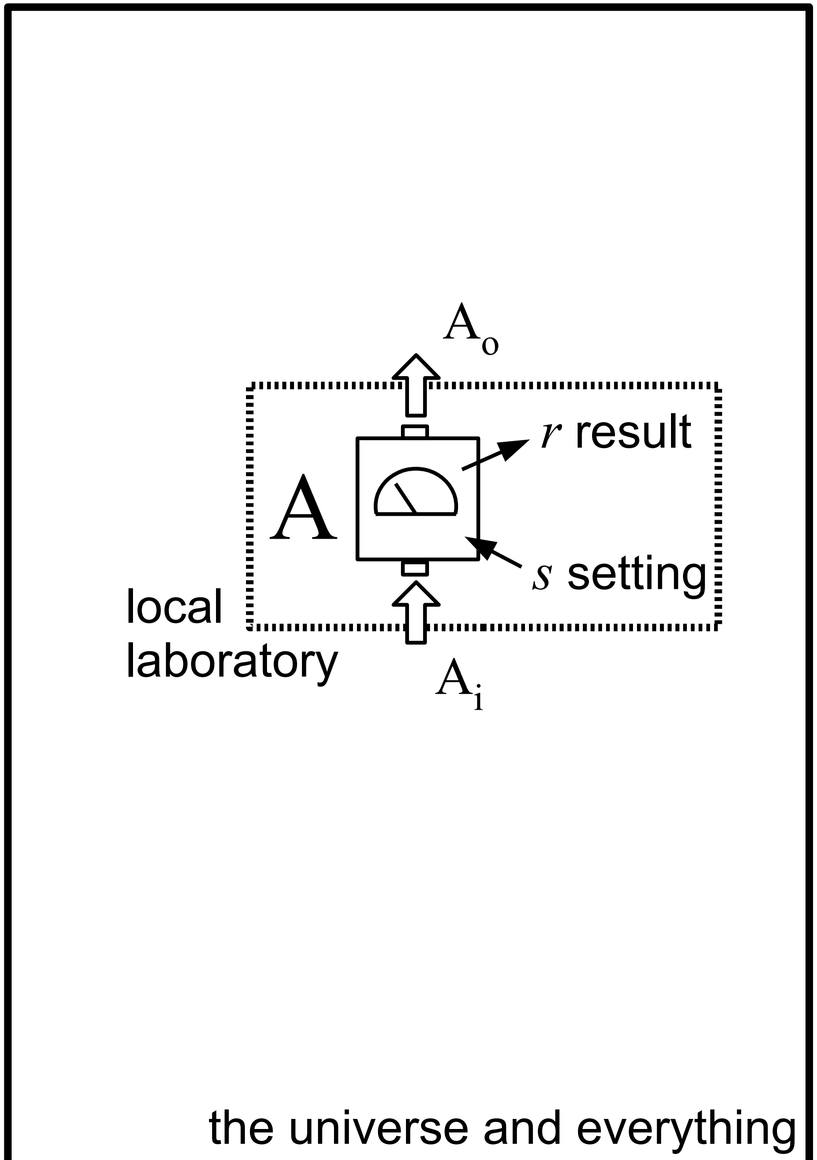
Example 1: preparing an ensemble $\{\rho^s\}_s$

$$\mathscr{M}^{sr} = \rho_o^s \otimes Tr_i \qquad (r \text{ fixed})$$

Example 2: projective measurement $\{\Pi^r\}_r$

$$\mathscr{M}^{sr}(\rho) = Tr_i(\Pi^r \rho) \qquad (s \text{ fixed})$$

Ansatz I: via mathematical formalism



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Equivalent representation: Choi operator

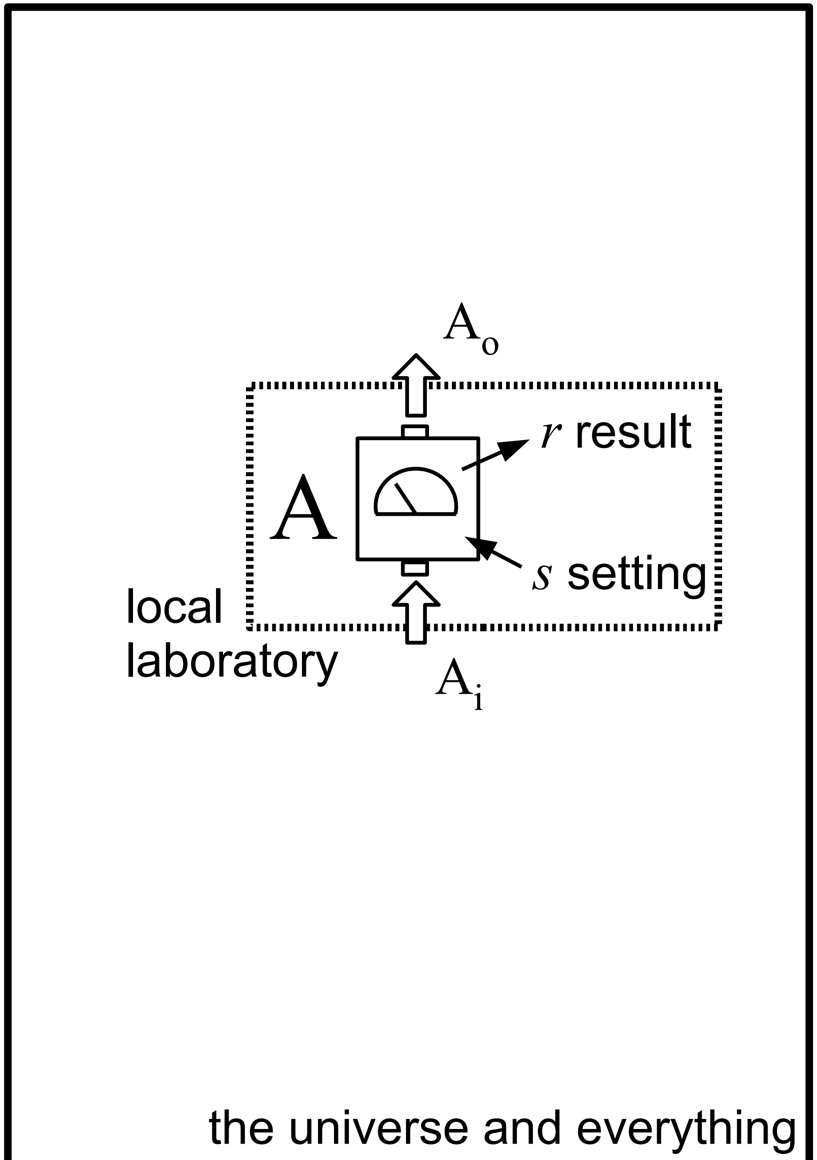
$$M^{sr} \in \mathcal{L}(\mathcal{H}_i \otimes \mathcal{H}_o):$$

$$\mathcal{M}^{sr}(\rho) = Tr_i\left[M^{sr}_{io} \rho_i^T\right] = \tilde{\rho}_o^{sr}$$

transpose

[Choi, Jamiolkowski]

Ansatz I: via mathematical formalism



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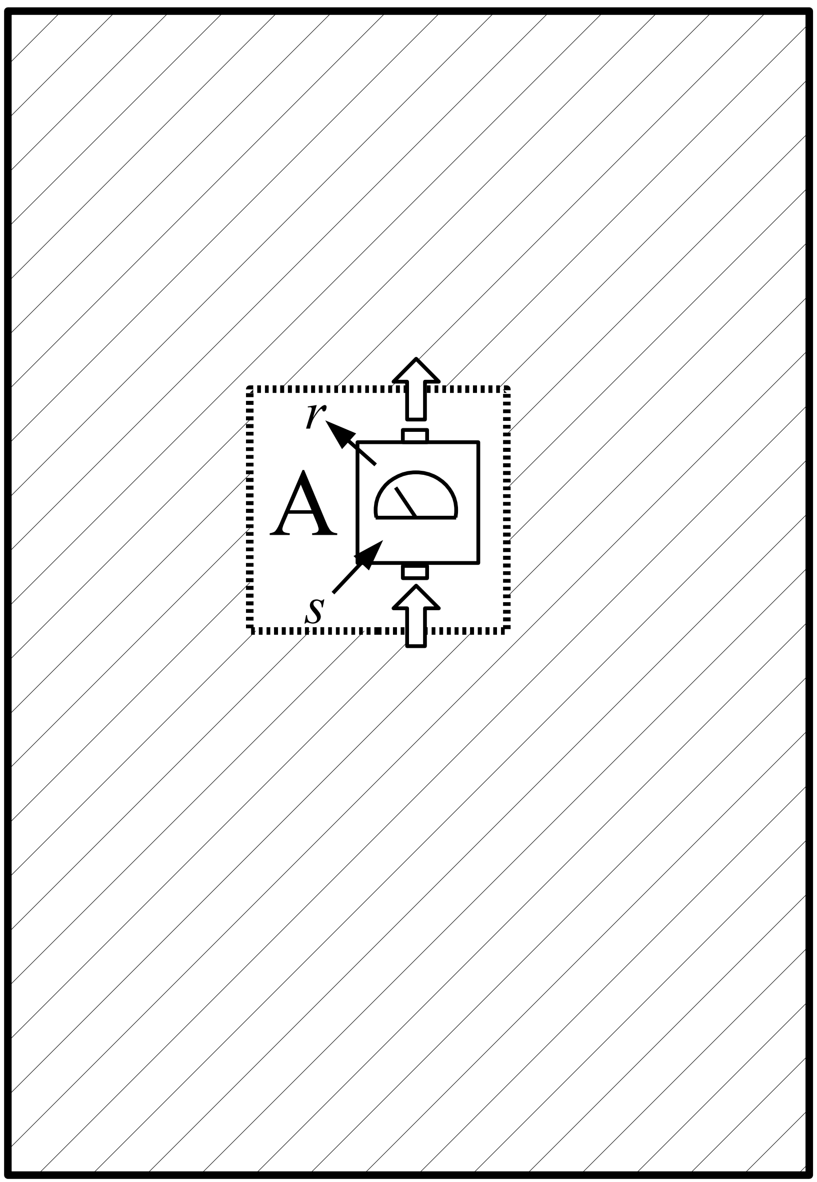
transpose

$$M^{sr} \geq 0, \quad Tr_o[\sum_r M_{io}^{sr}] = \boldsymbol{I}_i$$

[Choi, Jamiolkowski]

Ansatz I: via mathematical formalism



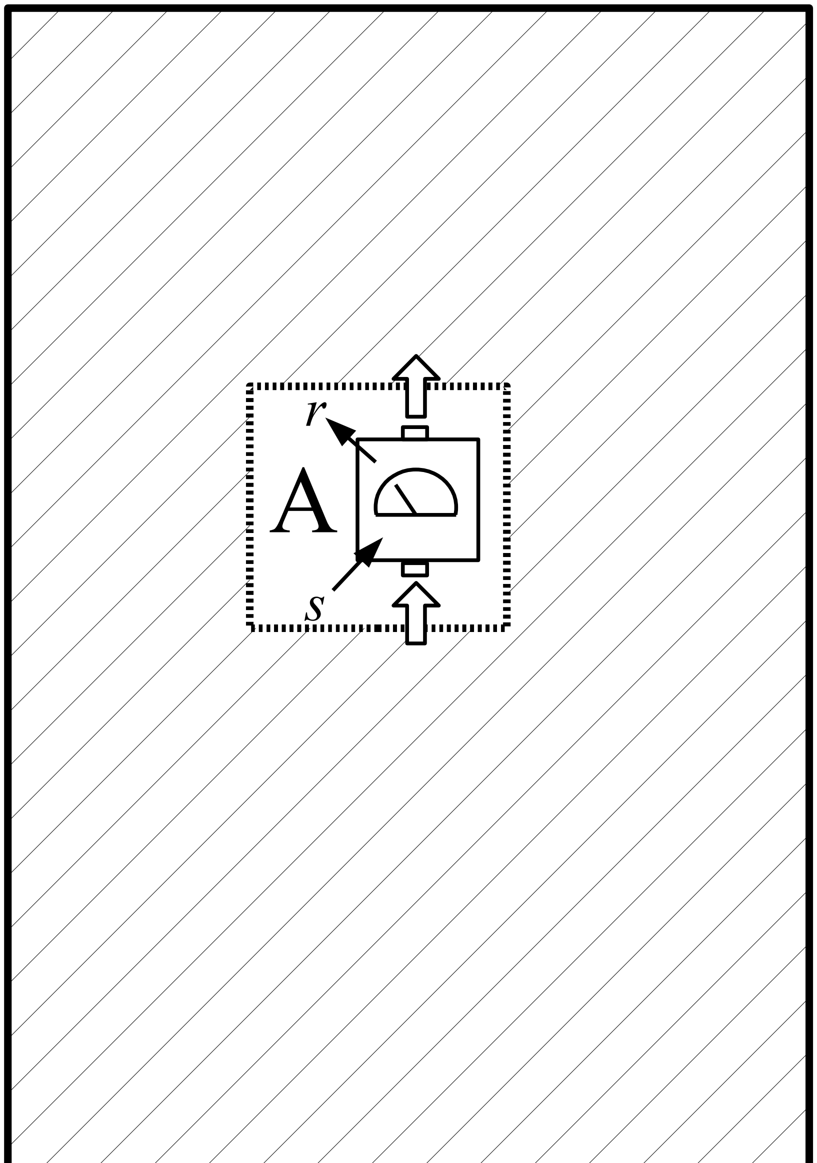
The environment is also described by an operator:

$$P(r|s) = Tr\left[M_{io}^{sr} W_{io}\right]$$

Example: environment prepares a state

$$W_{io} = \rho_i \otimes \boldsymbol{I}_o$$

Ansatz I: via mathematical formalism



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Example: environment prepares a state

$$W_{io} = \rho_i \otimes \boldsymbol{I}_o$$

Physical constraints: probabilities must be

- non-negative

$$Tr\left[M^{sr} W\right] \geq 0 \quad \forall M \quad \Rightarrow \quad W \geq 0$$

- normalized

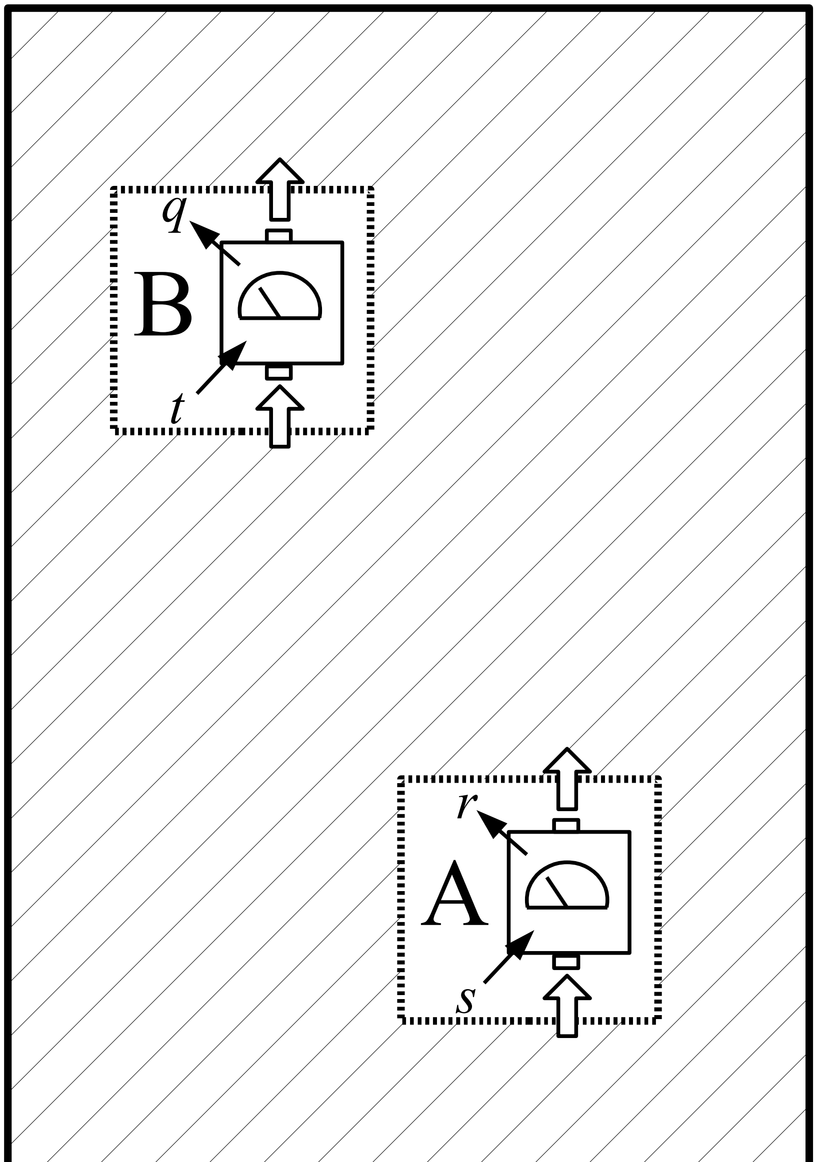
$$Tr\left[\sum_r M^{sr} W\right] = 1 \quad \forall M$$

Counter-example: 'looking for' a particular state

$$W_{io} = \rho_i \otimes |\psi\rangle\langle\psi|_o$$

$$Tr\left[\sum_r M^{sr} W\right] = 0 \quad if \quad M^{00} = \frac{1}{d} \boldsymbol{I}_i \otimes |\phi\rangle\langle\phi|_o \,, \quad |\phi\rangle \perp |\psi\rangle$$

Ansatz I: via mathematical formalism



The environment also specifies all **relations between quantum systems**:

$$P(r,q|s,t) = Tr\left[M^{sr}_{Ai,Ao} \otimes \tilde{M}^{tq}_{Bi,Bo} W_{Ai,Ao,Bi,Bo} \right]$$

Constraints:

$$Tr\left[\bar{M}^{sr}_{Ai,Ao,Bi,Bo} W \right] \geq 0 \quad \forall \bar{M} \quad \Rightarrow \quad W \geq 0$$

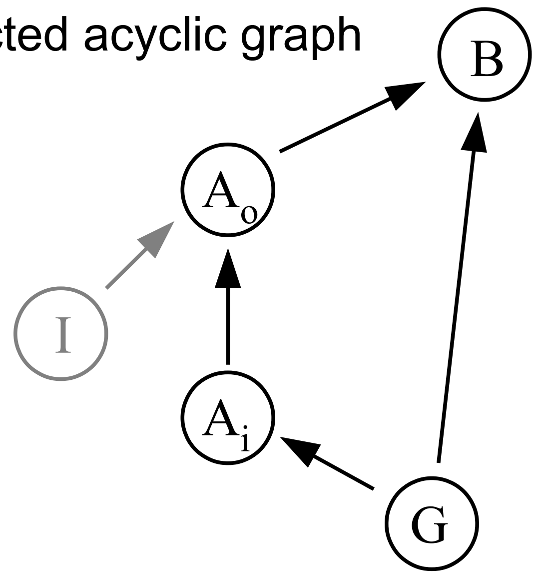
$$Tr\left[\sum_{r,q} M^{sr}_{Ai,Ao} \otimes \tilde{M}^{tq}_{Bi,Bo} W \right] = 1 \quad \forall M, \tilde{M}$$

Various related formalizations:

- quantum combs [Chiribella et al]
- process matrix [Oreshkov et al]
- causal map [Ried et al]

Ansatz II: modification of classical causal models

directed acyclic graph



+

conditionals

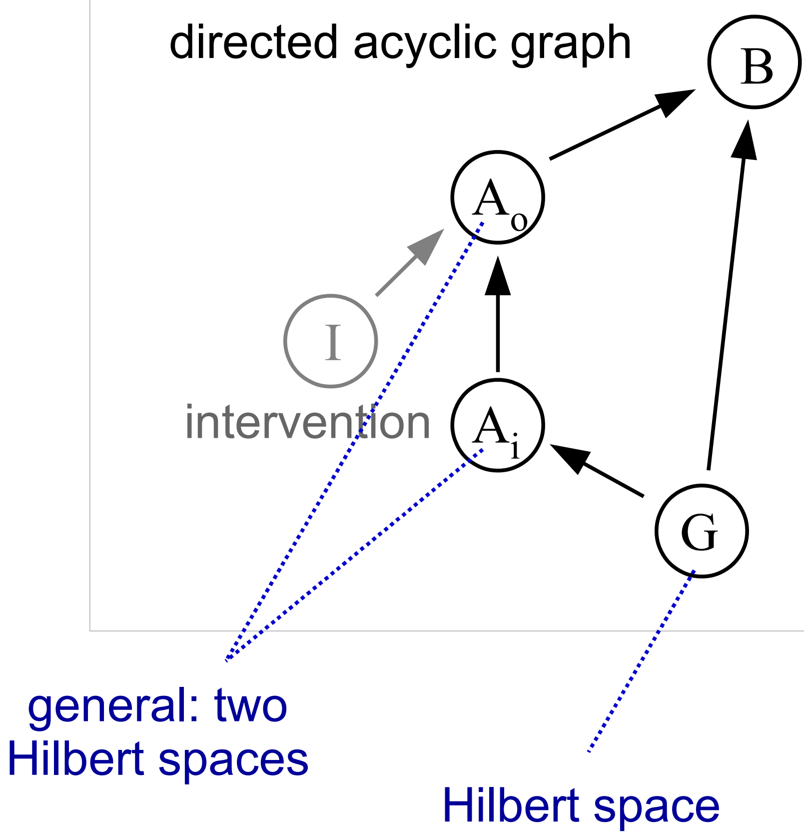
$$P(G)$$
$$P(A|G)$$
$$P(B|AG)$$

belief propagation

$$P(B) = \sum_A P(B|A)\,P(A)$$

[Leifer&Spekkens]

Ansatz II: modification of classical causal models



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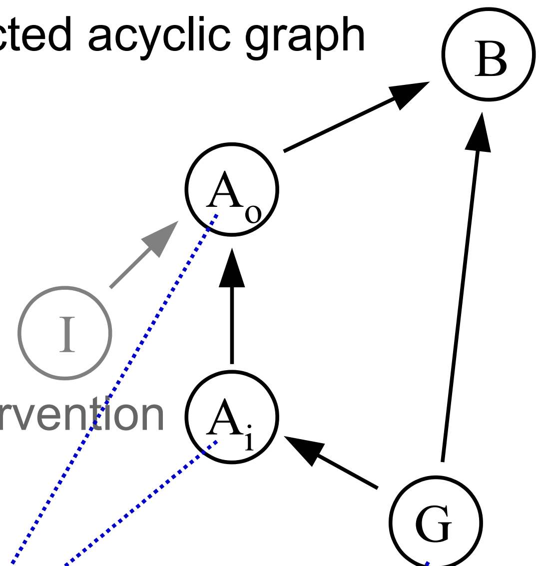

+ conditionals

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density operator

$$\rho_G \geq 0\ ,\quad Tr\,\rho_G = 1$$

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general: two Hilbert spaces

Hilbert space

[Leifer&Spekkens]

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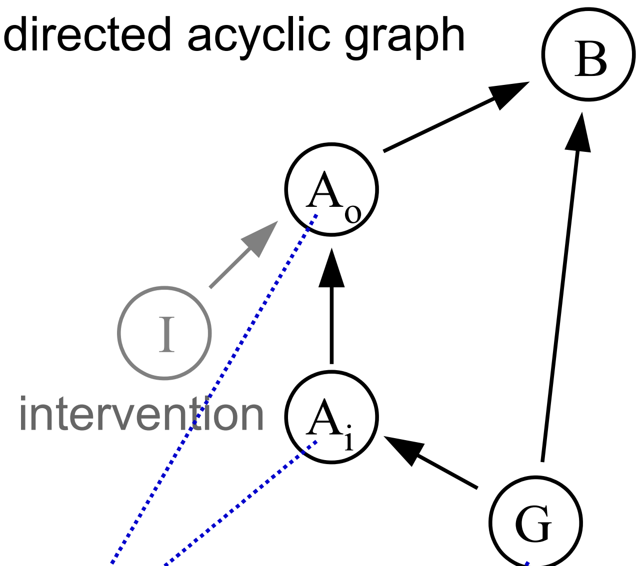

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quantum belief propagation

$\rho_B = Tr_A[\rho_{B|A}\rho_A]$

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[Leifer&Spekkens]

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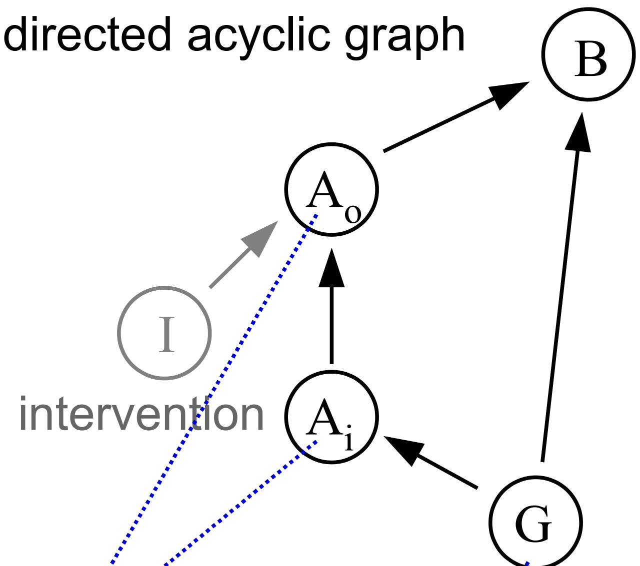

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$$\rho_G \geq 0 \ , \quad Tr\,\rho_G = 1$$

quantum belief propagation

$$\rho_B = Tr_A\left[\rho_{B|A}\rho_A\right]$$

general: two Hilbert spaces

Hilbert space

quantum (causal) conditional

$$\rho_{B|A} \equiv \left[\mathscr{E}_{A\to B} \otimes \mathscr{I}_A\right]\left(|\Phi^+\rangle\langle\Phi^+|\right)^{T_A}$$

$$\rho_{B|A}^{T_A} \geq 0 \ , \quad Tr_B\,\rho_{B|A} = \boldsymbol{I}_A$$

[Leifer&Spekkens]

General causal maps and the relation to process matrices

Causal maps

Local interventions described by

$$\rho_{o|i}^{sr} \in \mathscr{L}\left(\mathscr{H}_i \otimes \mathscr{H}_o\right): \ \tilde{\rho}_o^{sr} = Tr_i\left[\rho_{o|i}^{sr} \rho_i\right]$$

$$\rho_{o|i}^{T(i)} \geq 0 \ , \quad Tr_o\left[\sum_r \rho_{o|i}^{sr}\right] = \boldsymbol{I}_i$$

General causal maps and the relation to process matrices

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Probabilities of outcomes given settings

$$P(r, q|s, t) = Tr\left[\rho_{Ao|Ai}^{sr} \otimes \tilde{\rho}_{Bo|Bi}^{tq} \tau_{Ai, Bi|Ao, Bo}\right]$$

General causal maps and the relation to process matrices

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Mathematical properties

$$\tau^{T(Ai, Bi)}_{Ai, Bi|Ao, Bo} \geq 0$$

$$Tr\left[\sum_{r, q} \rho^{sr}_{Ao|Ai} \otimes \tilde{\rho}^{tq}_{Bo|Bi}\, \tau_{Ai, Bi|Ao, Bo}\right] = 1$$

$$\forall \rho_{Ao|Ai},\ \tilde{\rho}_{Bo|Bi}$$

General causal maps and the relation to process matrices

Causal maps

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$$\forall \rho_{Ao|Ai},\ \tilde{\rho}_{Bo|Bi}$$

Process matrices

Local interventions described by

$$M^{sr} \in \mathscr{L}(\mathscr{H}_i \otimes \mathscr{H}_o) : Tr_i\left[M^{sr}_{io}\rho^T_i\right] = \tilde{\rho}^{sr}_o$$

$$M^{sr} \geq 0, \quad Tr_o\left[\sum_r M^{sr}_{io}\right] = \boldsymbol{I}_i$$

Probabilities of outcomes given settings

$$P(r, q|s, t) = Tr\left[M^{sr}_{Ai, Ao} \otimes \tilde{M}^{tq}_{Bi, Bo} W\right]$$

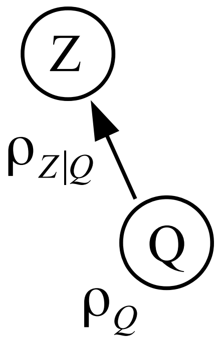
Mathematical properties

$$W \geq 0$$

$$Tr\left[\sum_{r,q} M^{sr}_{Ai, Ao} \otimes \tilde{M}^{tq}_{Bi, Bo} W\right] = 1 \quad \forall M, \tilde{M}$$

Inference for quantum systems

Case 1: retrodiction of cause given effect



Classical Bayesian inversion:

$$P(Z,Q)=P(Z|Q)P(Q)$$

$$\rightarrow \; P(Q|Z)=\frac{P(Z,Q)}{P(Z)}$$

[Leifer&Spekkens]

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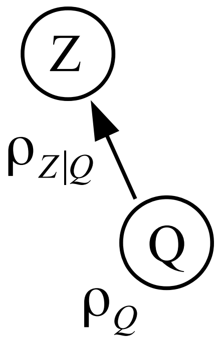

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Quantum version:

$$\rho_{Z,Q} = \rho_Q^{\frac{1}{2}} \rho_{Z|Q} \rho_Q^{\frac{1}{2}} \equiv \rho_{Z|Q} * \rho_Q$$

$$\rho_{Q|Z} = \left(\rho_Q^{\frac{1}{2}} \otimes \rho_Z^{-\frac{1}{2}}\right) \rho_{Z|Q} \left(\rho_Q^{\frac{1}{2}} \otimes \rho_Z^{-\frac{1}{2}}\right)$$

Note: same mathematical properties
as causal conditionals

$$\rho_{Q|Z}^{T(Z)} \geq 0$$

[Leifer&Spekkens, Horsman et al]

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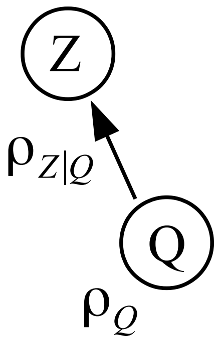

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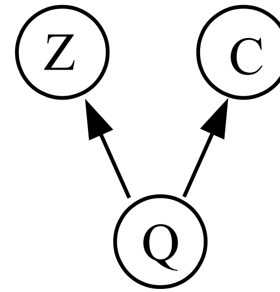
Note: same mathematical properties as causal conditionals

$$\rho_{Q|Z}^{T(Z)}\geq 0$$

[Leifer&Spekkens, Horsman et al]

Case 2: inference via a common cause

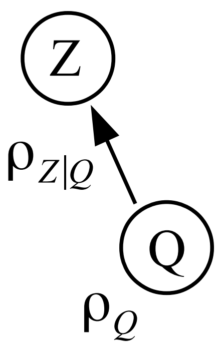


Classically,

$$\begin{aligned}P(C|Z)&=\sum_q P(C,Q=q|Z)\\&=\sum_q P(C|Q=q)\,P(Q=q|Z)\end{aligned}$$

Inference for quantum systems

Case 1: retrodiction of cause given effect



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Note: same mathematical properties as causal conditionals

$$\rho_{Q|Z}^{T(Z)}\geq 0$$

[Leifer&Spekkens, Horsman et al, Ried et al]

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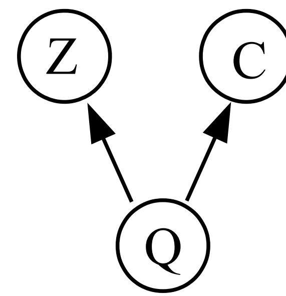


Classically,

$$P(C|Z)=\sum_q P(C,Q=q|Z)$$
$$=\sum_q P(C|Q=q)\,P(Q=q|Z)$$

Quantum version:

$$\rho_{C|Z}=Tr_Q\left(\rho_{C|Q}\rho_{Q|Z}\right)$$

Note:

- different mathematical properties

$$\rho_{C|Q}^{T(Q)}\geq 0\ ,\ \ \rho_{Q|Z}^{T(Z)}\geq 0\ \Rightarrow\ \rho_{C|Z}\geq 0$$

$\rightarrow$ conditionals reflect causal structure

- no simple form for joint state:

$$\rho_{C|Q}*\left(\rho_{Z|Q}*\left(\rho_Q\otimes\rho_Z^{-1}\right)\right)$$
$$\neq\left(\rho_{C|Q}*\rho_{Z|Q}\right)*\left(\rho_Q\otimes\rho_Z^{-1}\right)\neq\ldots$$

Causal Structure in a Quantum World

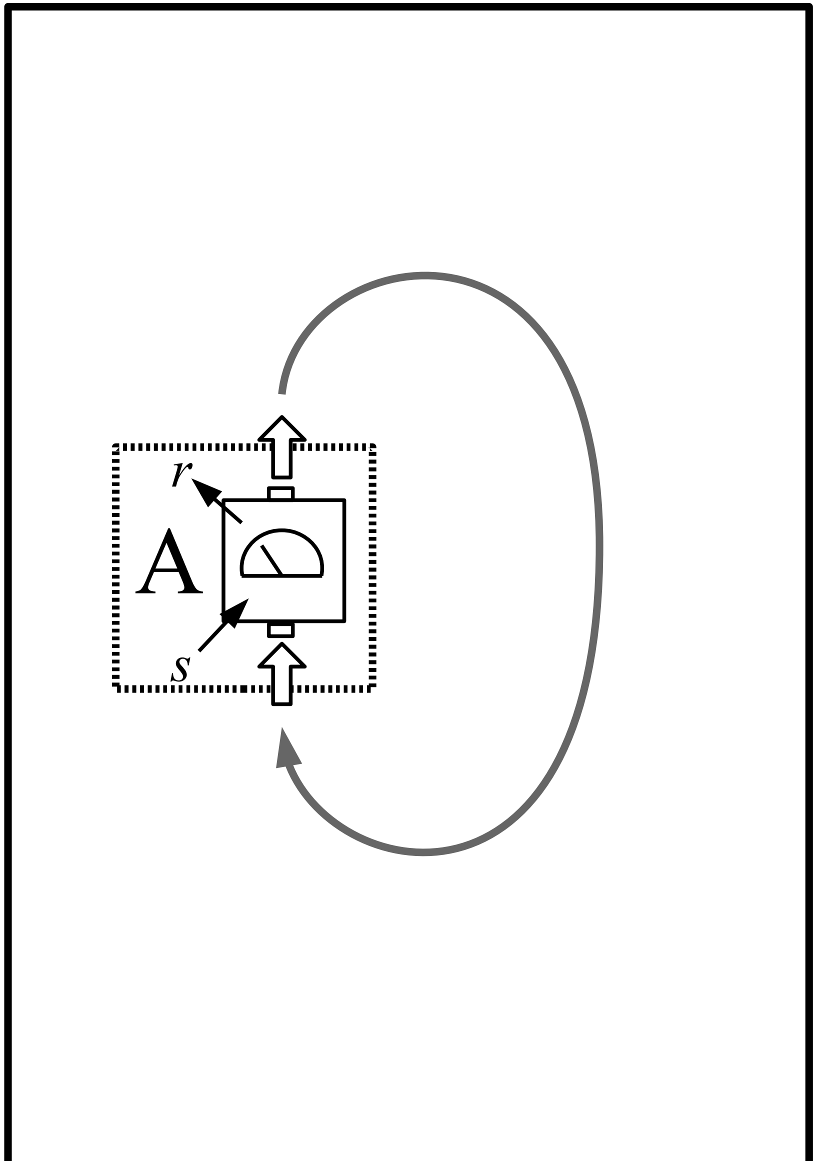
Given an operator relating several quantum systems,

- Can it be decomposed into separate causal relations?
- What kinds of causal relations can there be?
- How to classify the possible causal relations?

Causal Structure in a Quantum World

Causal Loops

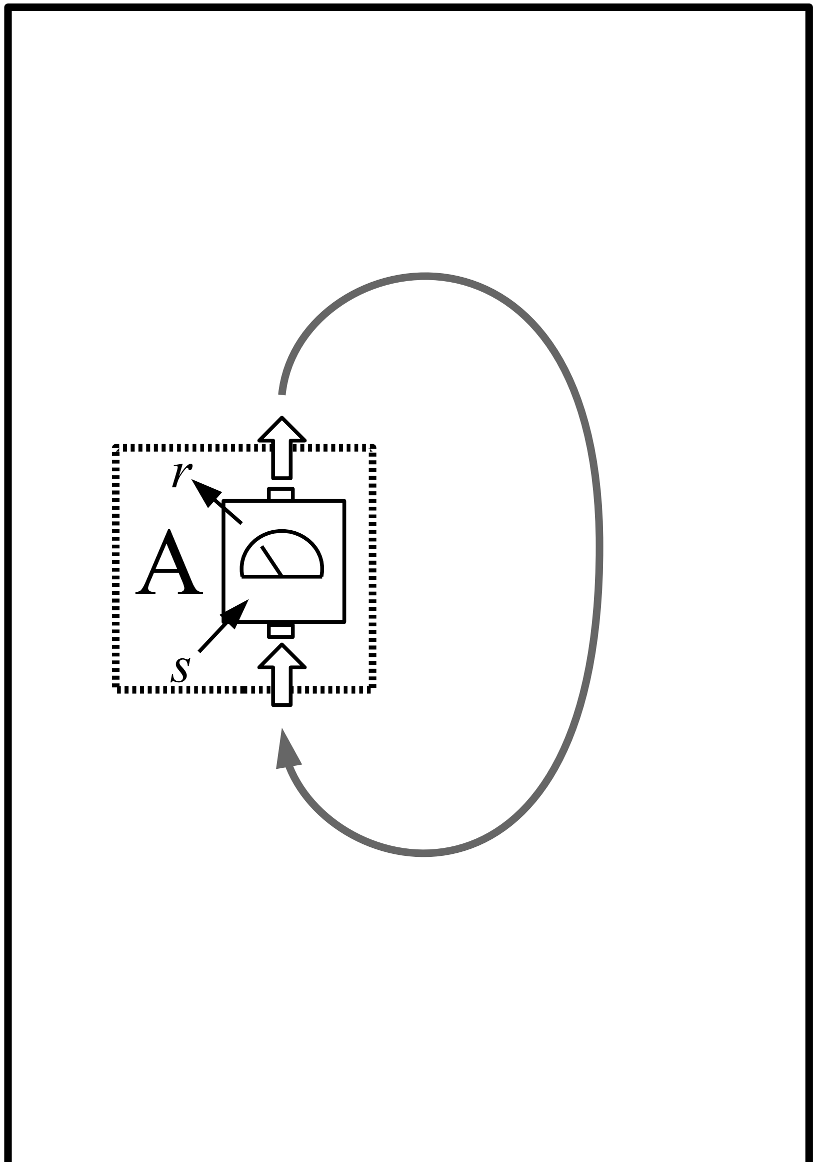
Causal loops revisited



Case 1

$$W = \boldsymbol{I} + Z \otimes Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Causal loops revisited



Case 1

$$W = \boldsymbol{I} + Z \otimes Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

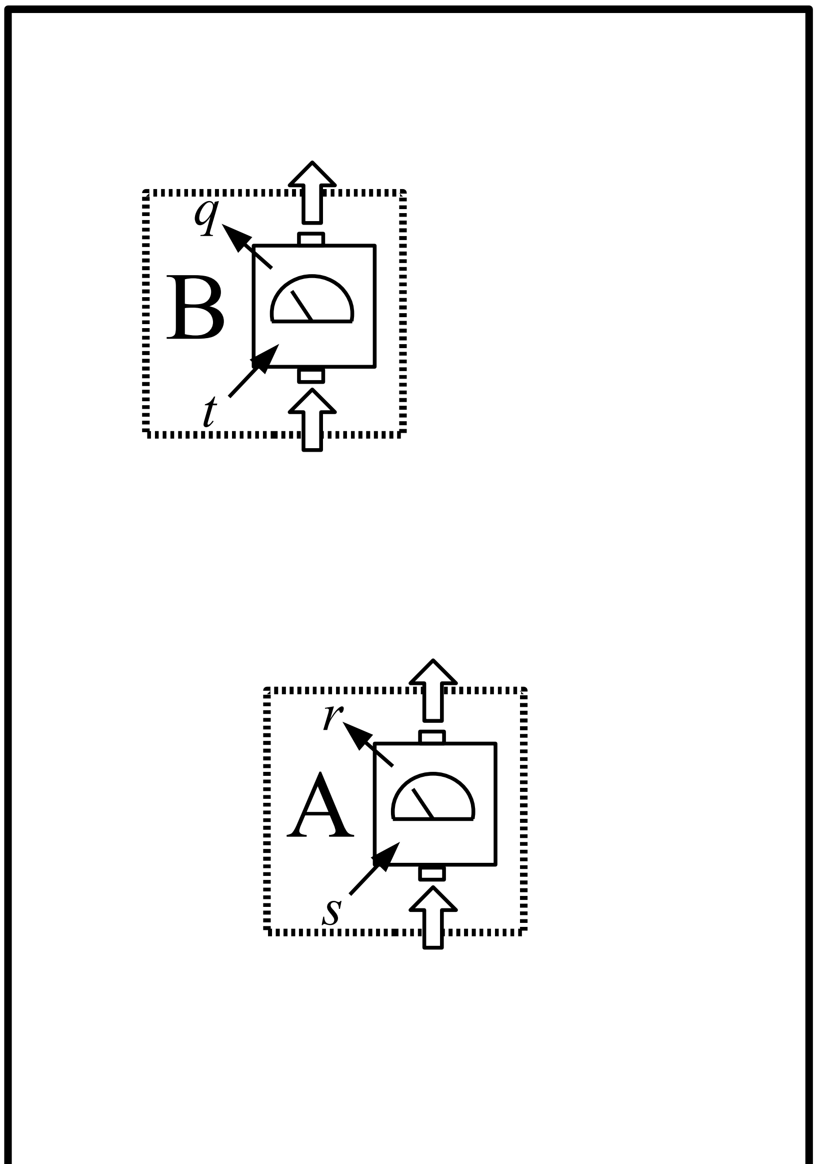
Alice measures in the Z basis, then flips the bit:

$$\begin{cases} M^{00} = |0\rangle\langle 0|_i \otimes |1\rangle\langle 1|_o \\ M^{01} = |1\rangle\langle 1|_i \otimes |0\rangle\langle 0|_o \end{cases} \quad \text{(single } s\text{)}$$

$$\Rightarrow \sum_r P(r|s=0) = Tr[\sum_r M_{io}^{0r} W_{io}] = 0$$

$\Rightarrow$ Does not satisfy physical constraints.

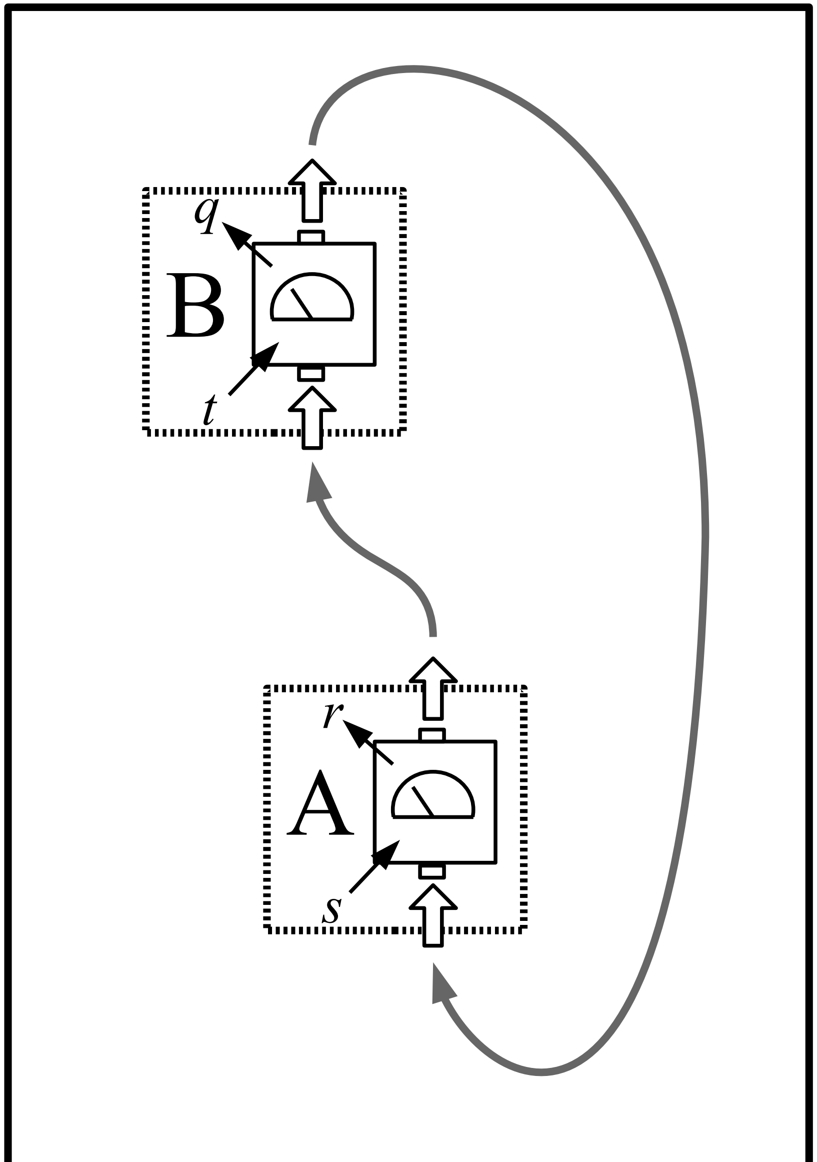
Causal loops revisited



Case 2

$$W = \left(\boldsymbol{I} + Z \otimes Z \right)_{Ao\,Bi} \otimes \left(\boldsymbol{I} + Z \otimes Z \right)_{Bo\,Ai}$$

Causal loops revisited



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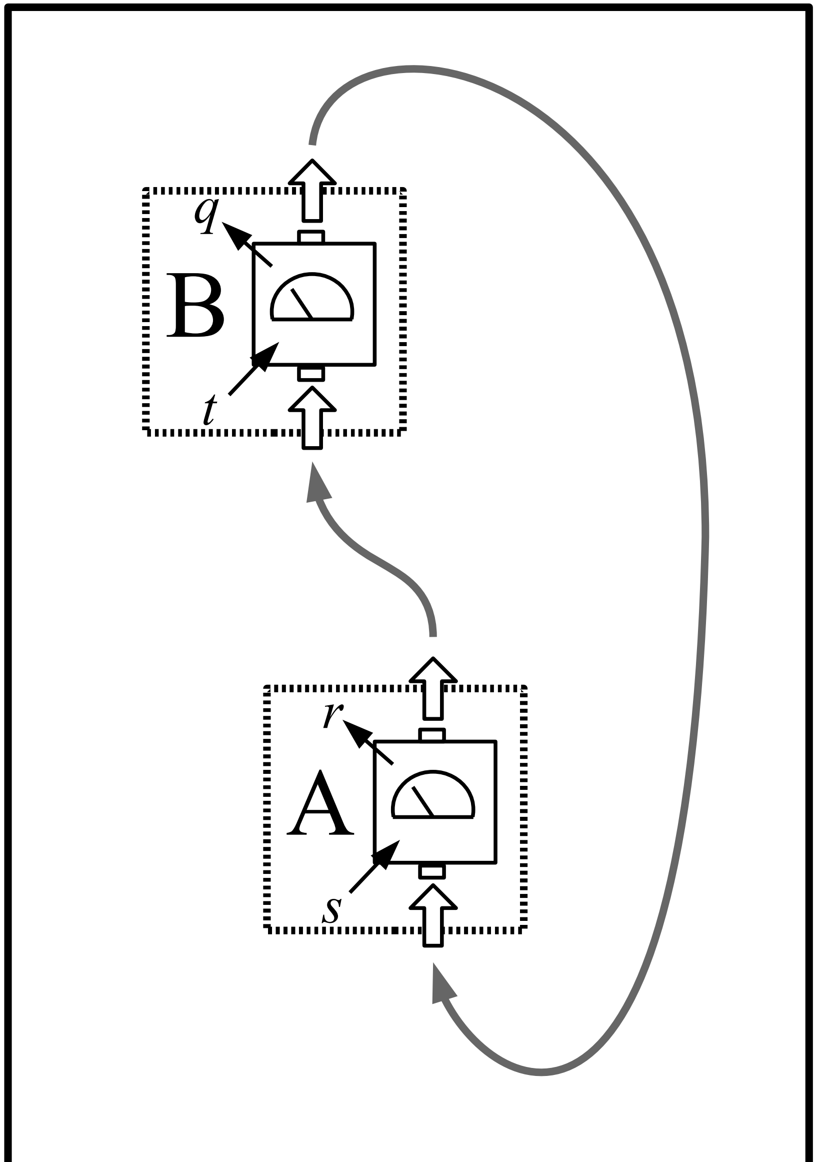
Causal loops revisited



Case 2

$$W = \left(\boldsymbol{I} + Z \otimes Z\right)_{Ao\,Bi} \otimes \left(\boldsymbol{I} + Z \otimes Z\right)_{Bo\,Ai}$$

Alice measures and flips, Bob just measures:

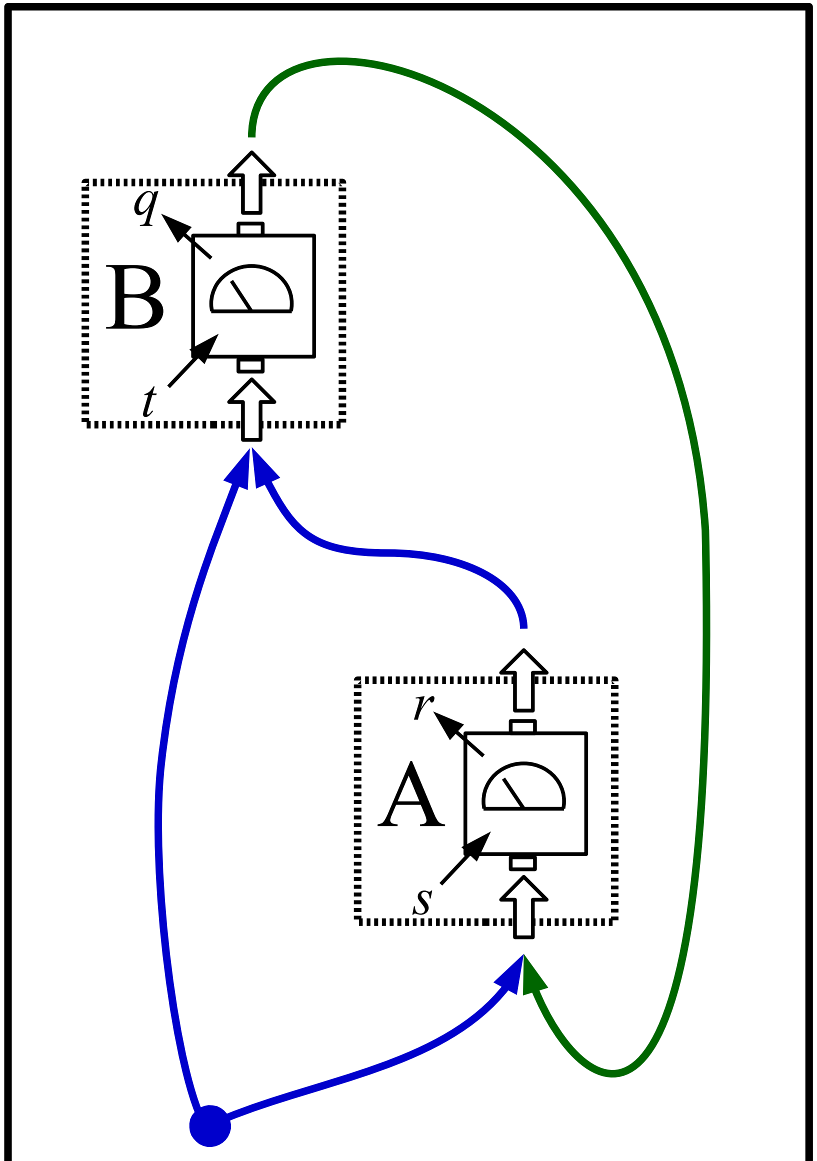
$$\begin{cases} M^{00} = |0\rangle\langle 0|_{Ai} \otimes |1\rangle\langle 1|_{Ao} \\ M^{01} = |1\rangle\langle 1|_{Ai} \otimes |0\rangle\langle 0|_{Ao} \end{cases} \quad \begin{cases} \tilde{M}^{00} = |0\rangle\langle 0|_{Bi} \otimes |0\rangle\langle 0|_{Bo} \\ \tilde{M}^{01} = |1\rangle\langle 1|_{Bi} \otimes |1\rangle\langle 1|_{Bo} \end{cases}$$

$$\Rightarrow \quad \sum_{r,q} P(rq|st) = Tr\left[\sum_{r,q} M^{sr}_{Ai,\,Ao} \otimes \tilde{M}^{tq}_{Bi,\,Bo} W\right] = 0$$

$\Rightarrow$ Does not satisfy physical constraints.

Note: Constraints on probabilities rule out (at least some types of) causal loops.

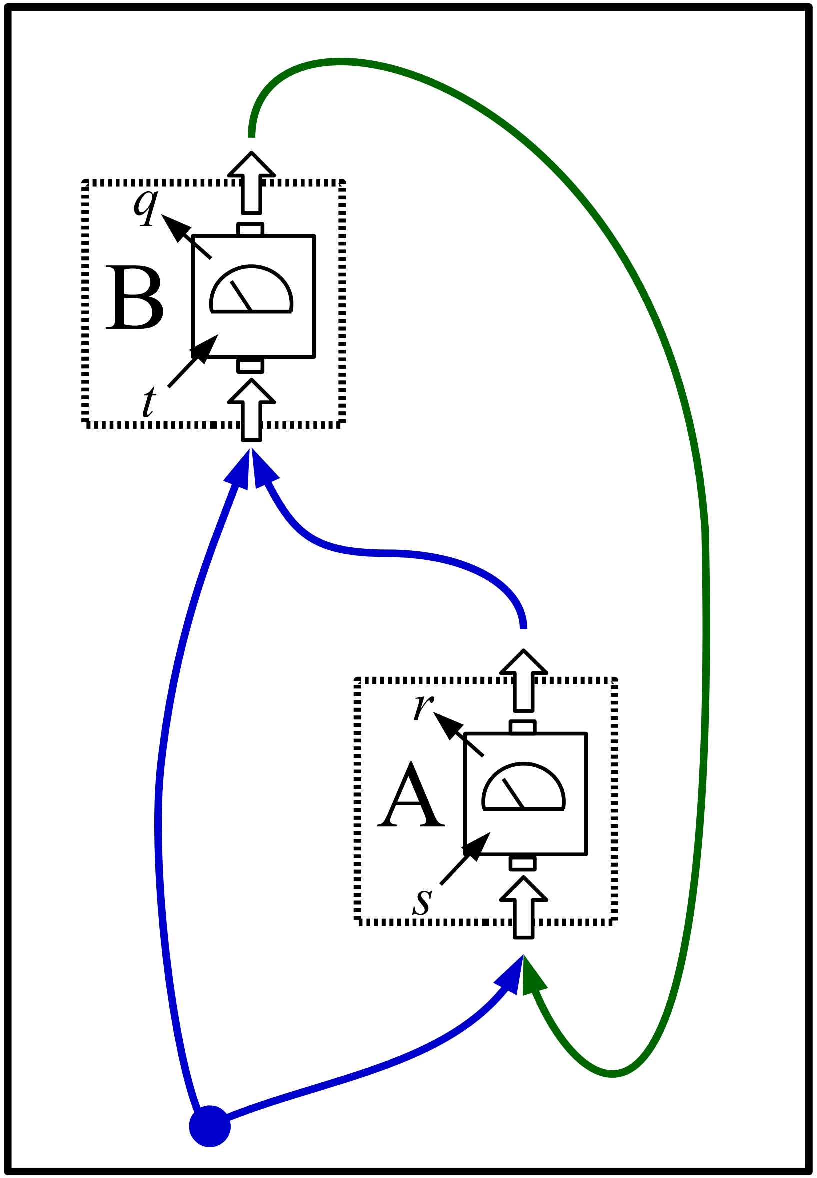
Causal loops revisited



Case 3

$$W = \frac{1}{2} W_{B<A} + \frac{1}{2} W_{A<B}$$
$$= \frac{1}{8}\left(\boldsymbol{I} + \frac{1}{\sqrt{2}} Z_{Bo} \otimes Z_{Ai} \right) + \frac{1}{8}\left(\boldsymbol{I} + \frac{1}{\sqrt{2}} Z_{Bi} \otimes X_{A_1} \otimes Z_{Ao} \right)$$

Causal loops revisited



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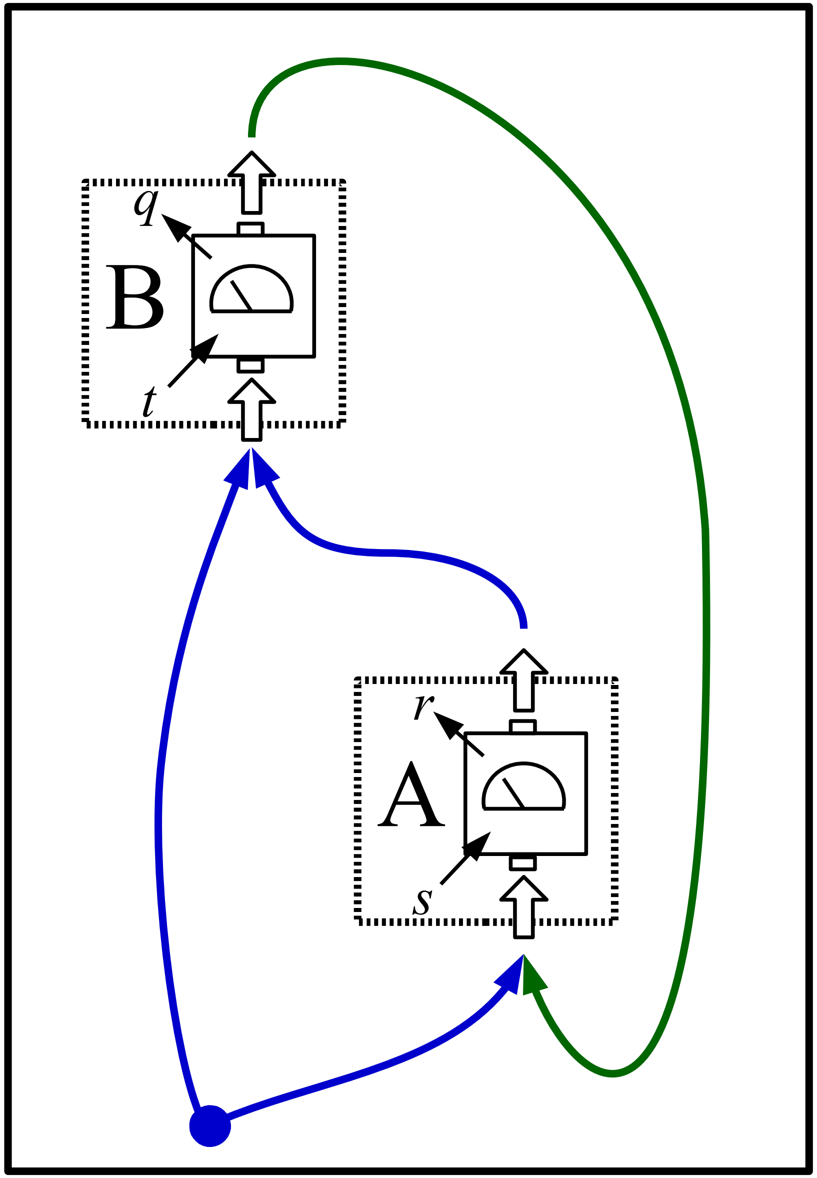
check positivity:

$$\lambda_{B<A} = \frac{1}{4}\left(1 \pm \frac{1}{\sqrt{2}}\right) \quad ; \qquad \lambda_{A<B} = \frac{1}{4}\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

Causal loops revisited



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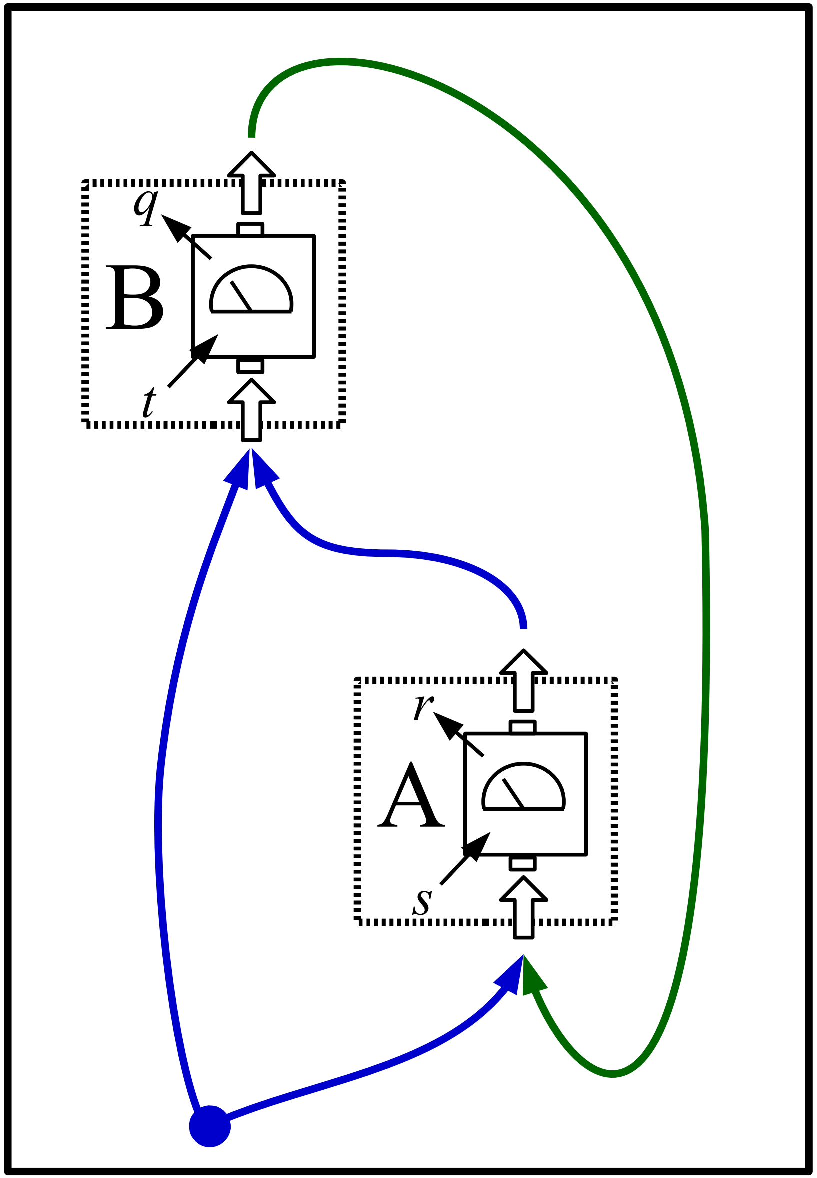
check normalization:

$$Tr\left[\sum_{r,q} M^{sr}_{Ai,Ao} \otimes \tilde{M}^{tq}_{Bi,Bo} W \right] = 1 \quad \forall\, M, \tilde{M}$$
$$\left(M^{sr} \geq 0, \quad Tr_o\left[\sum_r M^{sr}_{io}\right] = \boldsymbol{I}_i \right)$$

Causal loops revisited

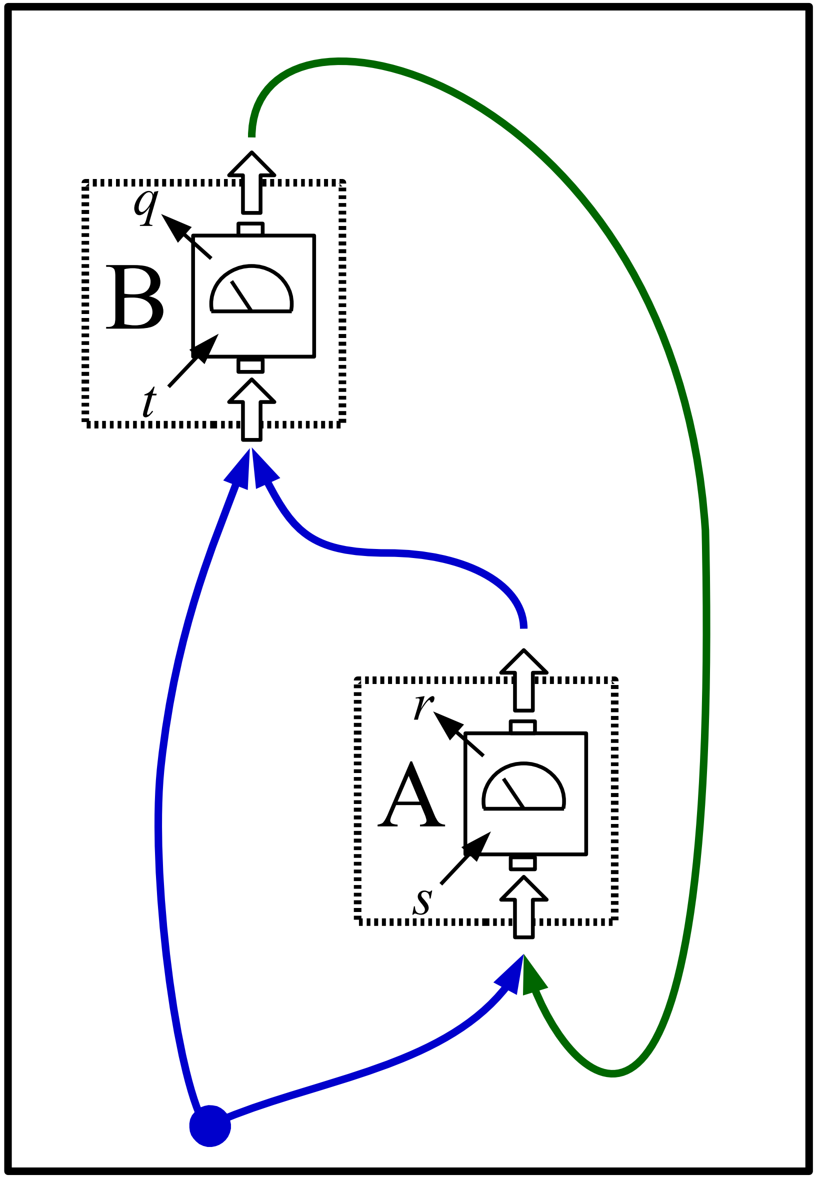


Case 4

$$W = \frac{1}{4}\left(\boldsymbol{I} + \frac{1}{\sqrt{2}} Z_{Bo} \otimes Z_{Ai} + \frac{1}{\sqrt{2}} Z_{Bi} \otimes X_{Ai} \otimes Z_{Ao} \right)$$
$$= \frac{1}{8}\left(\boldsymbol{I} + \frac{2}{\sqrt{2}} Z_{Bo} \otimes Z_{Ai} \right) + \frac{1}{8}\left(\boldsymbol{I} + \frac{2}{\sqrt{2}} Z_{Bi} \otimes X_{Ai} \otimes Z_{Ao} \right)$$

[Oreshkov et al]

Causal loops revisited



Case 4

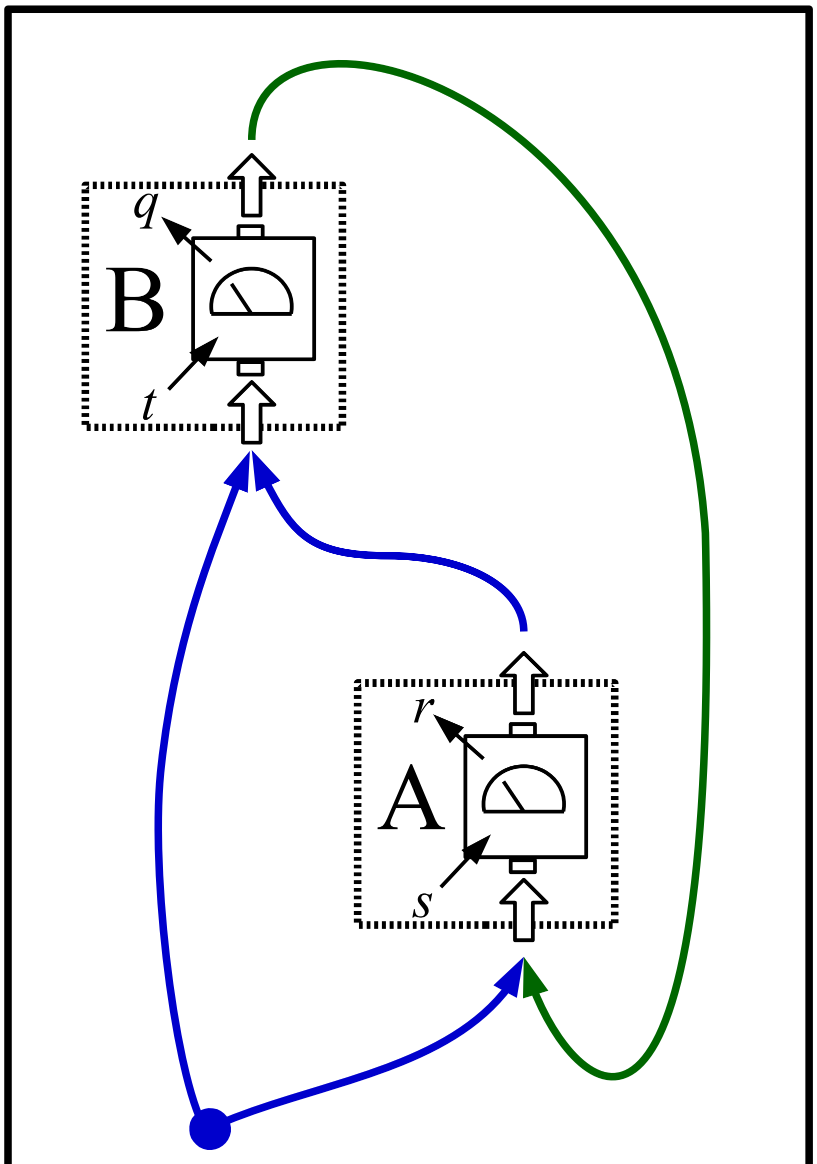
$$W=\frac{1}{4}\left(\boldsymbol{I}+\frac{1}{\sqrt{2}}Z_{Bo}\otimes Z_{Ai}+\frac{1}{\sqrt{2}}Z_{Bi}\otimes X_{Ai}\otimes Z_{Ao}\right)$$
$$=\frac{1}{8}\left(\boldsymbol{I}+\frac{2}{\sqrt{2}}Z_{Bo}\otimes Z_{Ai}\right)+\frac{1}{8}\left(\boldsymbol{I}+\frac{2}{\sqrt{2}}Z_{Bi}\otimes X_{Ai}\otimes Z_{Ao}\right)$$

- naive attempt at convex decomposition fails:

$$\lambda_{B<A}=\frac{1}{4}\left(1\pm\sqrt{2}\right)\quad;\qquad\lambda_{A<B}=\frac{1}{4}\left(1\pm\sqrt{2}\right)$$

[Oreshkov et al]

Causal loops revisited



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$$= \frac{1}{8}\left(\boldsymbol{I} + \frac{2}{\sqrt{2}} Z_{Bo} \otimes Z_{Ai}\right) + \frac{1}{8}\left(\boldsymbol{I} + \frac{2}{\sqrt{2}} Z_{Bi} \otimes X_{Ai} \otimes Z_{Ao}\right)$$

- naive attempt at convex decomposition fails:

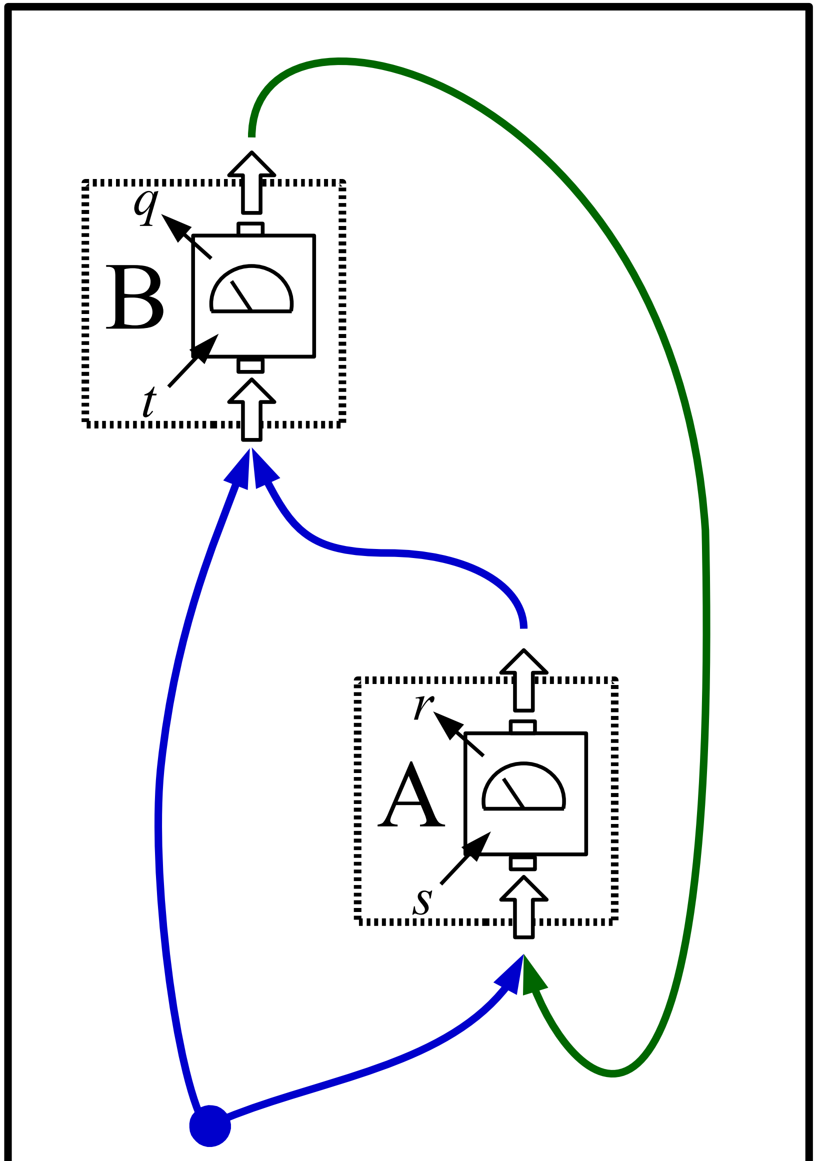
$$\lambda_{B<A} = \frac{1}{4}\left(1 \pm \sqrt{2}\right) \quad ; \qquad \lambda_{A<B} = \frac{1}{4}\left(1 \pm \sqrt{2}\right)$$

- yet W_q is a valid process:

$$\lambda_W \in \left\{0, \frac{1}{4}\right\} \geq 0$$
$$Tr\left[\sum_{r,q} M^{sr} \otimes \tilde{M}^{tq} W\right] = 1 \quad \forall M, \tilde{M}$$

[Oreshkov et al]

The causal separability game

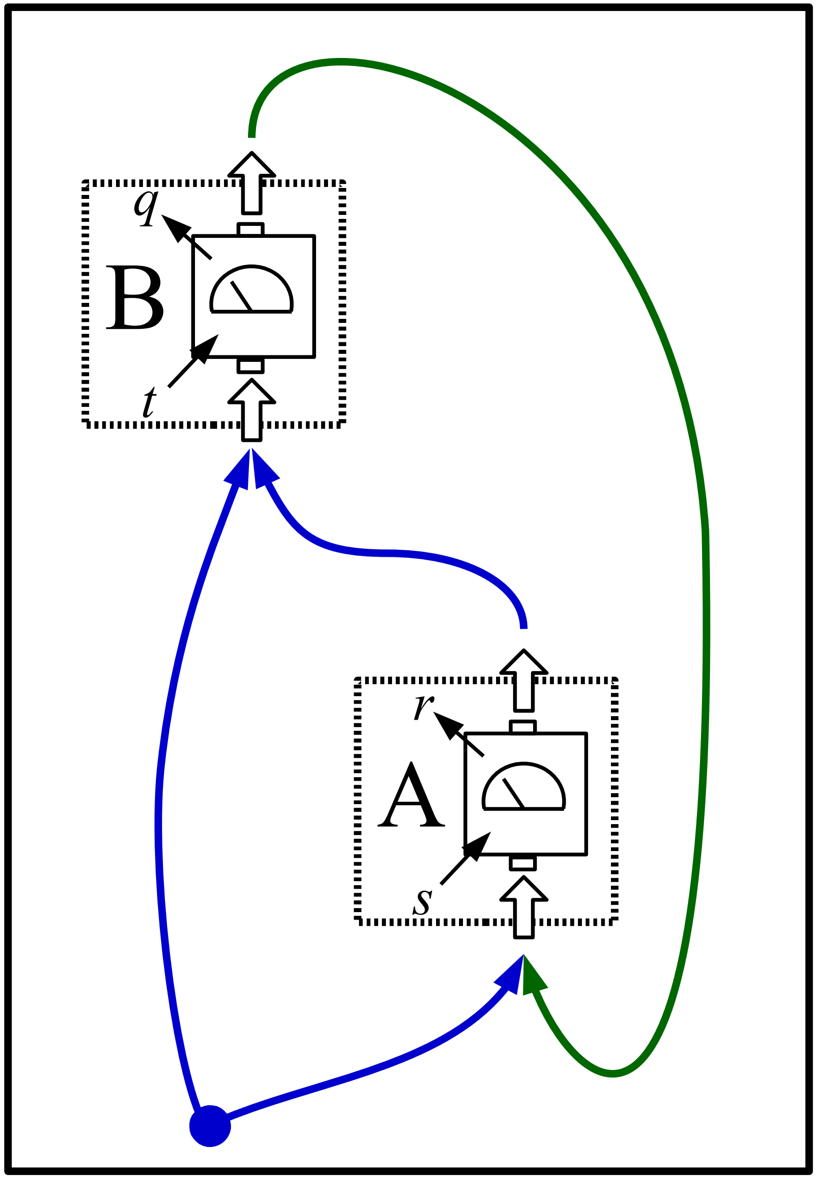


The task:

- if $j=0$, Alice must signal Bob: return $q=s$
- if $j=1$, Bob must signal Alice: return $r=t$

[Oreshkov et al]

The causal separability game



The task:

- if $j=0$, Alice must signal Bob: return $q=s$
- if $j=1$, Bob must signal Alice: return $r=t$

Success probability given a fixed causal order:

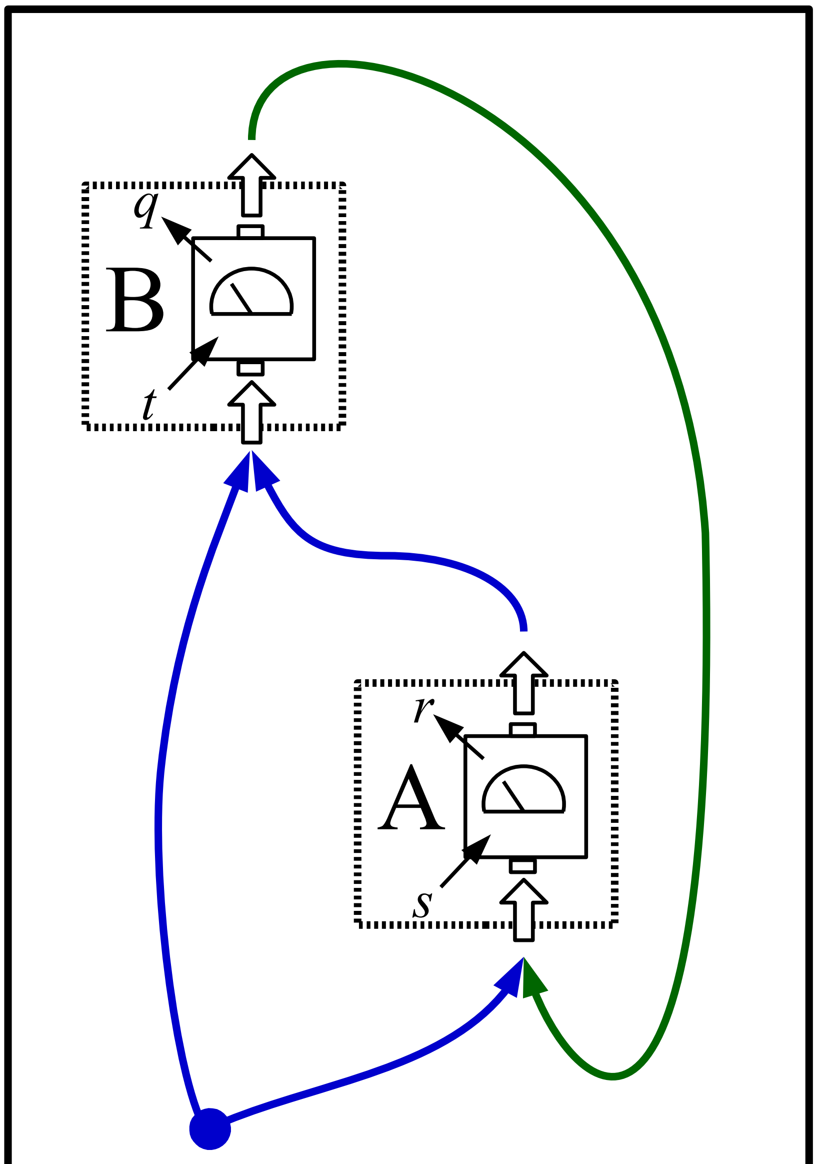
$$p_{suc} \leq \frac{3}{4}$$

Success probability using

$$W = \frac{1}{4}\left(\boldsymbol{I} + \frac{1}{\sqrt{2}} Z_{Bo} \otimes Z_{Ai} + \frac{1}{\sqrt{2}} Z_{Bi} \otimes X_{Ai} \otimes Z_{Ao}\right)$$

[Oreshkov et al]

The causal separability game



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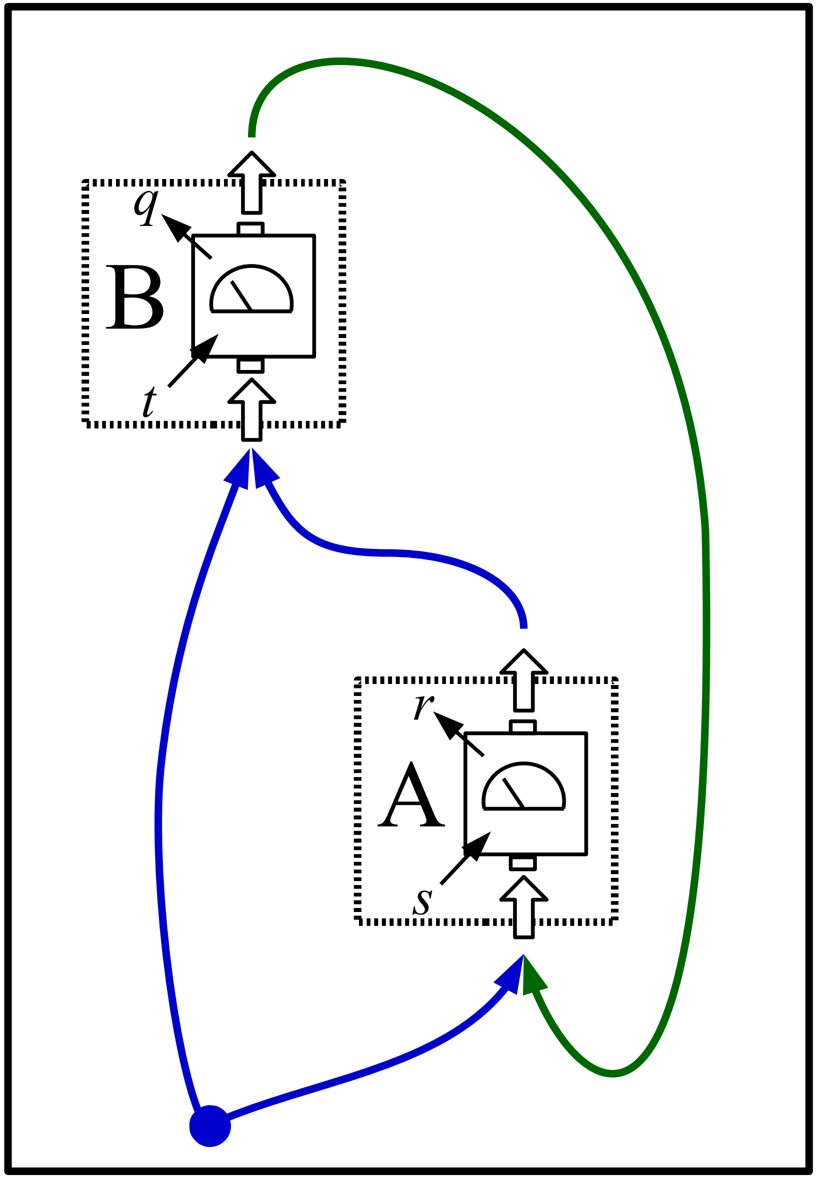
- if $j=0$, Alice measures in Z basis
- if $j=1$, Alice measures in X basis

$$\Rightarrow p_{suc} = \frac{2+\sqrt{2}}{4}$$

[Oreshkov et al]

Causal witnesses



More generally, one can find observables

S such that, for all W^{sep} of the form

$$W^{sep} = q\, W_{B<A} + (1-q)\, W_{A<B}$$

it holds that

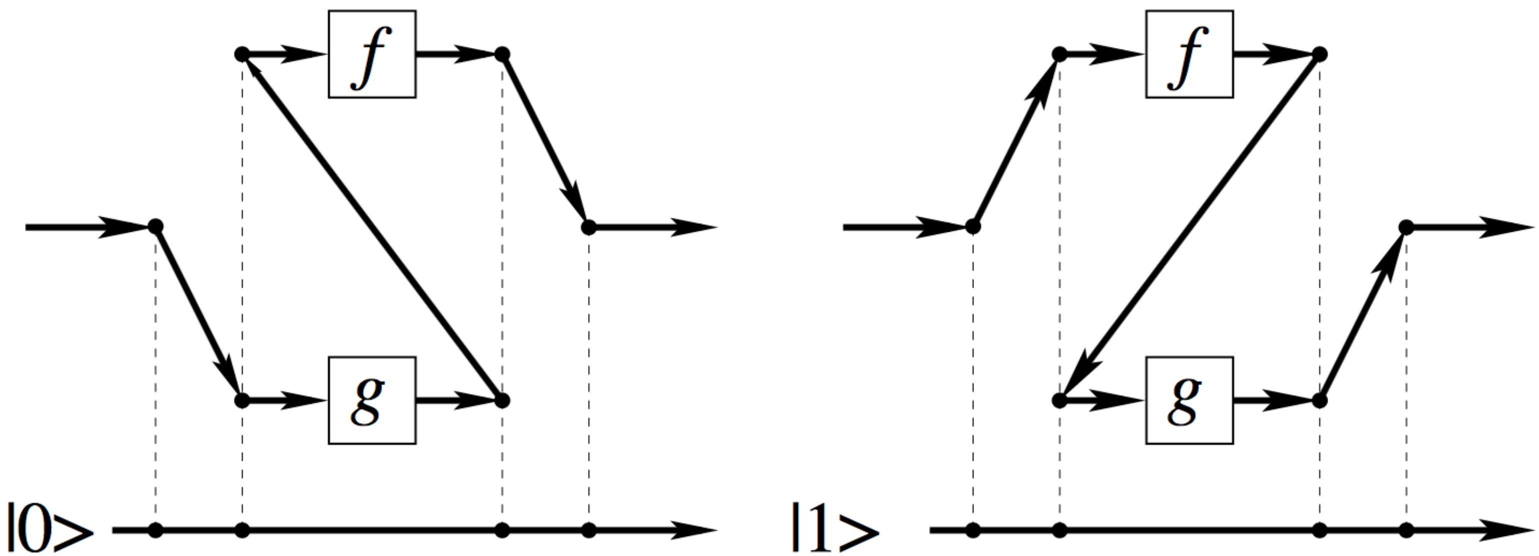
$$Tr[S\, W^{sep}] \geq 0$$

Compare with witnesses of entanglement:

$$Tr[\bar{S}\, \rho^{sep}] \geq 0 \quad \forall\ \rho^{sep} = \sum_j q_j\, \rho_A^j \otimes \rho_B^j$$

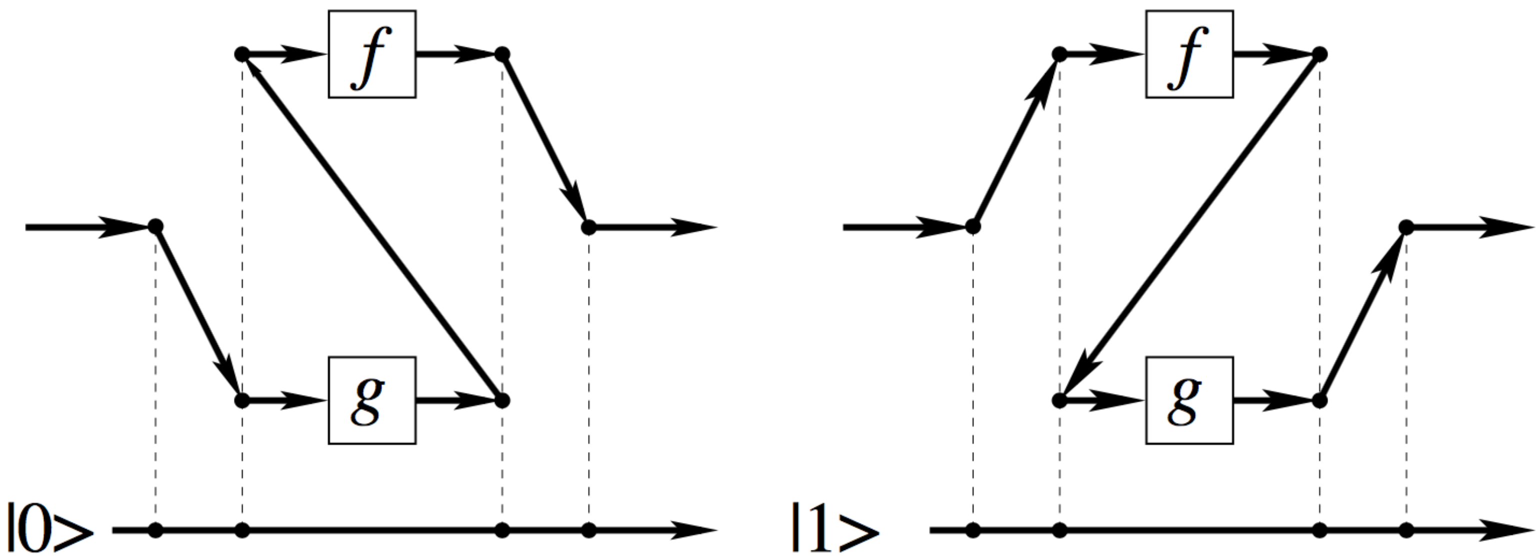
[Araújo et al]

The quantum switch



[Chiribella et al]

The quantum switch



- prepare a superposition of the control qubit
- post-select on still having a superposition afterwards

[Chiribella et al]

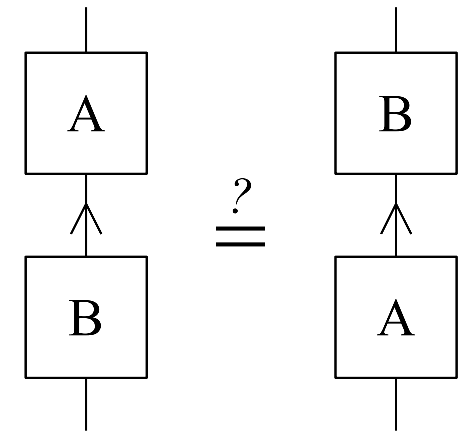
Applications of indefinite causal order

PHYSICAL REVIEW A **86**, 040301(R) (2012)

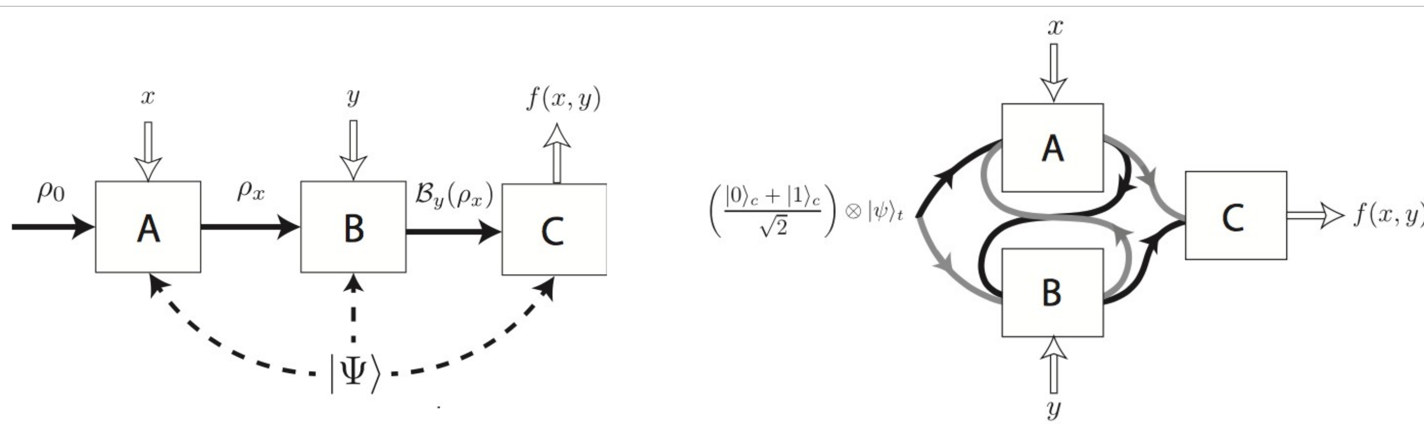
Perfect discrimination of no-signalling channels via quantum superpos

Giulio Chiribella
Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua U
(Received 23 September 2011; published 10 October 2012)

A no-signalling channel transforming quantum systems in Alice's and Bob's local la with two different causal structures: ($A \preceq B$) Alice's output causally precedes Bob's in output causally precedes Alice's input. Here I prove that two no-signalling channel distinguishable in any ordinary quantum circuit can become perfectly distinguishabl superposition of circuits with different causal structures.

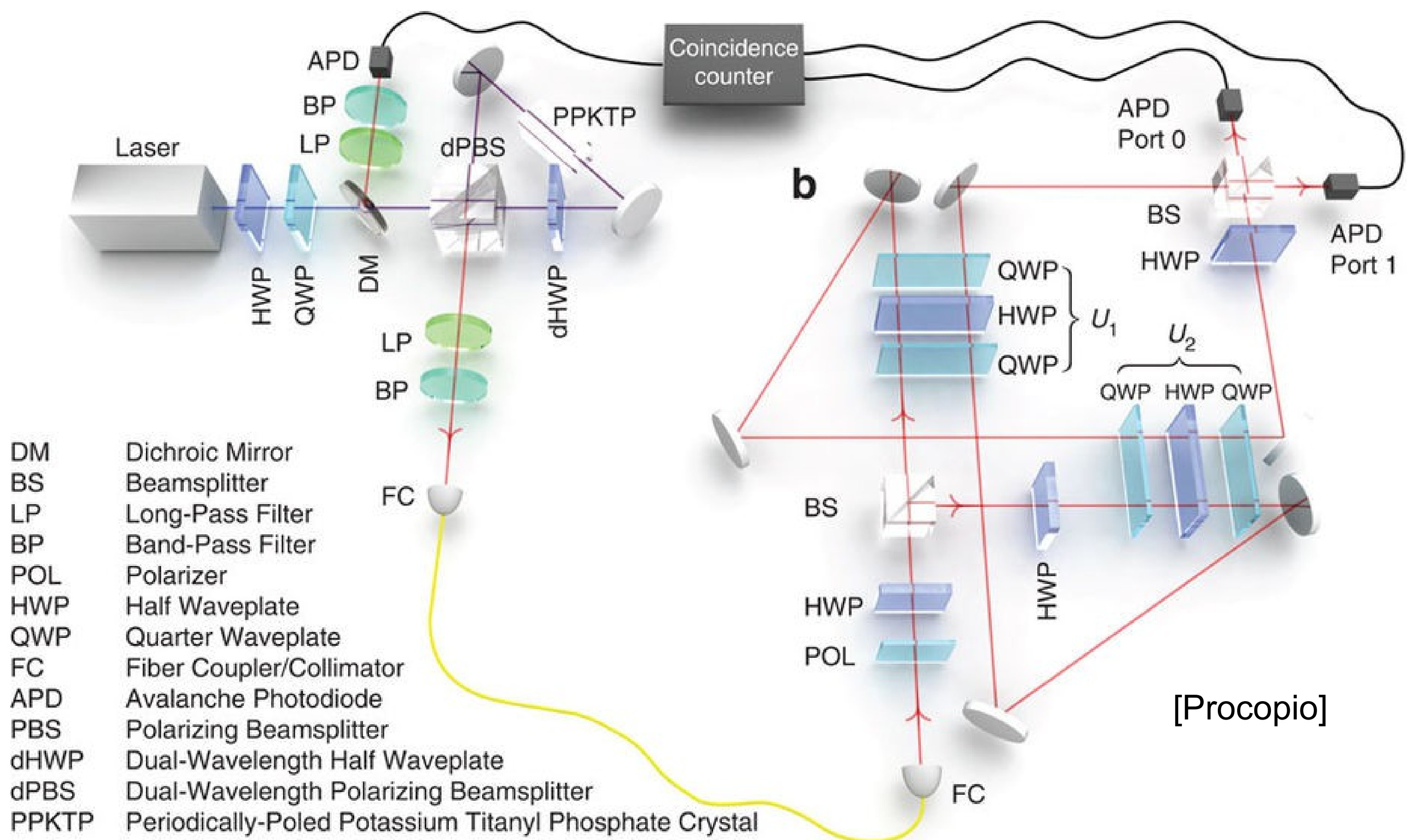


Testing permutation of unitaries: polynomial reduction in query complexity
[Colnaghi et al, Araújo et al]



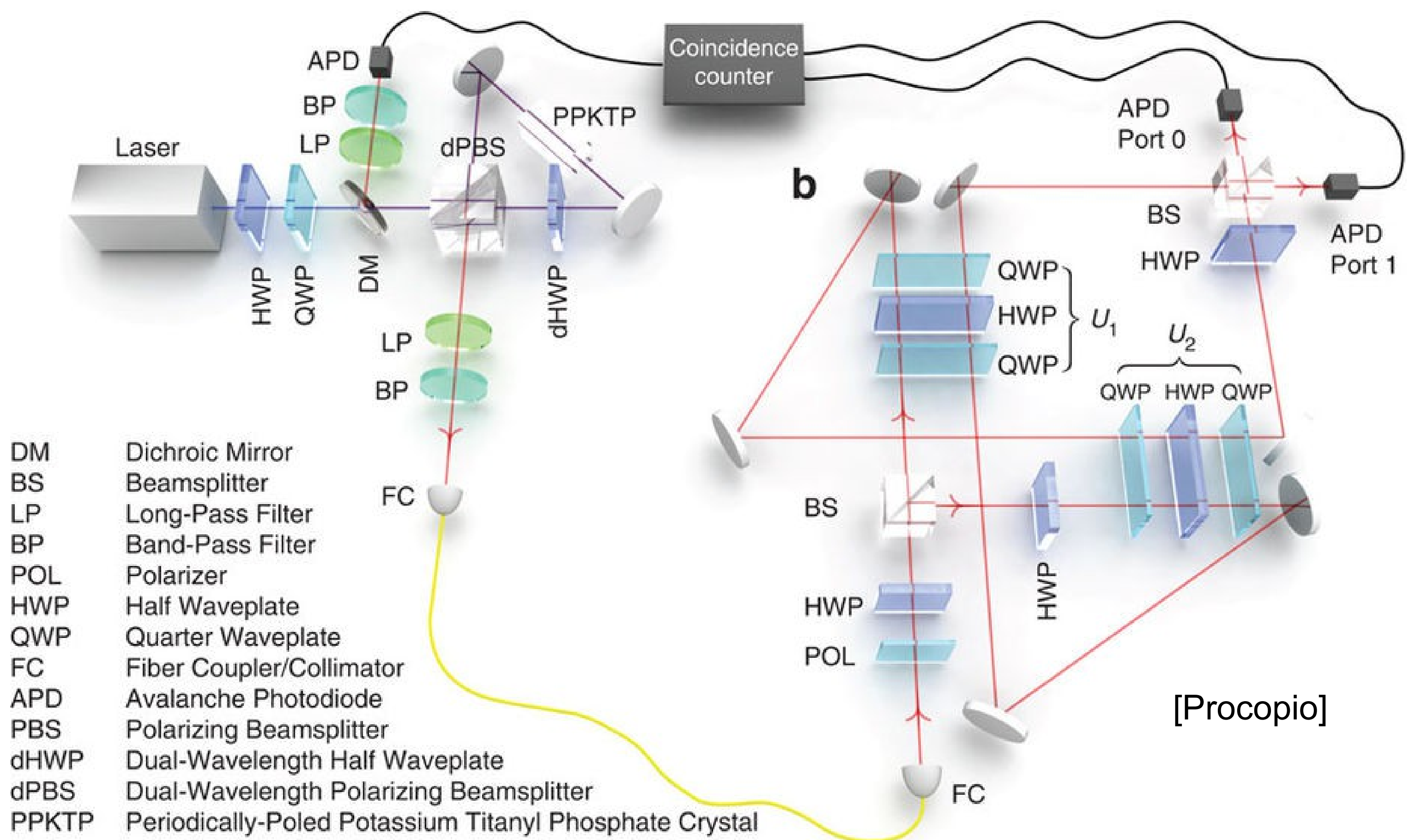
Exchange evaluation game: exponential advantage in communication complexity
[Guérin et al]

Experimental superposition of orders of quantum gates

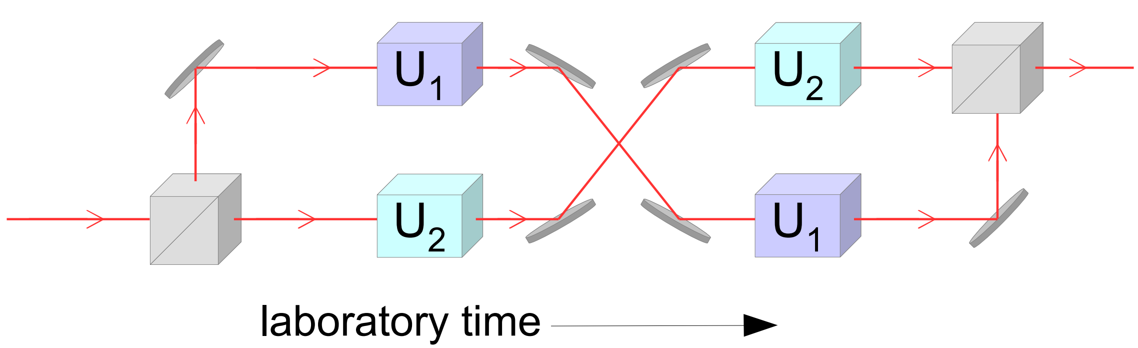


[Procopio]

Experimental superposition of orders of quantum gates



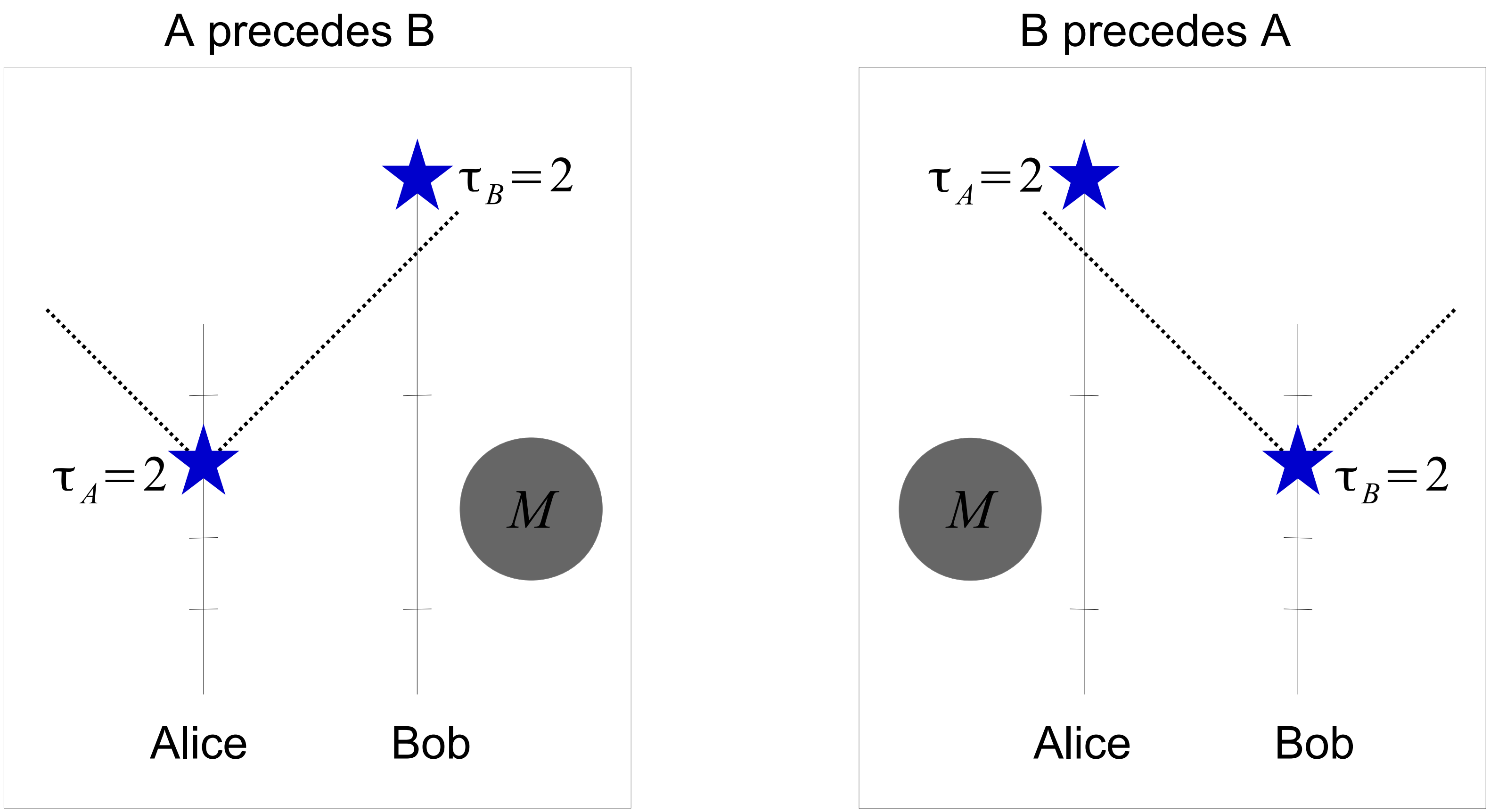
[Procopio]



How to count the number of uses of the black boxes?

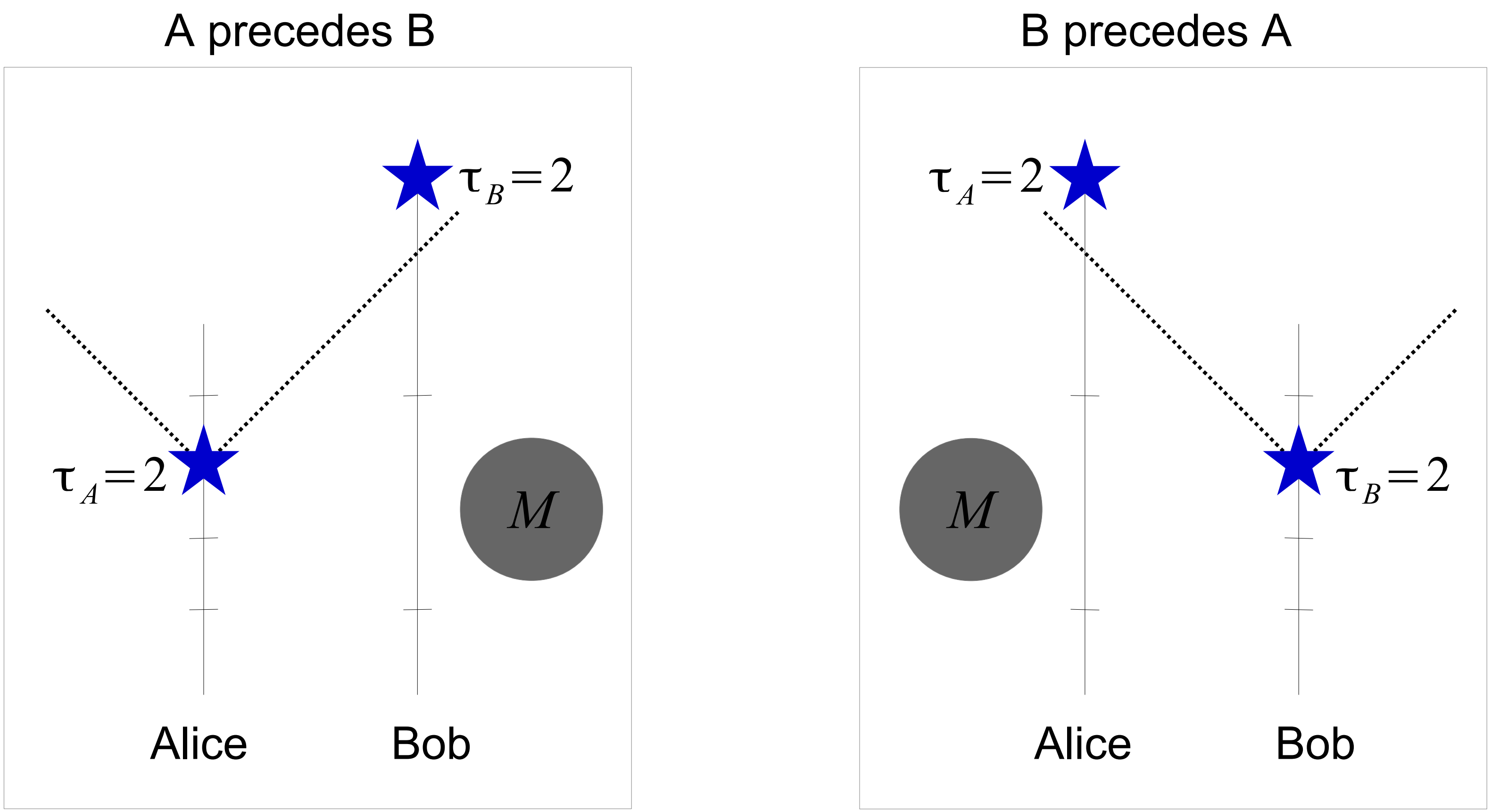
- points in spacetime
- internal degree of freedom

Superposition of causal structures using general relativity



[Feix&Brukner]

Superposition of causal structures using general relativity



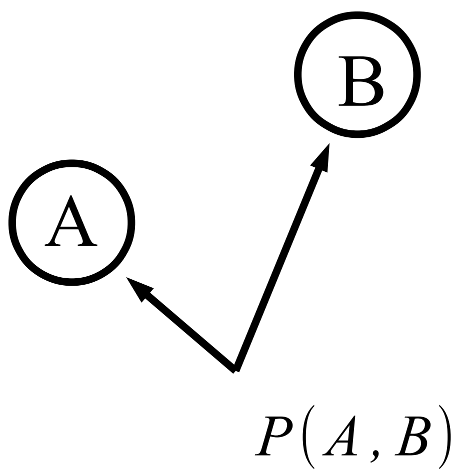
required parameters:
for a spatial superposition of $\Delta x = 10^{-3} m,\ M = 1\text{g}$ need time resolution of 10^{-27}
experimentally feasible:
spatial superposition of $\Delta x = 10^{-6} m,\ M \approx 10^{-21} g$, time resolution 10^{-18}

[Feix&Brukner]

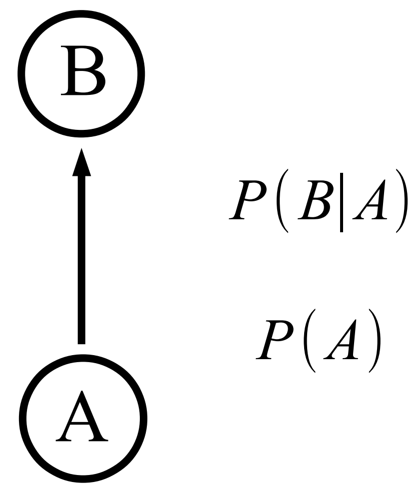
Causal Structure in a Quantum World

Combinations of causal relations

Testbed for combining causal relations
within a well-defined causal order:

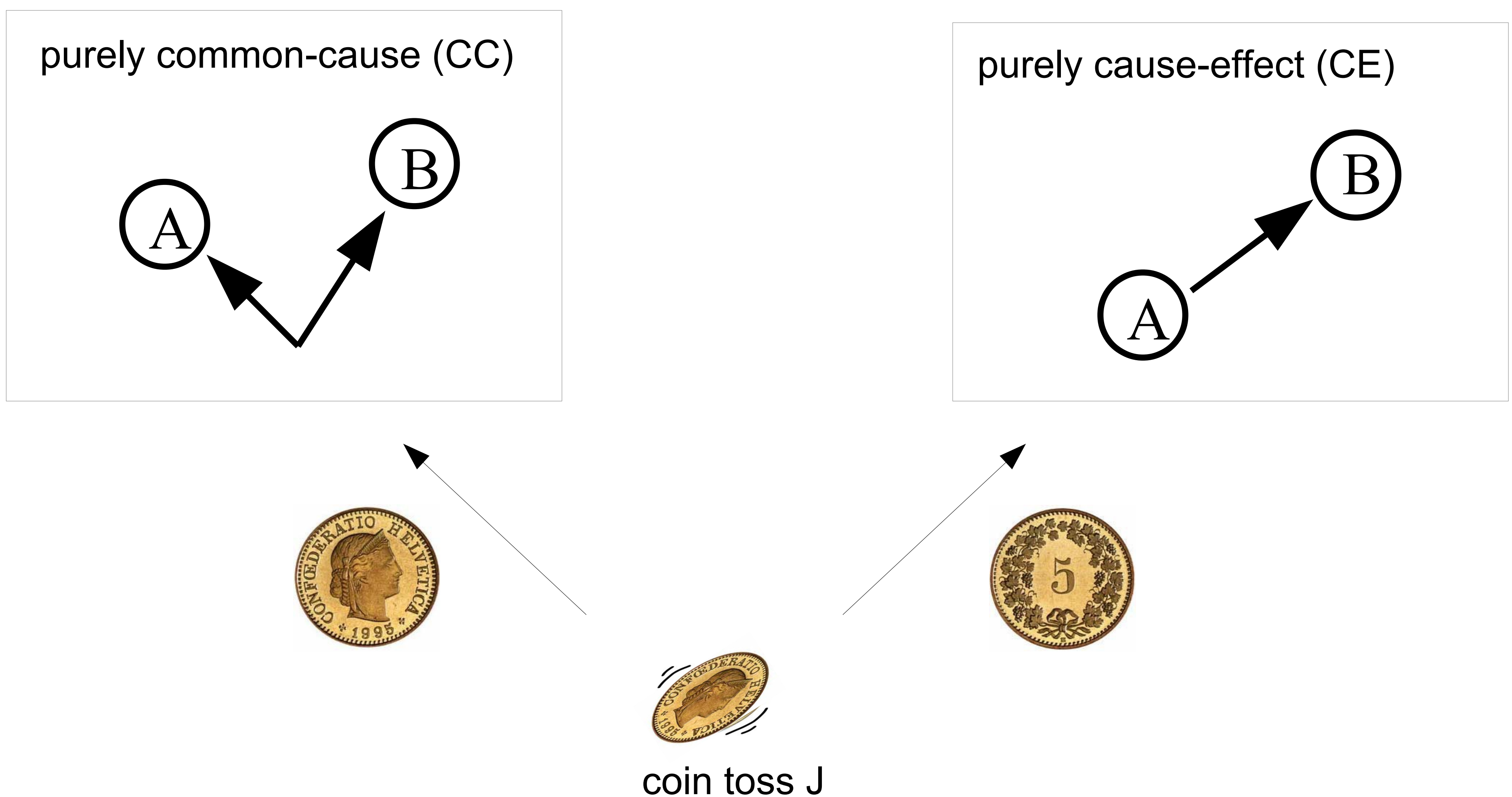


common cause

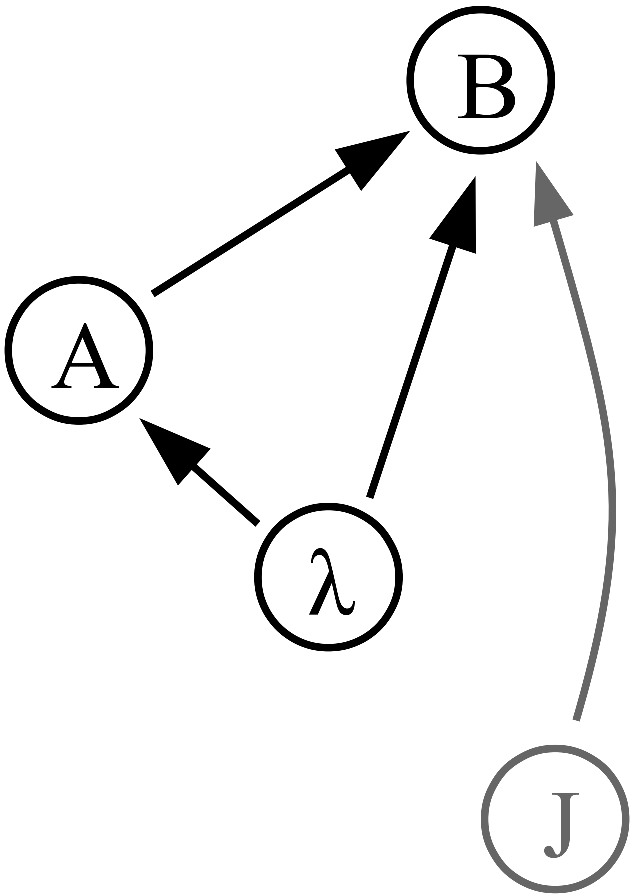


cause-effect

Mixing common-cause and cause-effect relations



Mixing common-cause and cause-effect relations



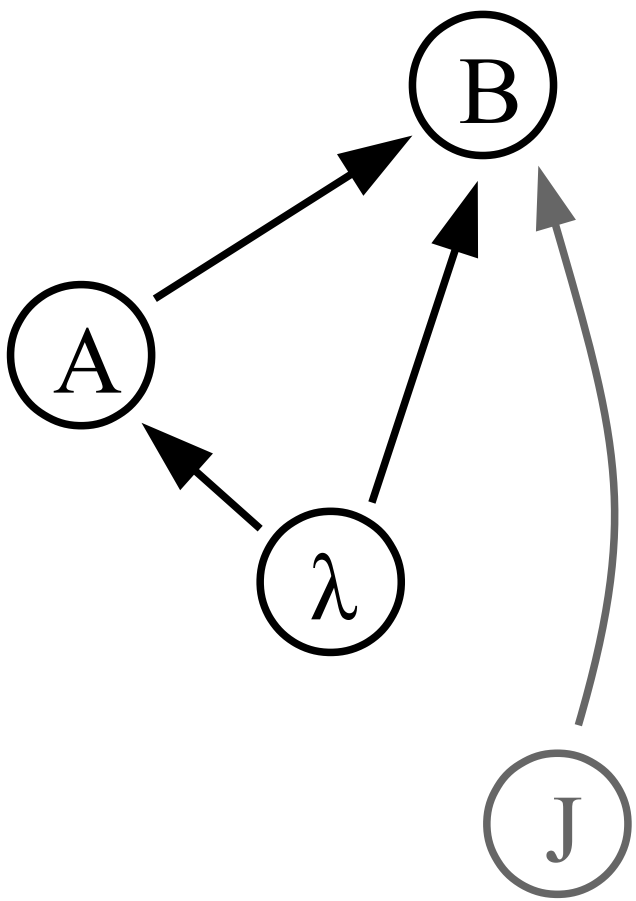
$P(B|A,\lambda,\text{heads})=P(B|\lambda)$

$\Rightarrow$ purely common-cause

$P(B|A,\lambda,\text{tails})=P(B|A)$

$\Rightarrow$ purely cause-effect

Mixing common-cause and cause-effect relations



Probabilistic mixture:

$P(B|A,\lambda,\text{heads})=P(B|\lambda)$

$\Rightarrow$ purely common-cause

$P(B|A,\lambda,\text{tails})=P(B|A)$

$\Rightarrow$ purely cause-effect

Physical mixture:

$P(B|A,\lambda,\text{heads})=P(B|A,\lambda)$

$P(B|A,\lambda,\text{tails})=P(B|A,\lambda)$

$\Rightarrow$ both CC and CE

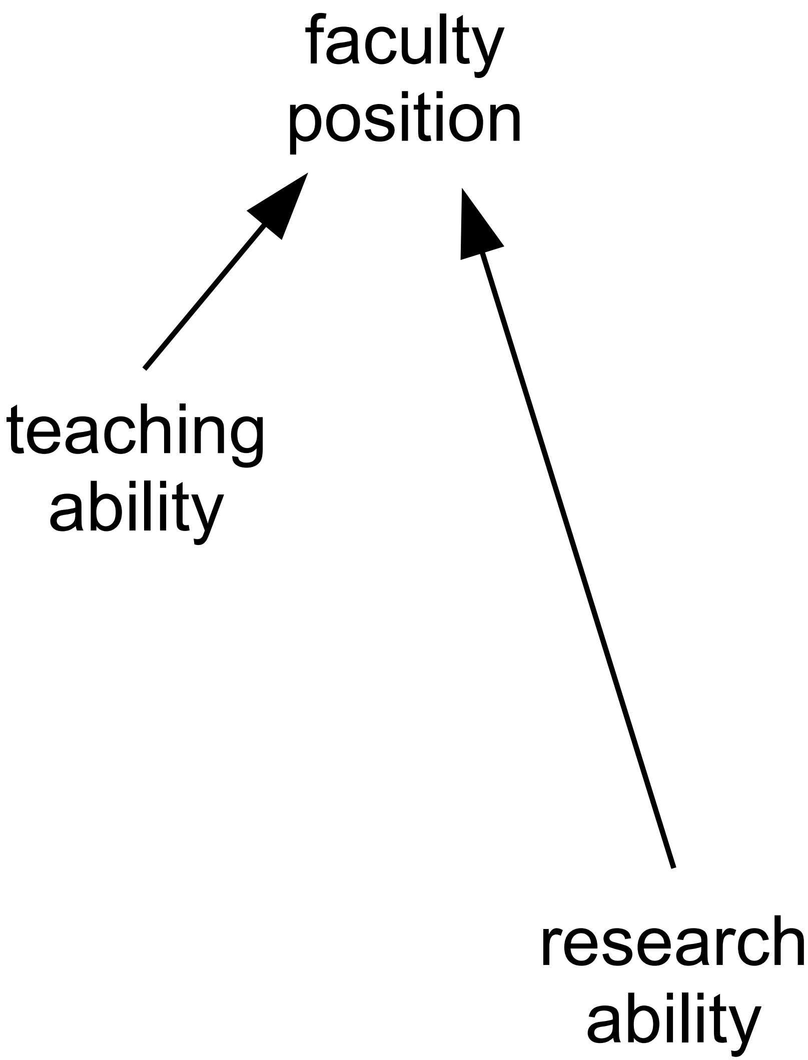
Note: both of these ways of combining causal relations are **classical**.

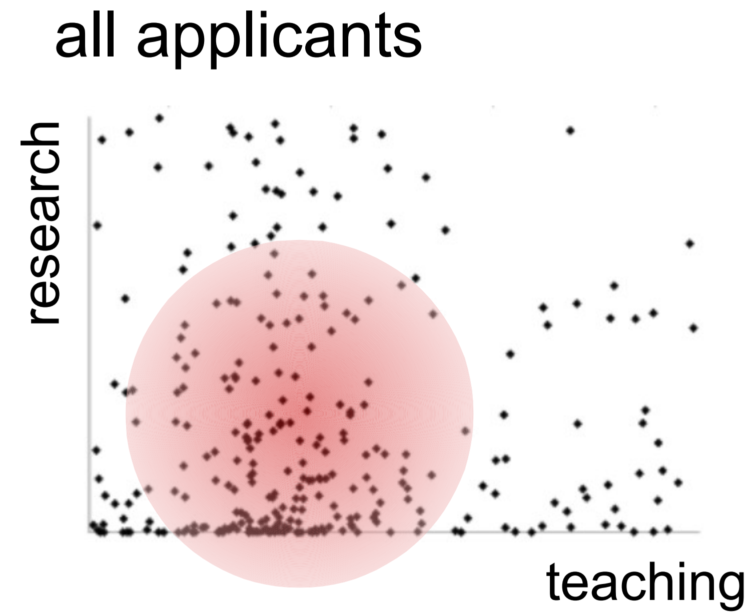
Probabilistic mixture:

Physical mixture:

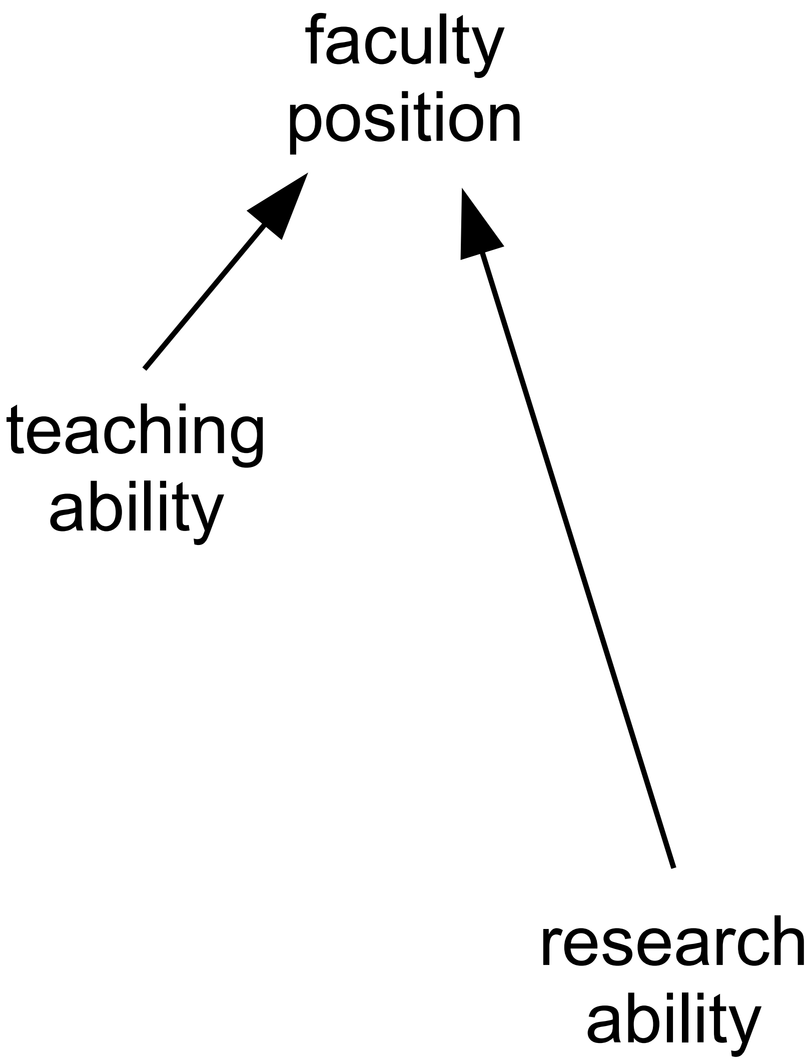
How to detect a combination of two causal influences?

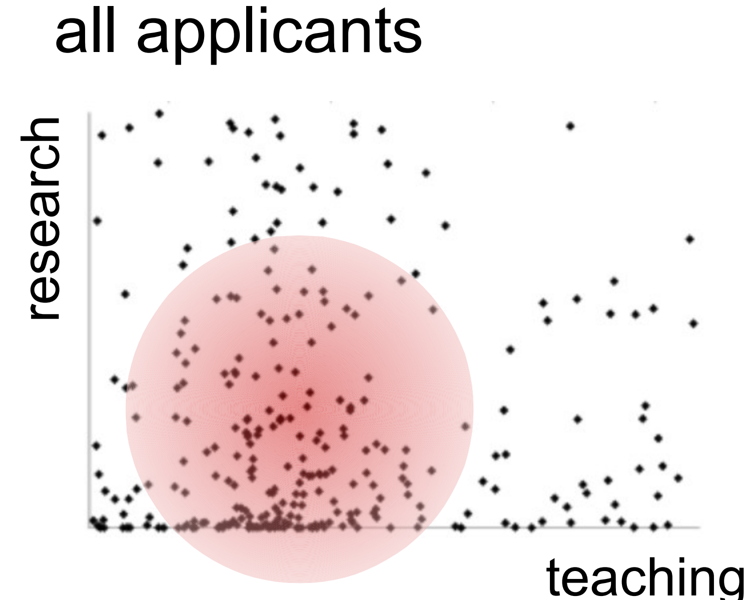
Berkson's paradox

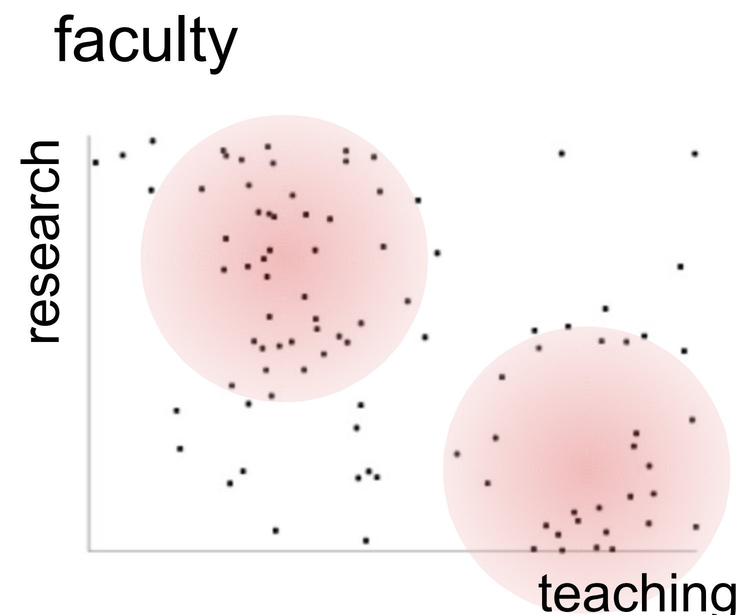




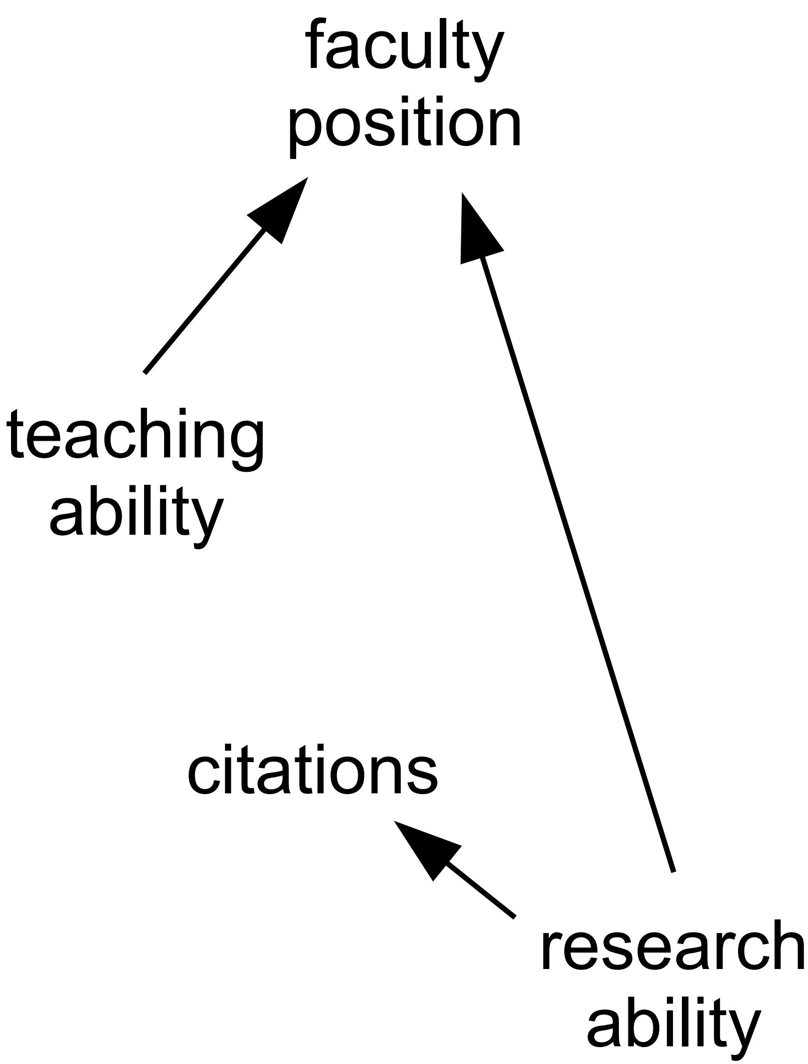
Berkson's paradox

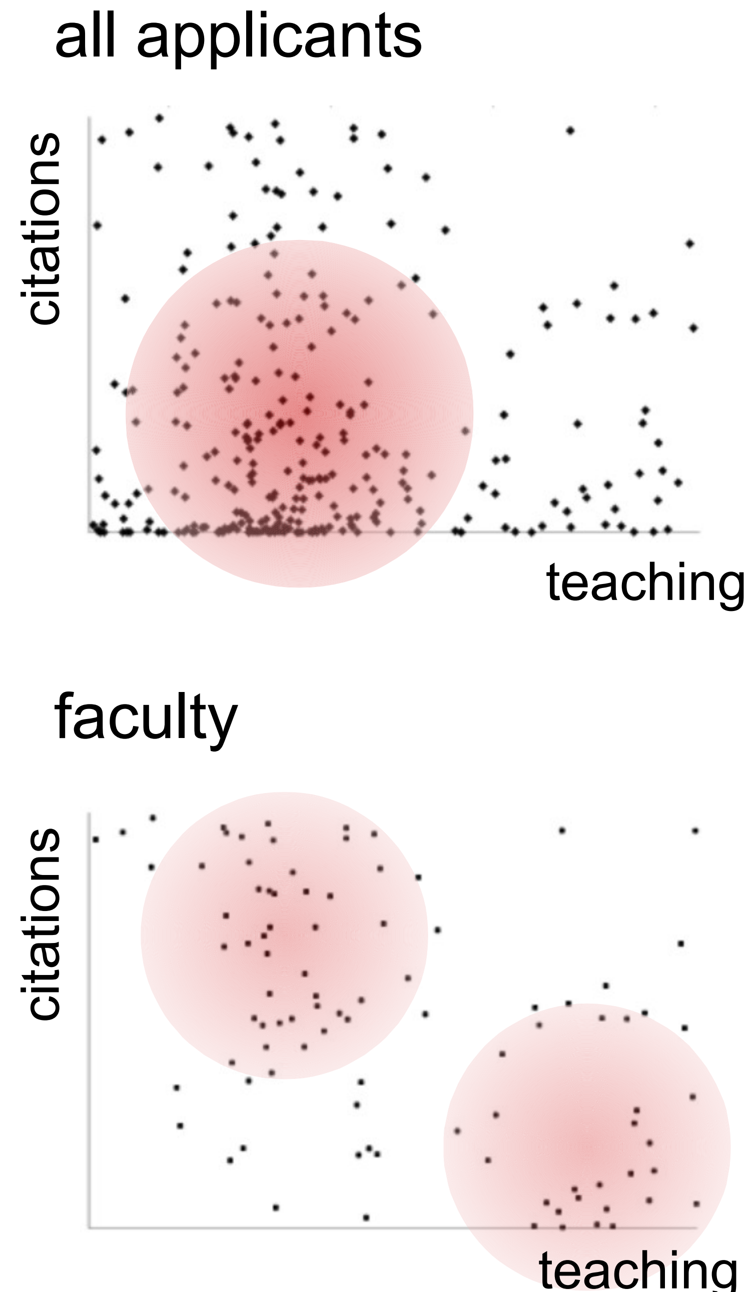


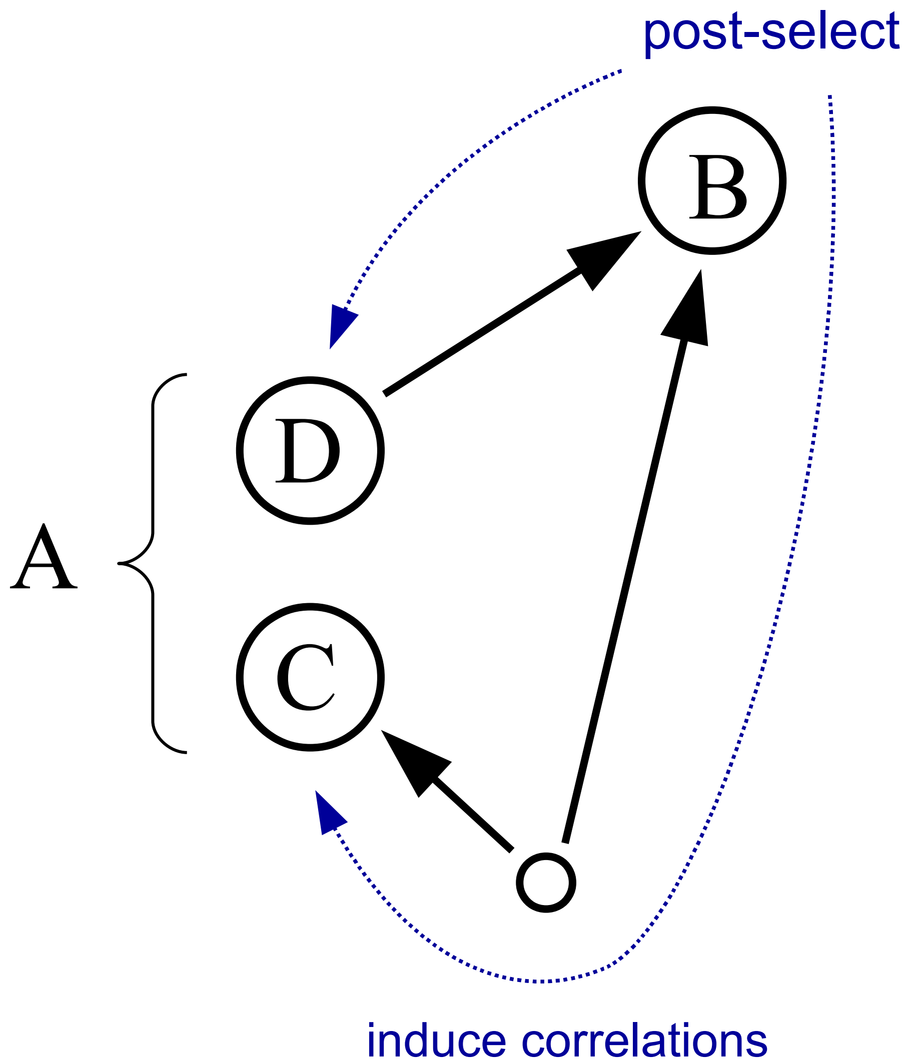




Berkson's paradox
(extended edition)







Berkson-type
induced correlations:

- classical

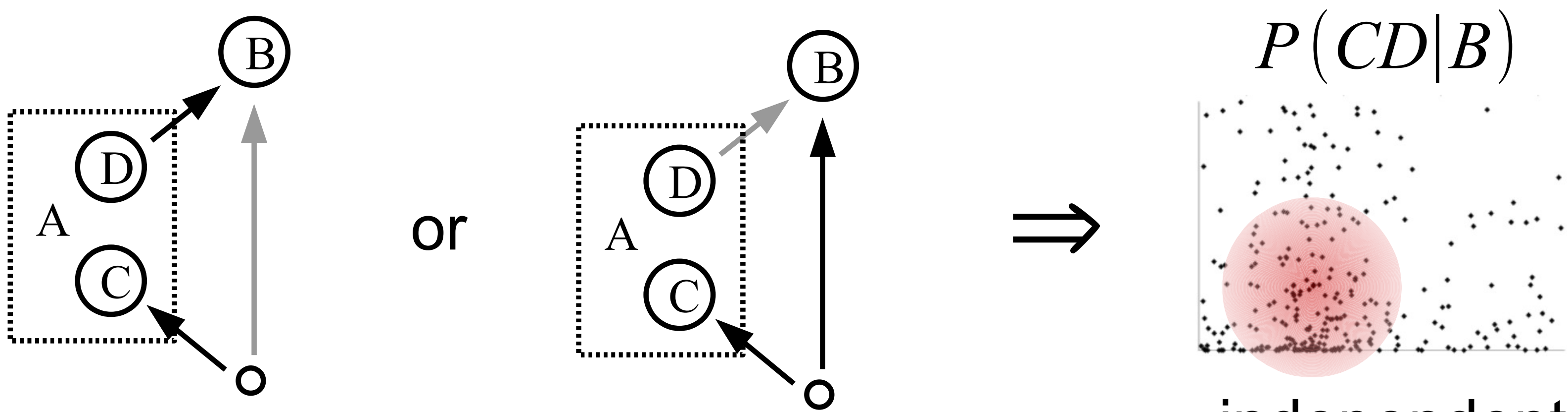
$$P(CD|B)$$

- quantum

$$\{E_B^b\} \Rightarrow \{\boldsymbol{\tau}_{CD}^b\}$$

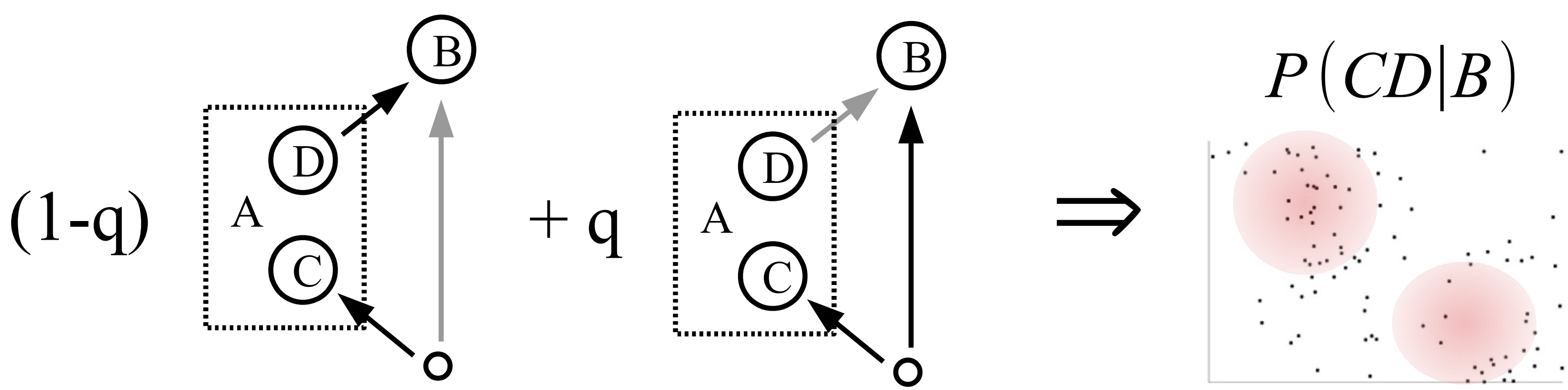
Distinguishing combinations of causal structures

purely cause-effect
or purely common-cause:



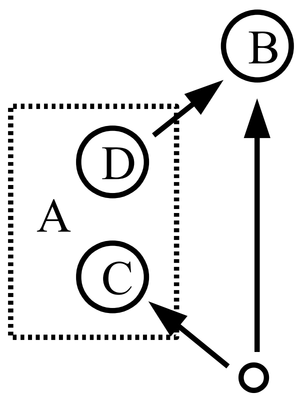
independent

probabilistic mixture:



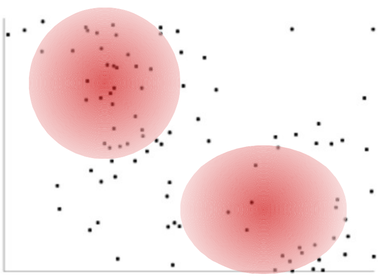
weak correlations

physical mixture (not probabilistic):



$\Rightarrow$

$P(CD|B)$

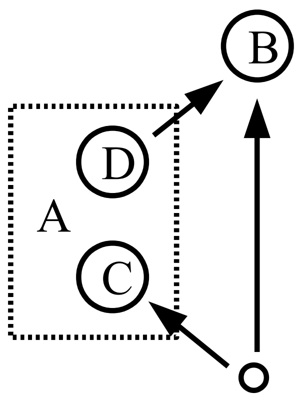


strong correlations

for example

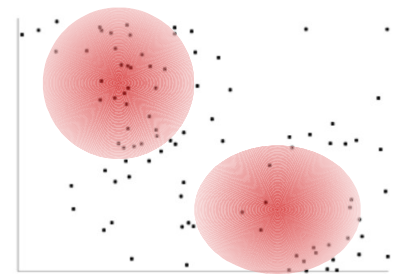
$$B = D \oplus \lambda$$
$$C = \lambda$$

physical mixture (not probabilistic):



$\Rightarrow$

$P\left(CD|B\right)$



strong correlations

for example

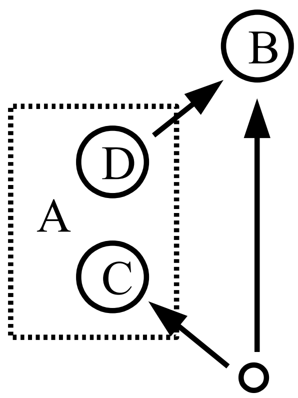
$$B = D \oplus \lambda$$
$$C = \lambda$$

then

$$B = 0$$
$$\Rightarrow C = D$$

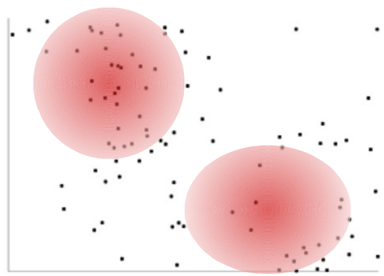
perfect correlation

physical mixture (not probabilistic):



$\Rightarrow$

$P\left(CD|B\right)$



strong correlations

for example

$$B = D \oplus \lambda$$
$$C = \lambda$$

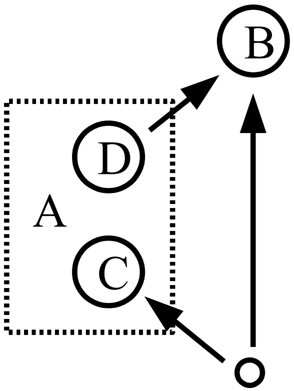
then

$$B = 0$$
$$\Rightarrow C = D$$

perfect correlation

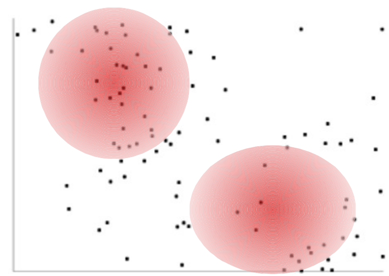
$\Rightarrow$ Conversely, strong correlations
rule out a probabilistic mixture.

physical mixture (not probabilistic):



$\Rightarrow$

$$P\left(CD|B\right)$$



strong correlations

for example

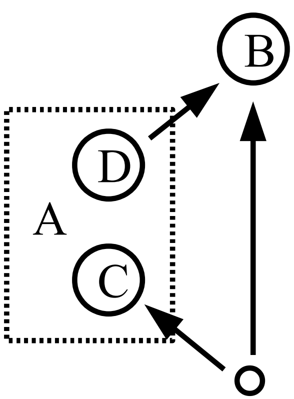
$$B = D \oplus \lambda$$
$$C = \lambda$$

then

$$B = 0$$
$$\Rightarrow C = D$$

perfect correlation

intrinsically quantum combination:

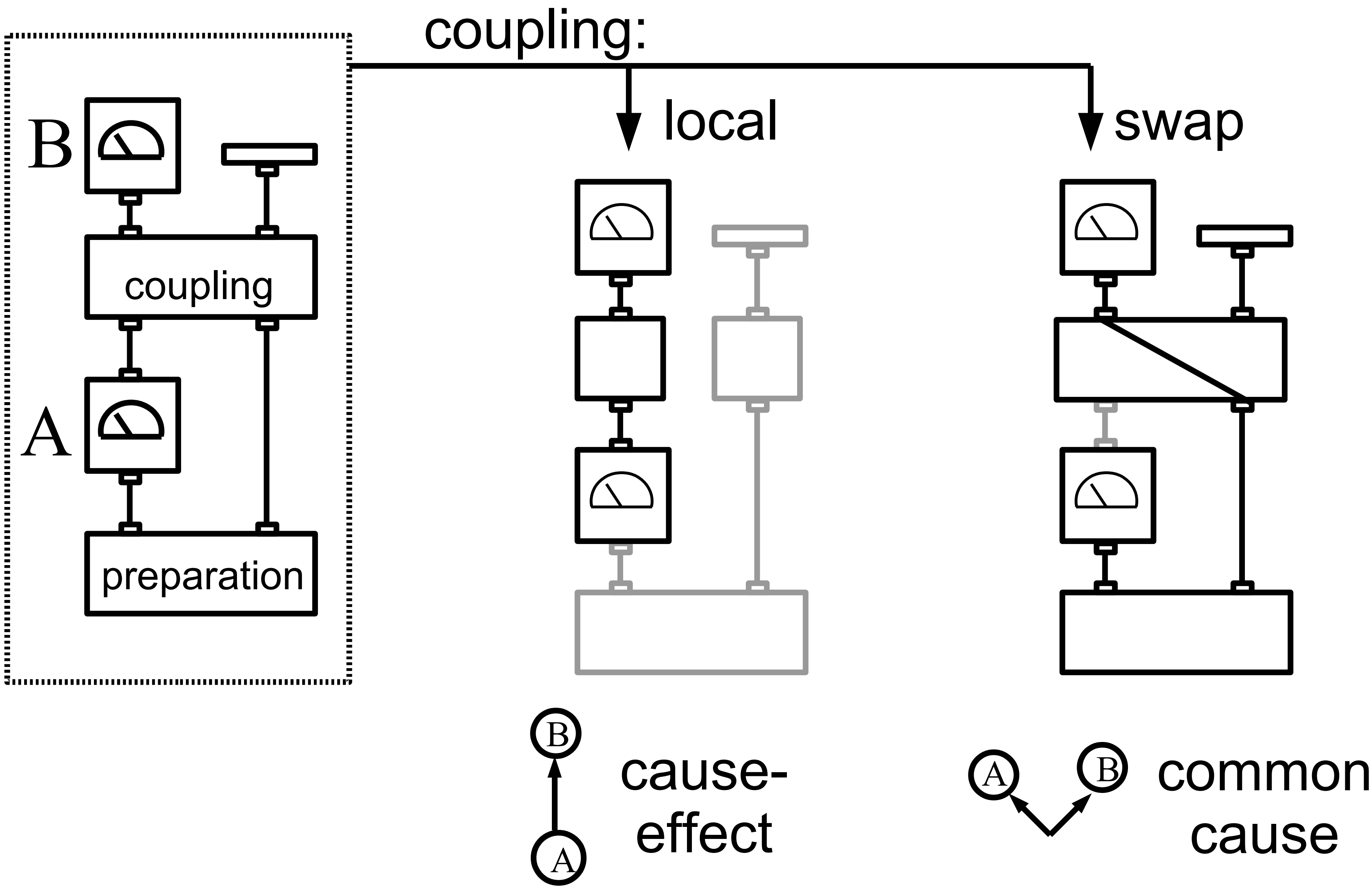


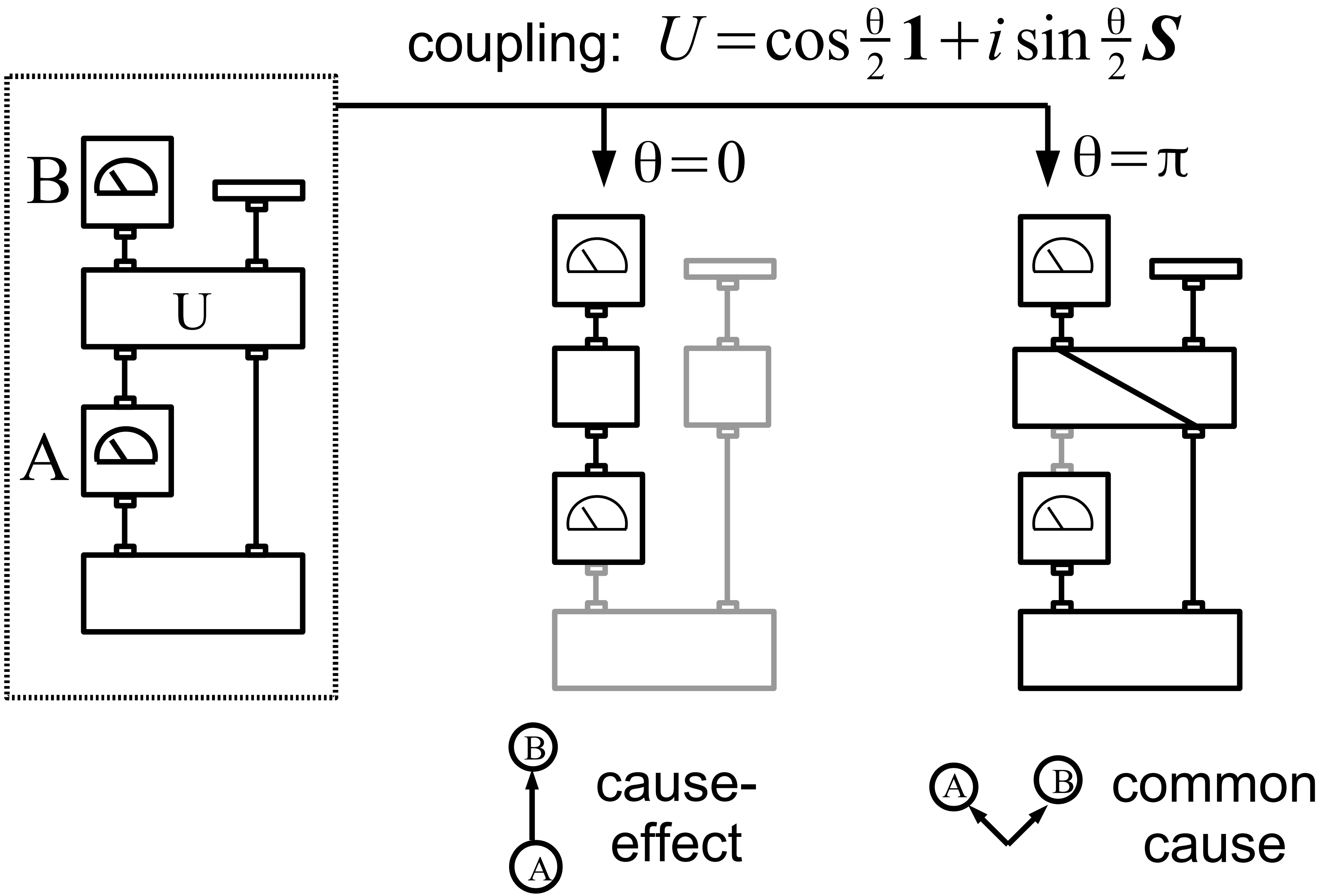
$\Rightarrow$

$$\tau\left(CD|B\right) \neq \Sigma_i \rho_C^{(i)} \otimes \rho_D^{(i)}$$

stronger-than-classical
correlations

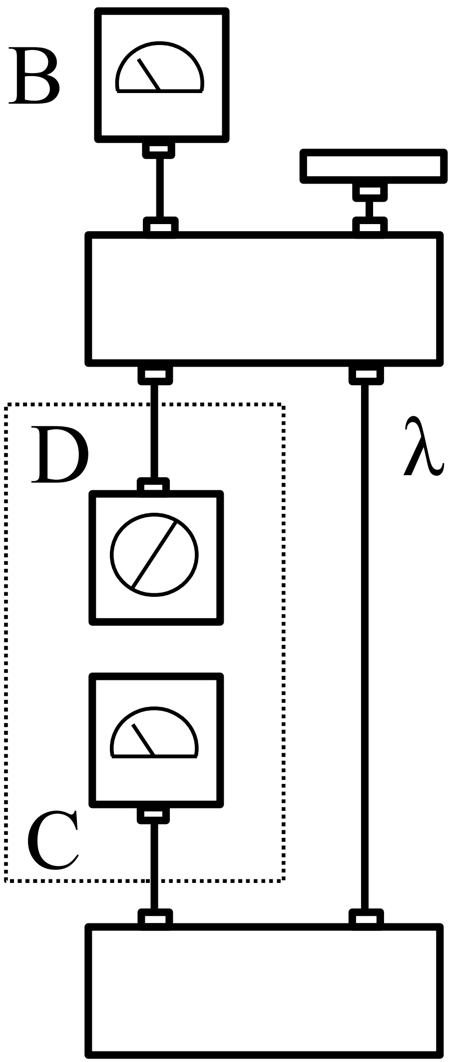
Two quantum variables with tunable causal relation





coupling: $U = \cos\frac{\theta}{2}\,\mathbf{1} + i\sin\frac{\theta}{2}\,\boldsymbol{S}$
B
A
U
$\theta = 0$
$\theta = \pi$
B
A
cause-effect
A
B
common cause

Berkson-type induced entanglement



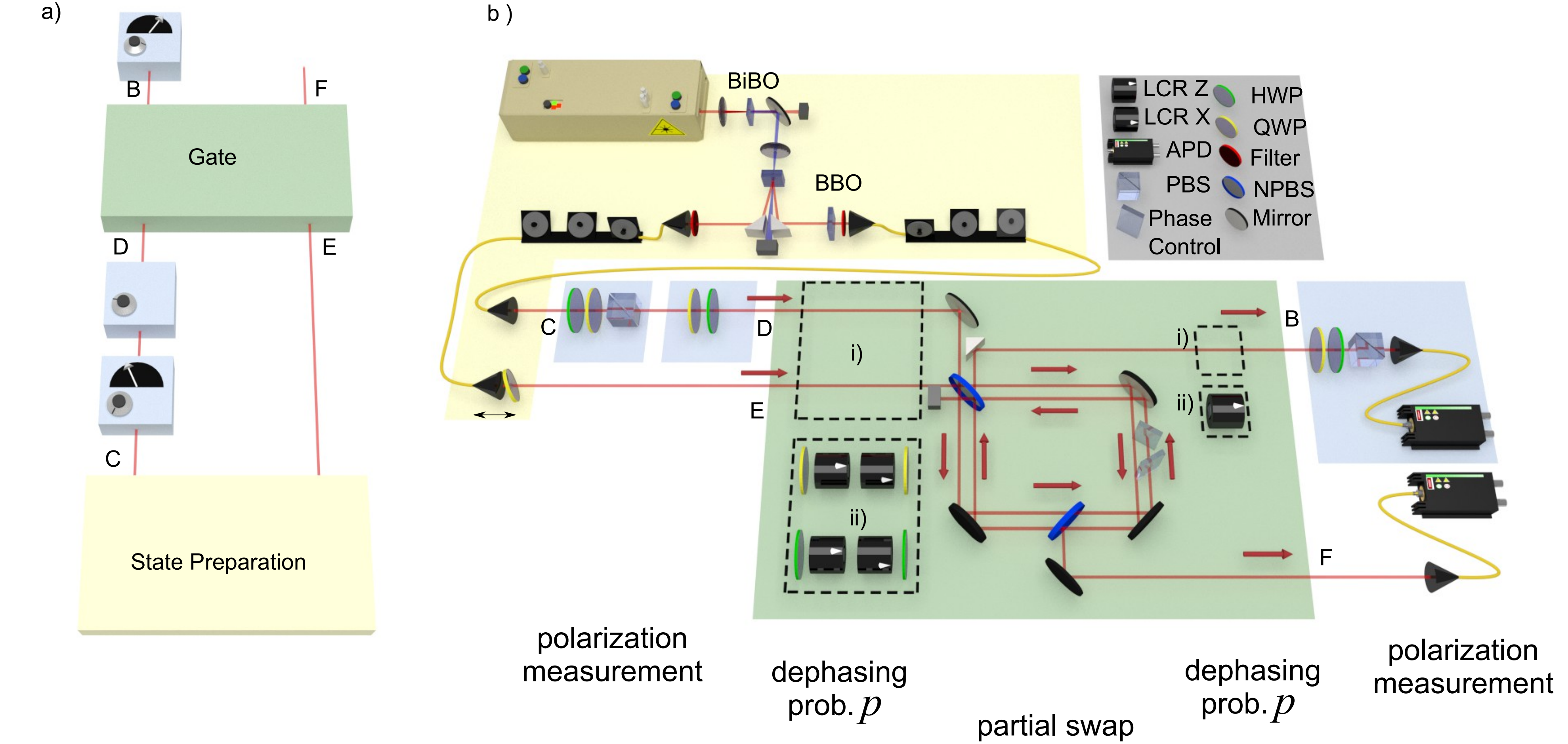
post-selection $\rho_B = |0\rangle\langle 0|$

induced state

$$\rho_{D\lambda}^{(0)} = U^\dagger \left(\rho_B \otimes \frac{1}{2}\mathbf{1} \right) U$$

$$= \frac{1}{2} U^\dagger \big[|00\rangle\langle 00| + |01\rangle\langle 01| \big] U$$

$$= \frac{1}{2} |00\rangle\langle 00| + \frac{1}{2} \underline{|\Psi^-\rangle\langle \Psi^-|}$$

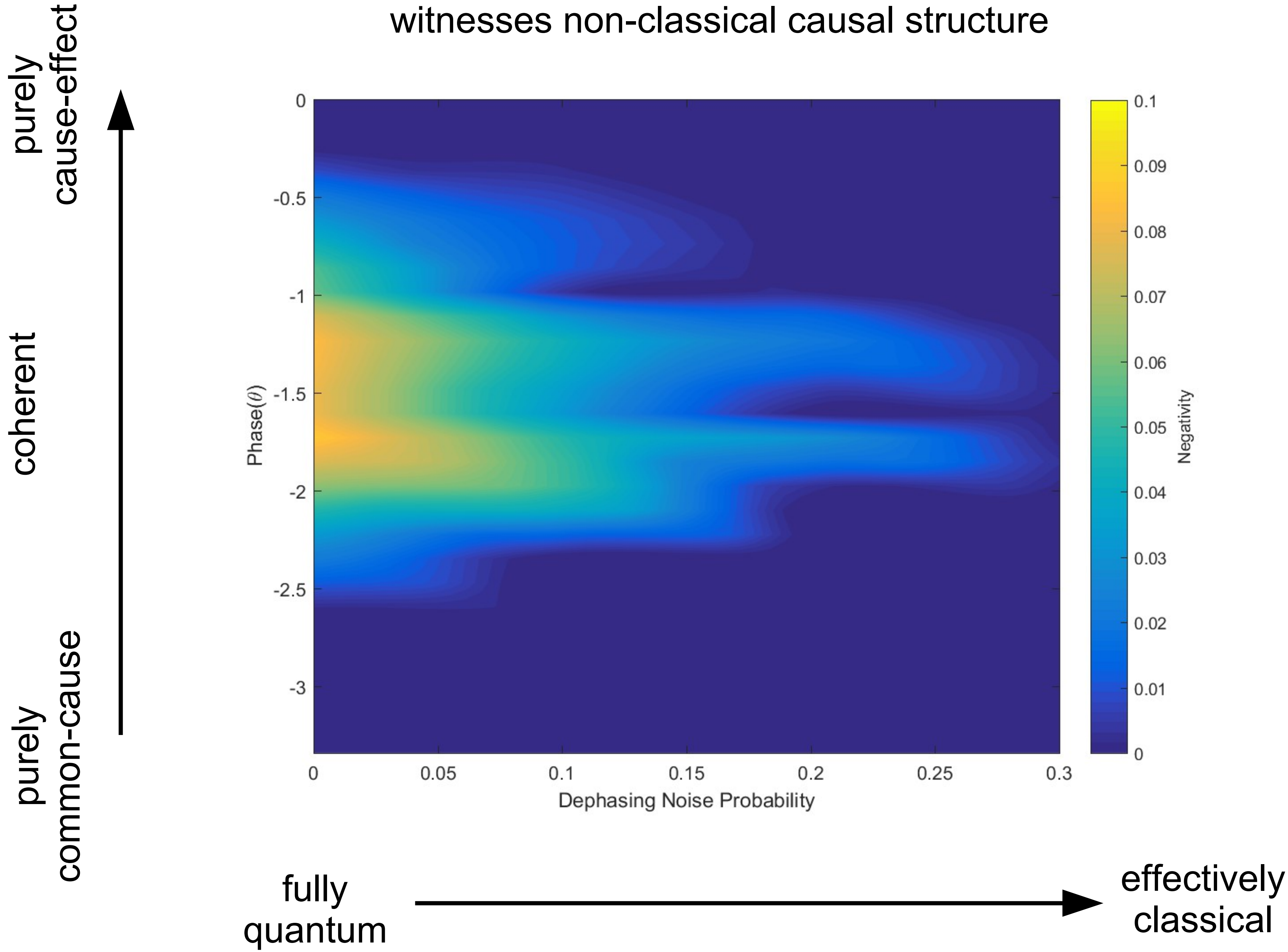


$$U = \cos\tfrac{\theta}{2}\,\mathbf{1} + i\sin\tfrac{\theta}{2}\,\boldsymbol{S}$$

[MacLean et al, Nat Comm 8]

Induced negativity

witnesses non-classical causal structure



Take-home messages and open questions

Take-home messages and open questions

Causal models framework: mathematical and conceptual toolbox

- classical: rigorous definition of causation, methods for inferring causal relations and deriving predictions from this information
- quantum: description of relations among quantum systems in terms of operators that can (at least in part) be interpreted causally, distinction between causation and inference

Causal models provide a clear language and context for analysing many counter-intuitive phenomena in quantum mechanics, such as the apparent retrocausality in delayed-choice experiments, propagation outside the lightcone in quantum field theory, and the tangle of assumptions the lead to Bell inequalities.

The conjunction of all the principles that hold in classical causal models (Reichenbach, no fine-tuning etc) is at odds with the predictions of quantum mechanics. However, it is difficult to determine which of these principles are violated. More work is needed to develop a convincing, consistent account of causality that allows one to give up any of these princples.

Take-home messages and open questions

Two proposals for how quantum mechanics might handle causal loops:

- allow generic causal loops but give up linearity, which leads to unusual information flow
- preserve linearity but allow only a restricted class of causal loops

Non-classical causal relations

- There are a few concrete examples of such scenarios, but a systematic account of all the possibilities is still outstanding.
- Some have been realized experimentally, but all experiments so far were embedded in a background spacetime with well-defined causal order. It would be interesting to overcome this limitation.
- Non-classical causal structures are known to be resources for certain tasks. What other advantages can be extracted from these phenomena and what fundamental insights does this entail?

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