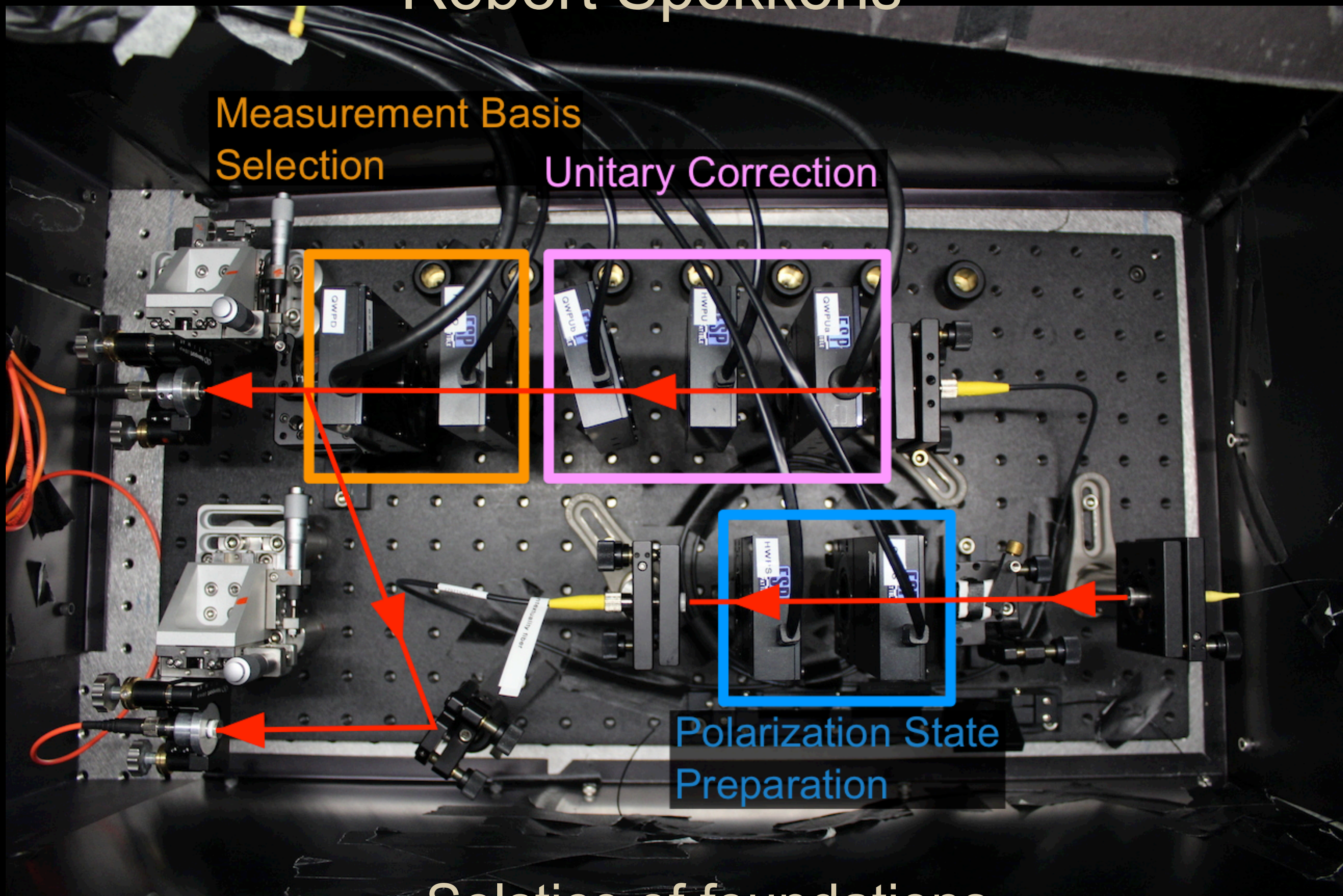
Experimental quantum foundations

Robert Spekkens



Solstice of foundations

June 19, 2017

What does a scientific theory aim to do?

Realism

It aims at a true description of physical objects and their attributes, and aims to provide successively better approximations to the truth over time. The realist endorses a correspondence theory of truth.

Empiricism

It aims at an efficient summary of our experience. The empiricist seeks to avoid false belief by building on top of what we cannot be mistaken about, such as statements about what we've observed directly.

Pragmatism

It drops the notion of truth as correspondence with reality altogether, and aims only to be useful to us in achieving various goals.

Empiricism/realism/pragmatism
as a philosophy of science

vs.

Empiricism/realism/pragmatism
as a methodological principle for devising new theories

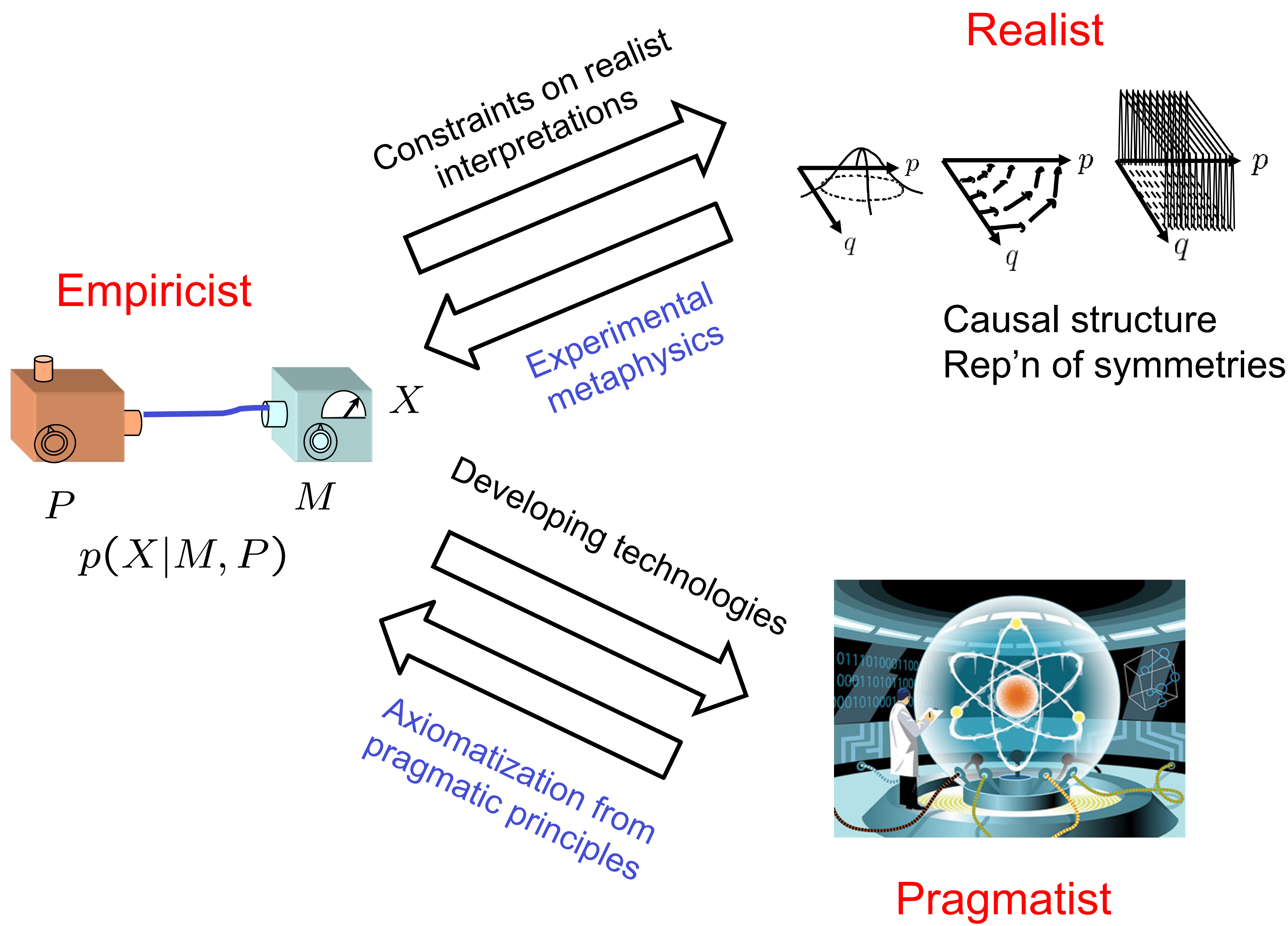
What is the historical scorecard for realism vs. empiricism vs pragmatism as methodological principles for devising new theories?

- Thermodynamics

- The atomic hypothesis

- Relativity theory

- Quantum theory


Realist
Constraints on realist interpretations
Experimental metaphysics
Causal structure
Rep'n of symmetries
Empiricist
X
P
M
$p(X|M,P)$
Developing technologies
Axiomatization from pragmatic principles
Pragmatist

Axiomatization from pragmatic principles
→ experimental consequences of pragmatic principles

Pragmatic principles such as:
- Second law
- No superluminal signalling
- Data processing inequality

are unlikely to be violated, so one would like to know the scope of physical theories that respect them

Variation of axioms a good way to probe alternatives to QT (contrast w/ Weinberg's proposed modification of QT)

Experimental metaphysics
→ Experimental consequences of ontological principles

Provide constraint on ontological possibilities for **all future theories of physics**

This is a precise sense in which experimental quantum foundations distinguishes itself from experiments in the rest of physics

Frameworks for describing theories

Realist

Ontological models

GPTs w/ symmetries

Process theories

Interventionist Causal models

Generalized probabilistic theories

Empiricist

Theory of Bayesian inference

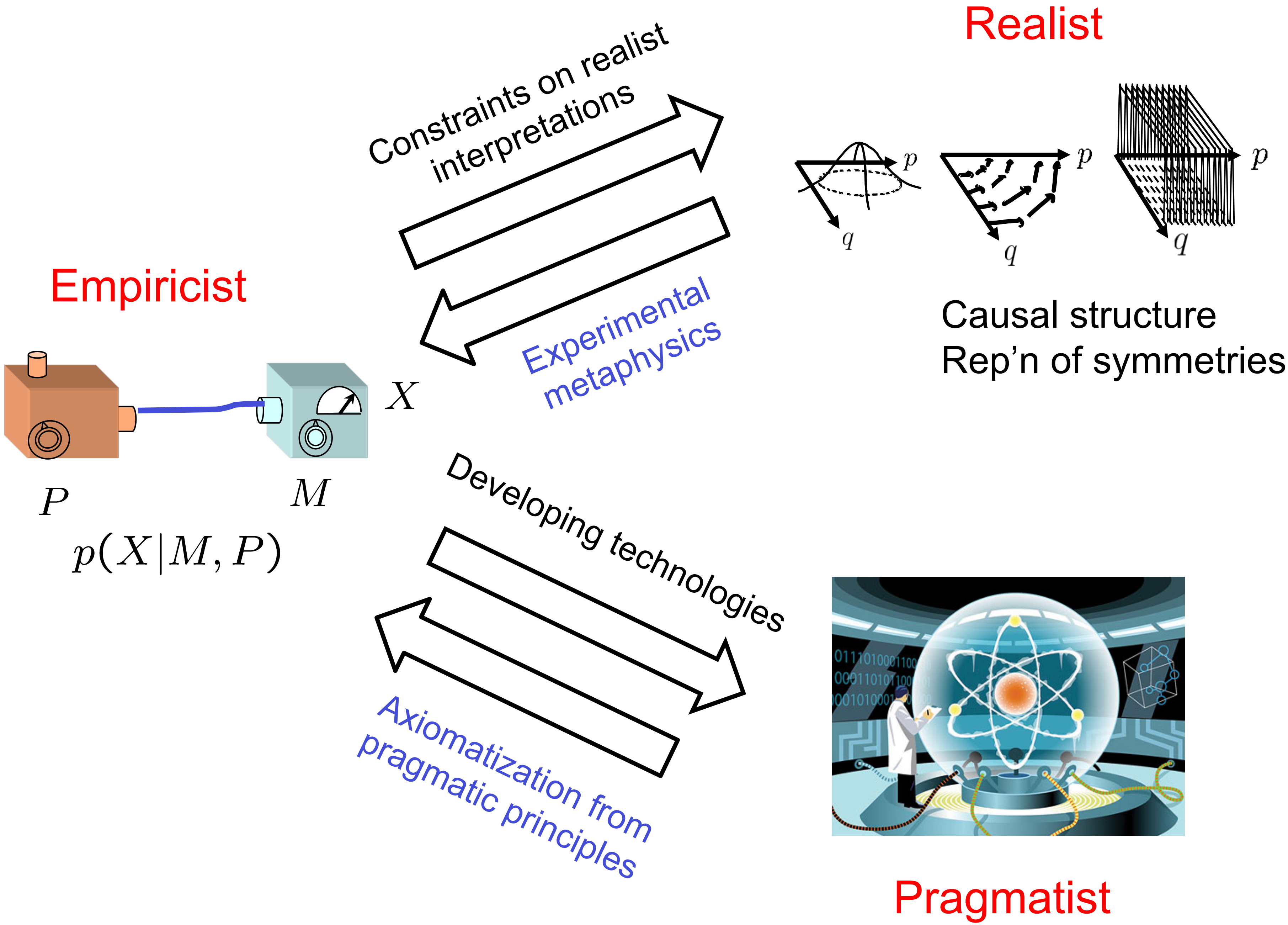
Device-independent paradigm

Thermodynamic

Resource theories

Information processing

Pragmatist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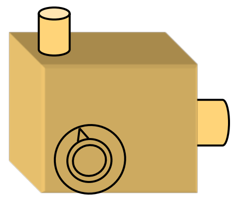
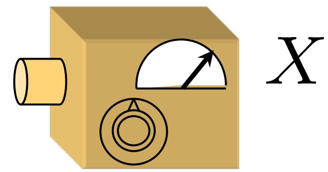
Pragmatist

The framework of generalized probabilistic theories (GPTs)

See: L. Hardy, quant-ph/0101012
J. Barrett, PRA 75, 032304 (2007)

The framework of generalized probabilistic theories





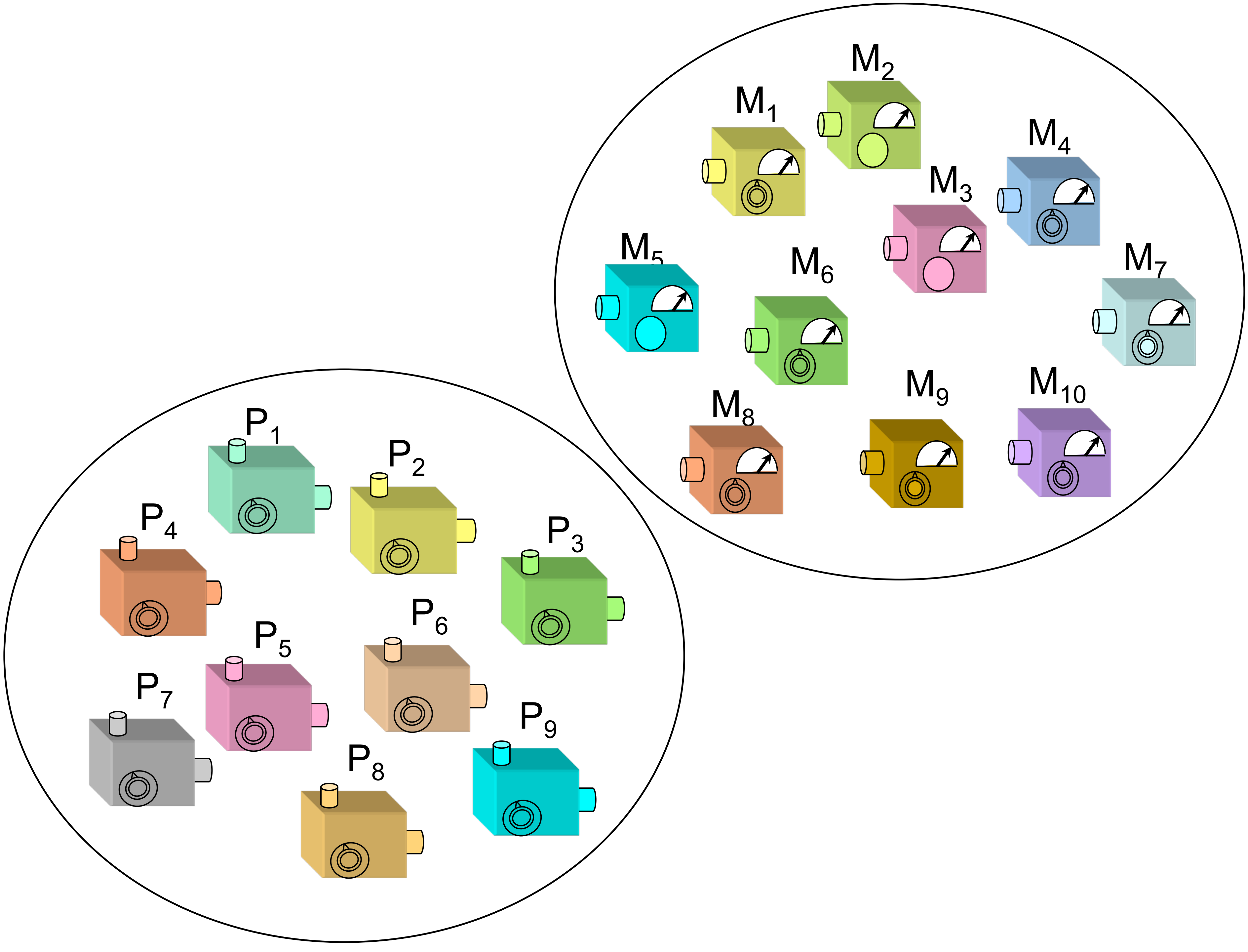
Preparation

P

Measurement

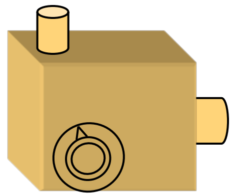
M

$$Pr(X|\mathsf{P},\mathsf{M})$$



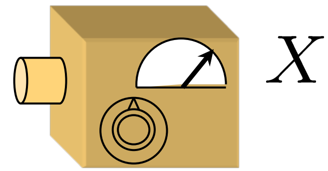
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

The framework of generalized probabilistic theories



Preparation

P



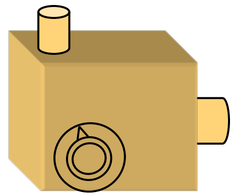
Measurement

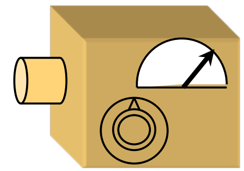
M

$$\mathbf{s}_P = \begin{pmatrix} \mathrm{Pr}(1|\mathrm{P},\mathrm{M}_1) \\ \mathrm{Pr}(2|\mathrm{P},\mathrm{M}_1) \\ \mathrm{Pr}(1|\mathrm{P},\mathrm{M}_2) \\ \mathrm{Pr}(2|\mathrm{P},\mathrm{M}_2) \\ \mathrm{Pr}(3|\mathrm{P},\mathrm{M}_2) \\ \vdots \end{pmatrix} \qquad \mathbf{r}_{M,X} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

$$Pr(X|\mathrm{P},\mathrm{M}) = \mathbf{r}_{M,X} \cdot \mathbf{s}_P$$

The framework of generalized probabilistic theories





pass or fail

Preparation

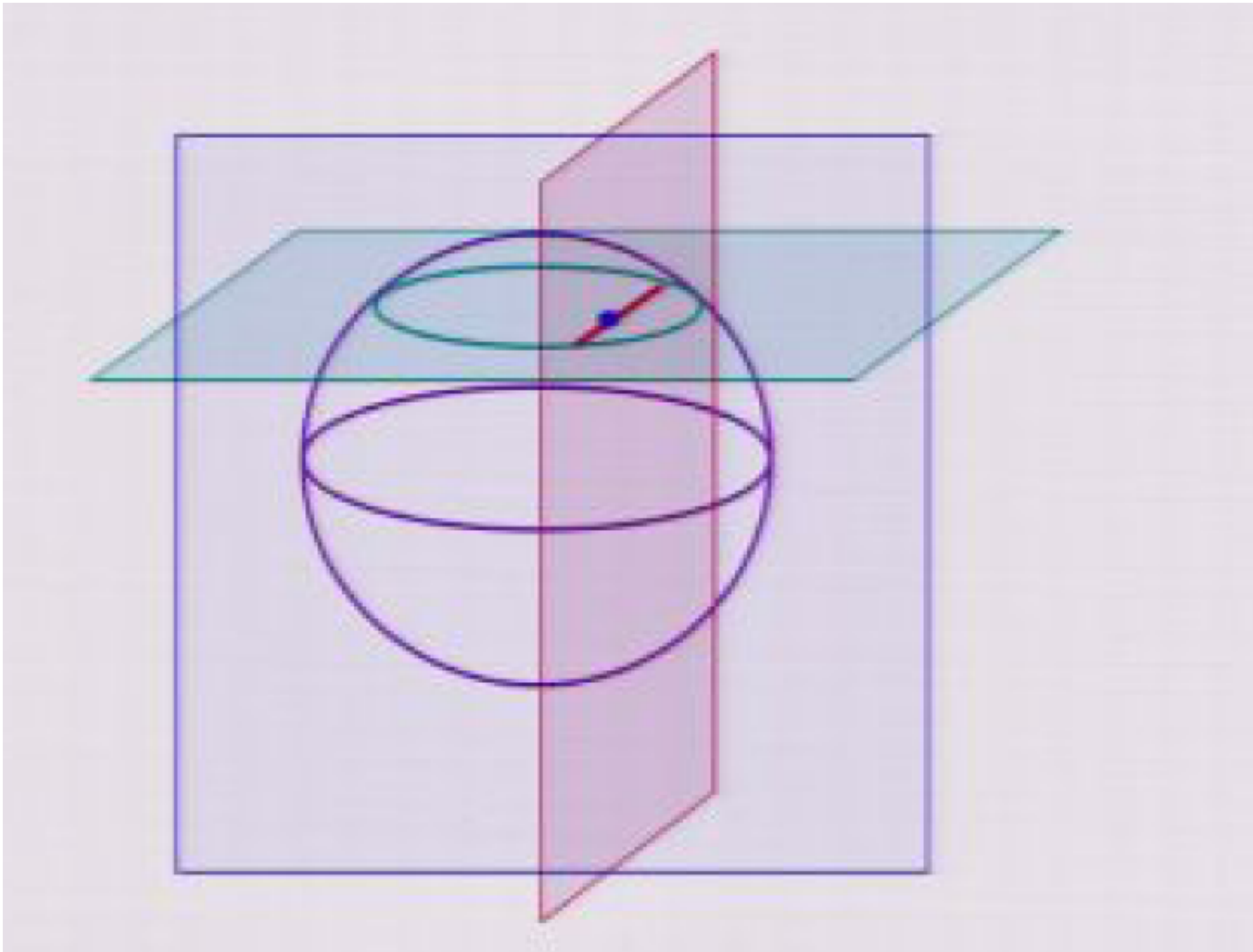
P

Measurement

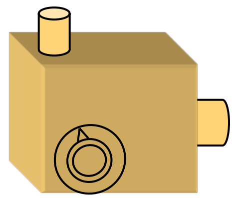
M

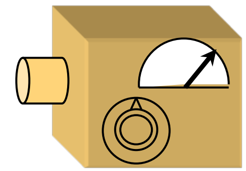
Suppose there are K measurements in a tomographically complete set (pass-fail mmts from which one can infer the statistics for all mmts)

State tomography for a single qubit



The framework of generalized probabilistic theories





pass or fail

Preparation

P

Measurement

M

Suppose there are K measurements in a tomographically complete set (pass-fail mmts from which one can infer the statistics for all mmts)

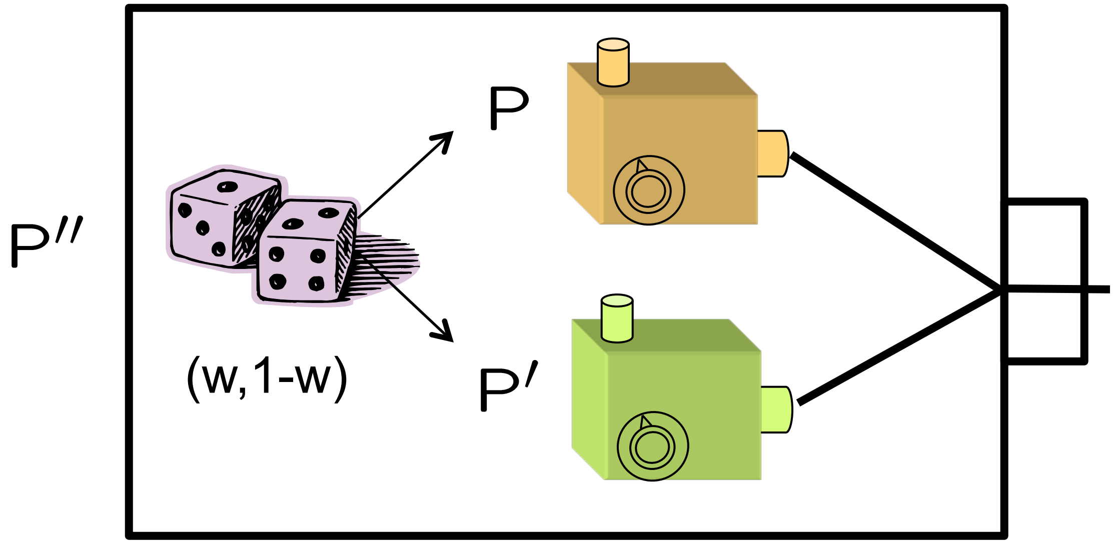
$$\mathbf{s}_P = \begin{pmatrix} \mathsf{Pr}(\mathsf{pass}|\mathsf{P},\mathsf{M}_1) \\ \mathsf{Pr}(\mathsf{pass}|\mathsf{P},\mathsf{M}_2) \\ \vdots \\ \mathsf{Pr}(\mathsf{pass}|\mathsf{P},\mathsf{M}_K) \end{pmatrix}$$

"operational state"

$$\mathsf{Pr}(\mathsf{pass}|\mathsf{P},\mathsf{M}) = f_{M,\mathsf{pass}}(\mathbf{s}_P)$$

What can we say about *f*?

Operational states form a convex set



$$\forall M, k : p(k|\mathsf{P''}, \mathsf{M}) = w\, p(k|\mathsf{P}, \mathsf{M}) + (1-w)\, p(k|\mathsf{P'}, \mathsf{M})$$

$$f(\mathbf{s}_{\mathsf{P''}}) = w\ f(\mathbf{s}_{\mathsf{P}}) + (1-w)\ f(\mathbf{s}_{\mathsf{P'}})$$

Also true for mmts in tomo. complete set, so $\mathbf{s}_{\mathsf{P''}} = w\ \mathbf{s}_{\mathsf{P}} + (1-w)\ \mathbf{s}_{\mathsf{P'}}$

Closed under convex combination –> a convex set

$$f(w\ \mathbf{s}_{\mathsf{P}} + (1-w)\ \mathbf{s}_{\mathsf{P'}}) = w\ f(\mathbf{s}_{\mathsf{P}}) + (1-w)\ f(\mathbf{s}_{\mathsf{P'}})$$

Convex linear

Convex linearity implies linearity

If f is convex linear on GPT states

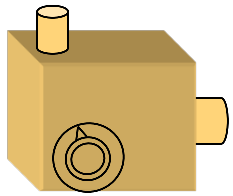
$$\mathrm{s} = \sum_i w_i \mathrm{s}_i \;\; \Rightarrow \;\; f(\mathrm{s}) = \sum_i w_i f(\mathrm{s}_i) \qquad 0 \leq w_i \leq 1 \text{ and } \sum_i w_i = 1$$

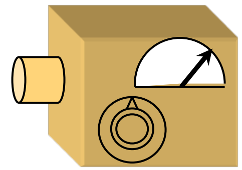
Then f is linear on GPT states

$$\mathrm{s} = \sum_i \alpha_i \mathrm{s}_i \;\; \Rightarrow \;\; f(\mathrm{s}) = \sum_i \alpha_i f(\mathrm{s}_i) \qquad \alpha_i \in \mathbb{R}$$

Therefore $\exists \mathbf{r} : f(\mathbf{s}) = \mathbf{r} \cdot \mathbf{s}$

The framework of generalized probabilistic theories





pass or fail

Preparation

P

Measurement

M

$\mathbf{s}_P \in S$

"operational states"

$\mathbf{r}_{M,\text{pass}} \in R$

"operational effects"

S = Convex set in $\mathbb{R}^K$

R = Interval of positive cone in $\mathbb{R}^K$

S and *R* characterize the GPT theory!

$$\Pr(\text{pass}|\text{P},\text{M}) = \mathbf{r}_{M,\text{pass}} \cdot \mathbf{s}_P$$

GPT characterization of classical theory

s can be any probability distribution

S = a simplex

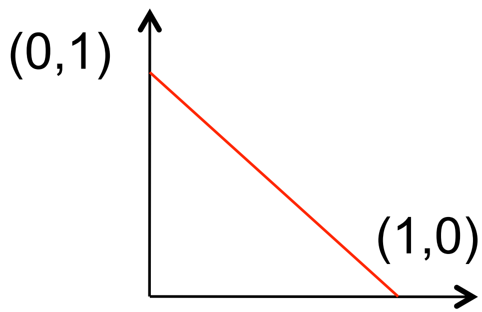
r can be any vector of conditional probabilities

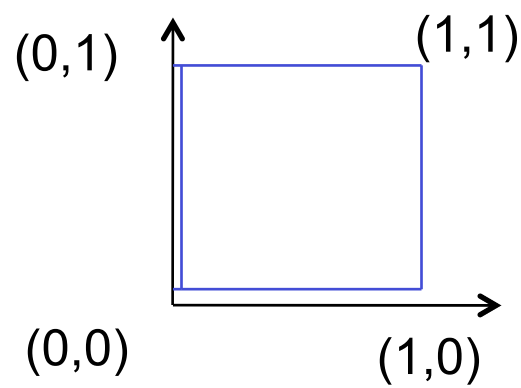
R = the unit hypercube

$$\mathbf{s} = (p(1), p(2))$$

$$\mathbf{r} = (p(\text{pass}|1), p(\text{pass}|2))$$

$$K = 2$$





$$p(\text{pass}) = \mathbf{r} \cdot \mathbf{s} = \sum_i p(i) p(\text{pass}|i)$$

GPT characterization of classical theory

s can be any probability distribution

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r can be any vector of conditional probabilities

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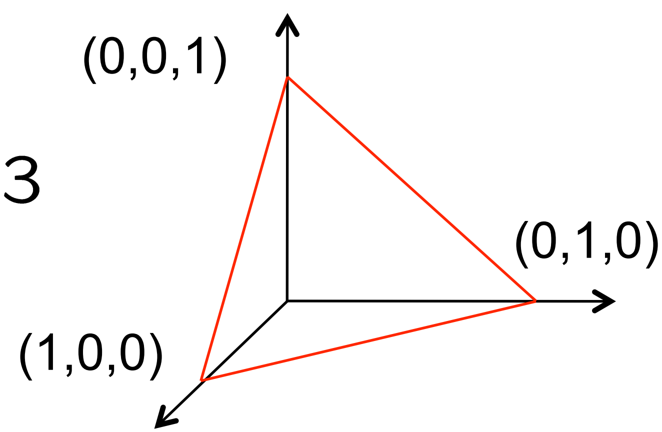
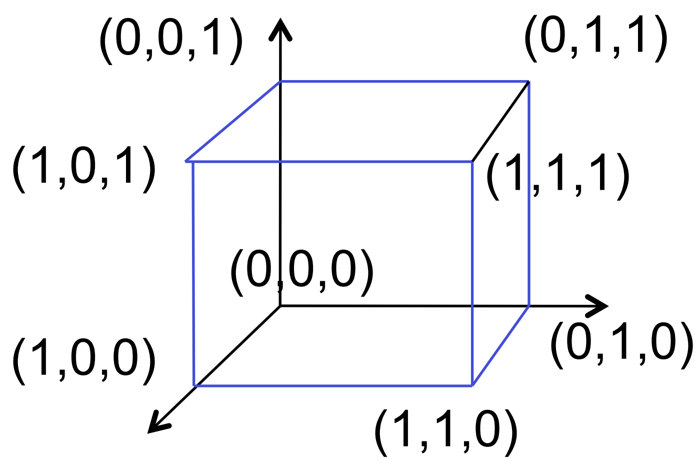
$$\mathbf{s} = (p(1), p(2), p(3)) \qquad \mathbf{r} = (p(\text{pass}|1), p(\text{pass}|2), p(\text{pass}|3))$$

$$K = 3$$





$$p(\text{pass}) = \mathbf{r} \cdot \mathbf{s} = \sum_i p(i)p(\text{pass}|i)$$

GPT characterization of classical theory

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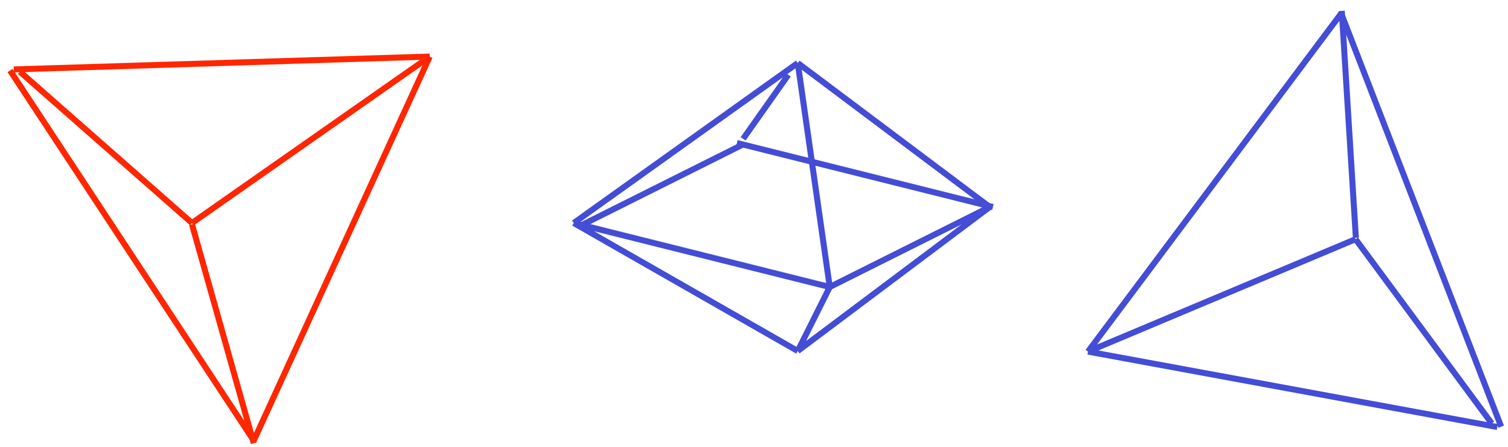
r can be any vector of conditional probabilities

R = the unit hypercube

$K = 4$



GPT characterization of quantum theory

Recall: The Hermitian operators on a Hilbert space of dimension *d* form a real Euclidean vector space of dimension d^2

s can represent any trace one positive operator

S = the convex set of such operators

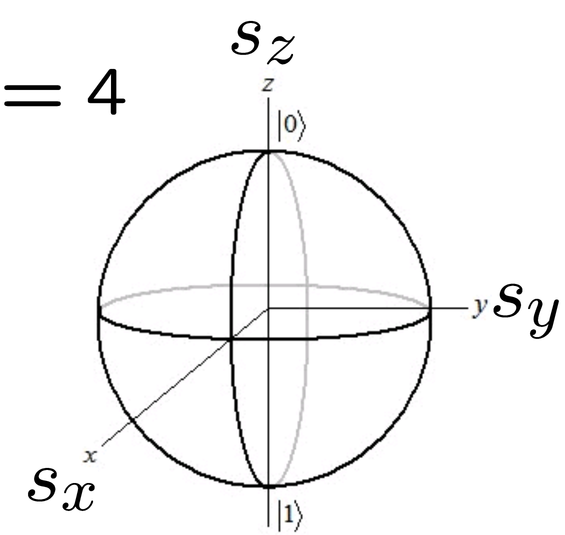
$$(s_0, s_x, s_y, s_z) = (\mathrm{Tr}(\rho I), \mathrm{Tr}(\rho\sigma_x), \mathrm{Tr}(\rho\sigma_y), \mathrm{Tr}(\rho\sigma_z))$$

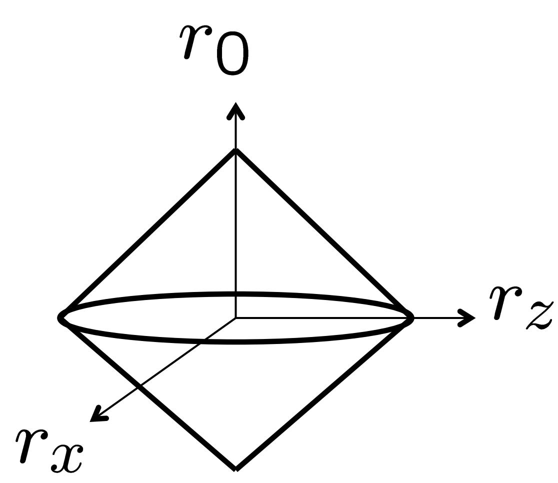
r can be any positive operator less than identity

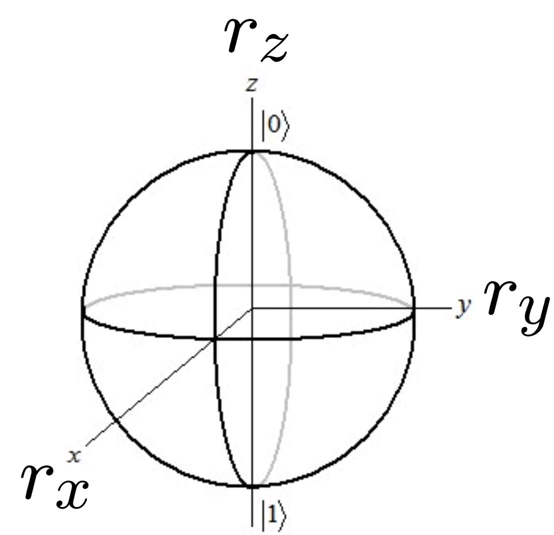
R = an interval of the positive cone of such operators

$$(r_0, r_x, r_y, r_z) = \tfrac{1}{2}(\mathrm{Tr}(EI), \mathrm{Tr}(E\sigma_x), \mathrm{Tr}(E\sigma_y), \mathrm{Tr}(E\sigma_z))$$

$K = 4$

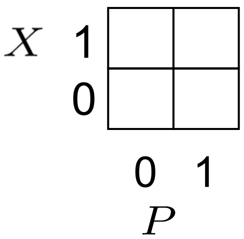






Toy theory

RWS, PRA 75, 032110 (2007)



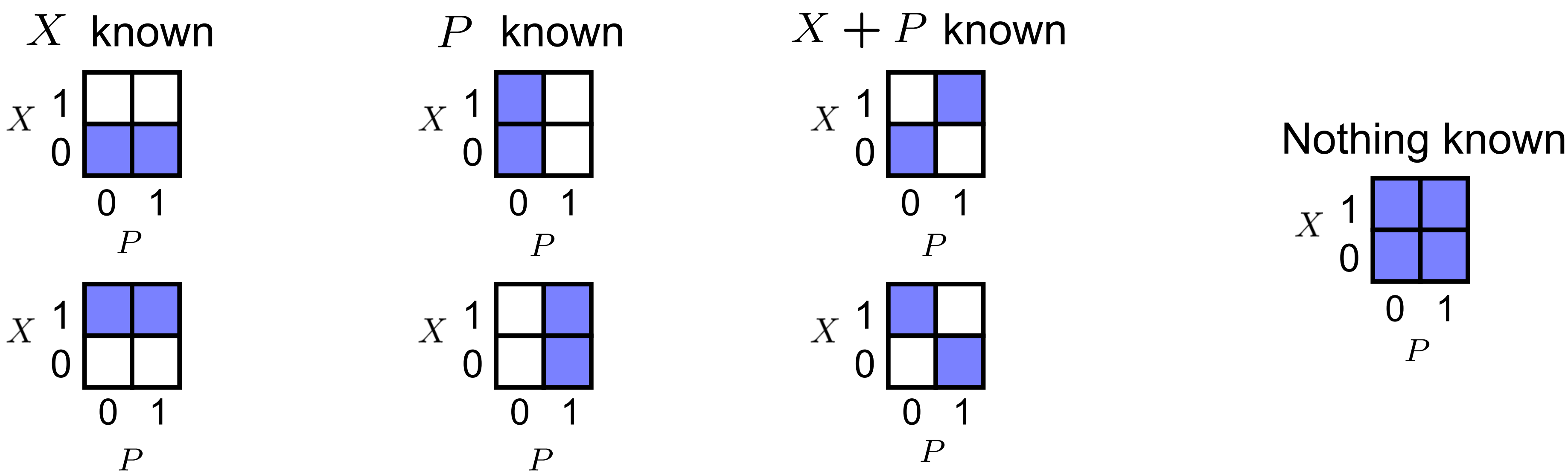
Canonical variables

$$aX + bP \qquad a, b \in \mathbb{Z}_2$$

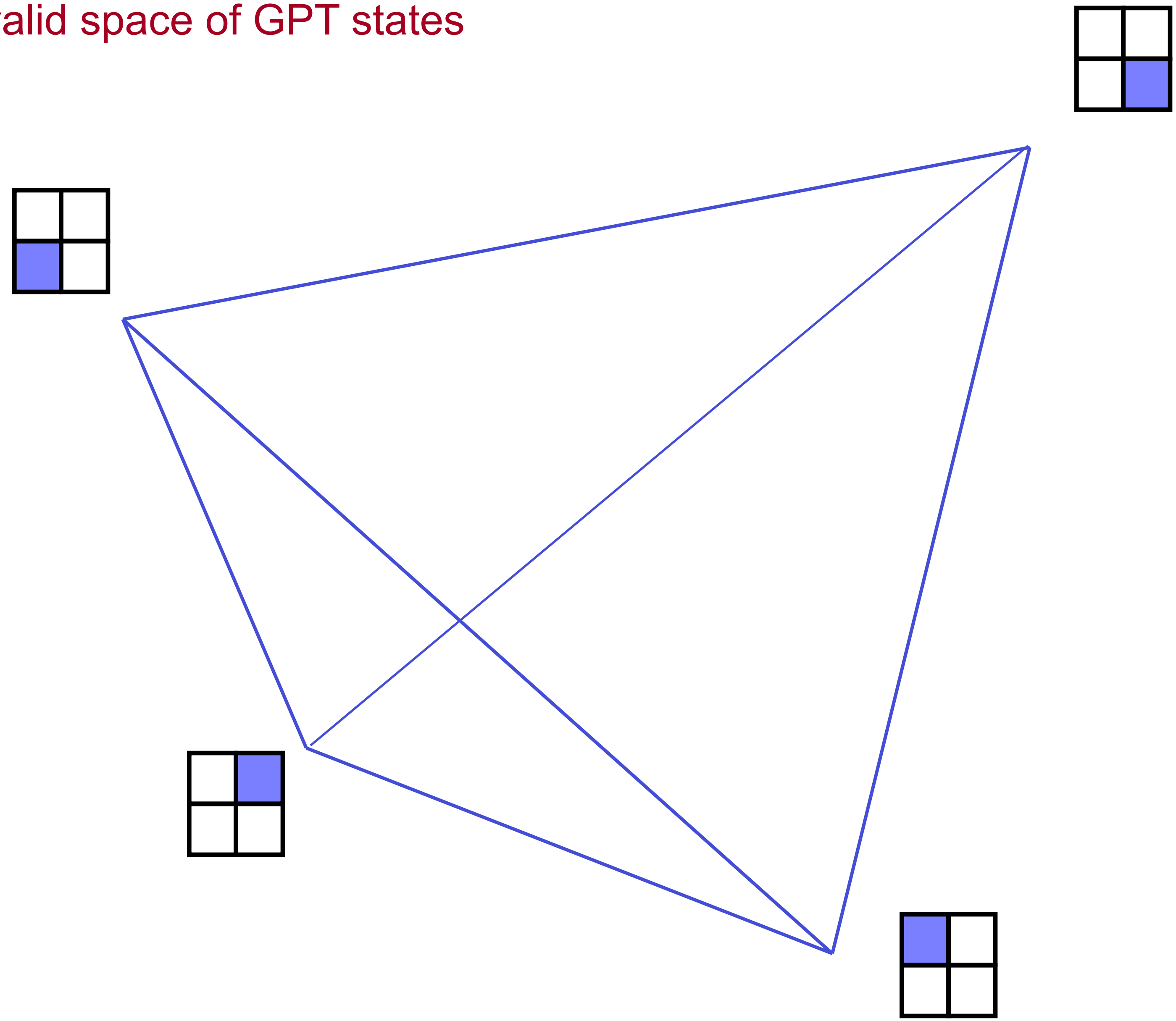
Addition is mod2

$$X, \quad P, \quad X + P$$

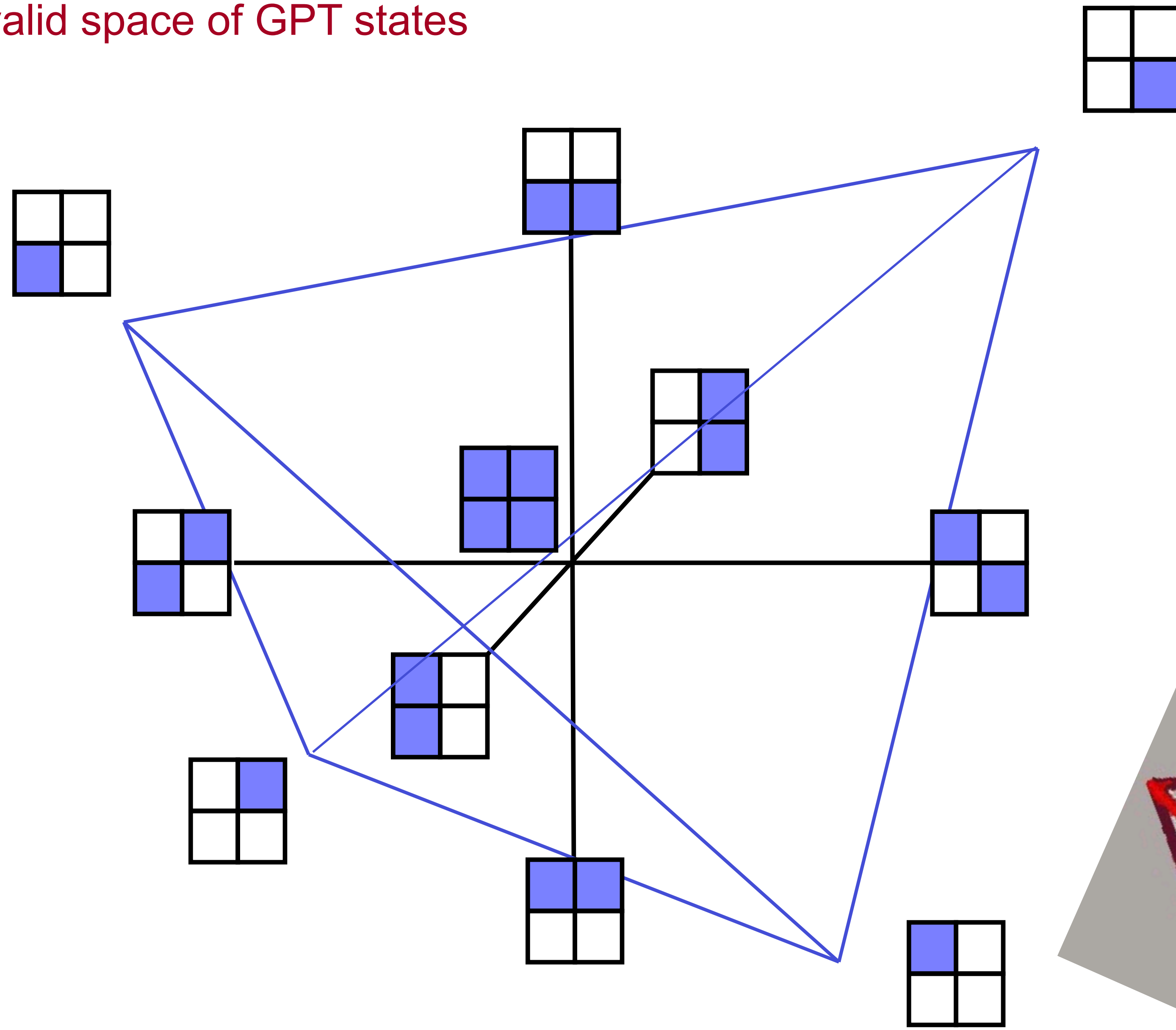
Probability distributions allowed by the epistemic restriction

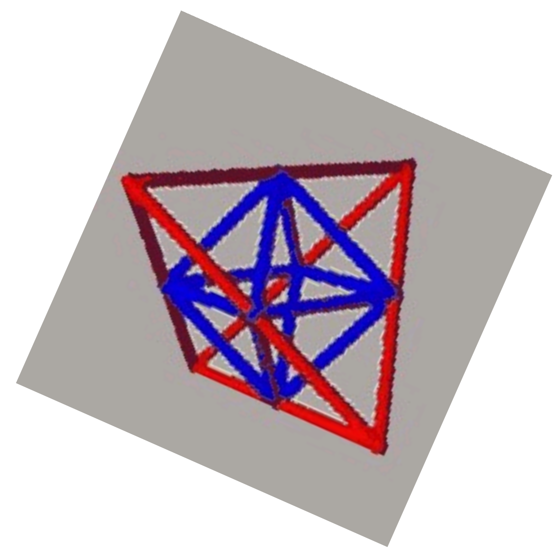


The valid space of GPT states

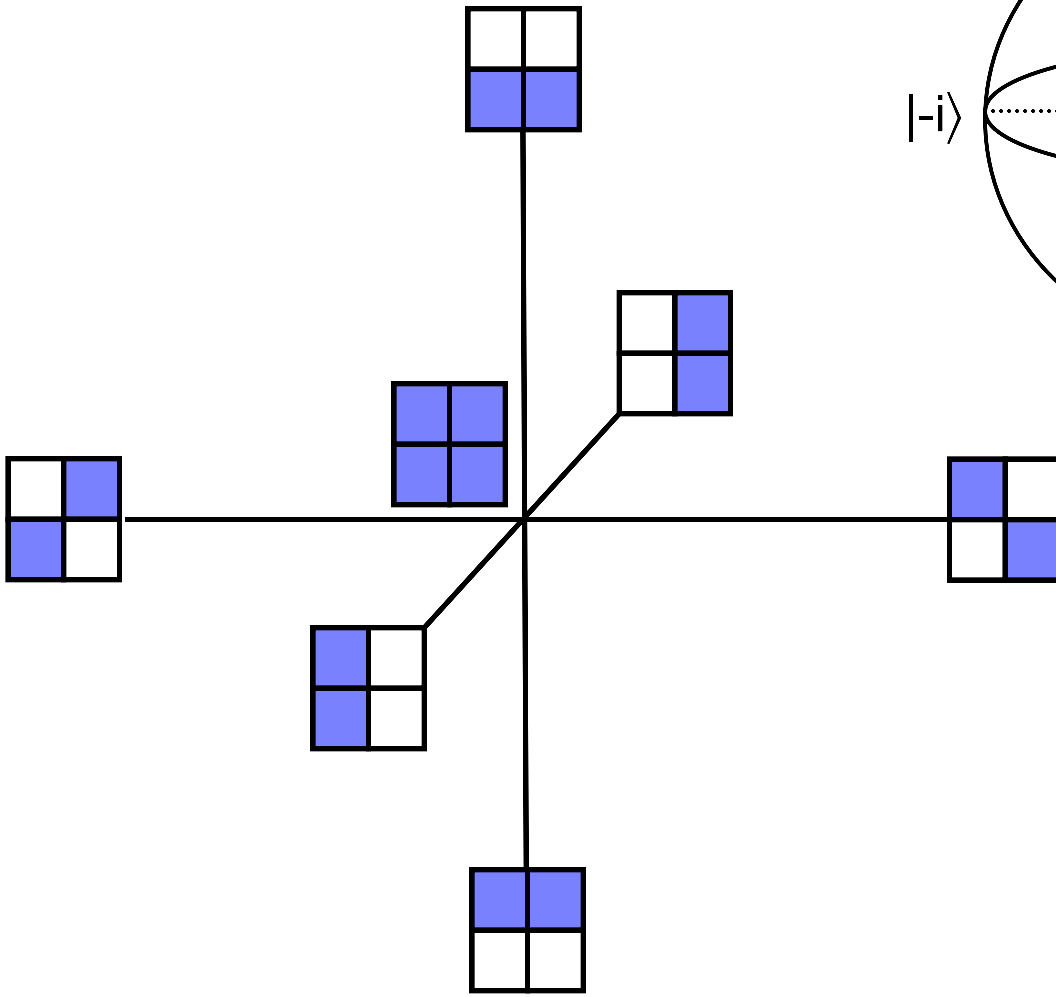


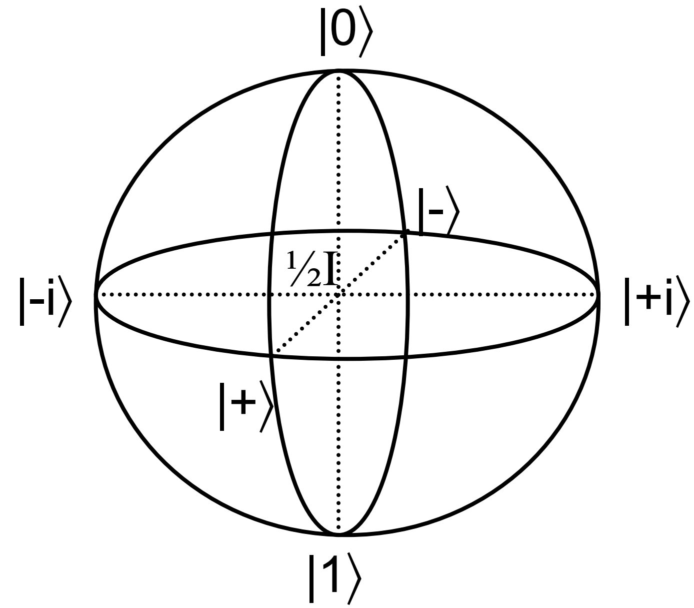
The valid space of GPT states





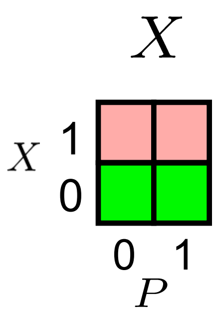
The valid space of GPT states



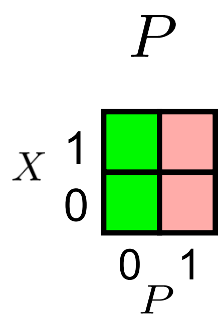


Valid measurements:

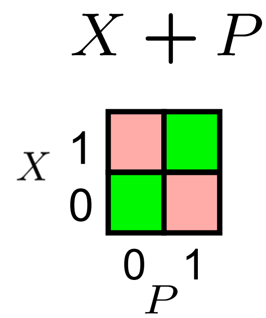
Any commuting set of canonical variables



$\{|0\rangle, |1\rangle\}$

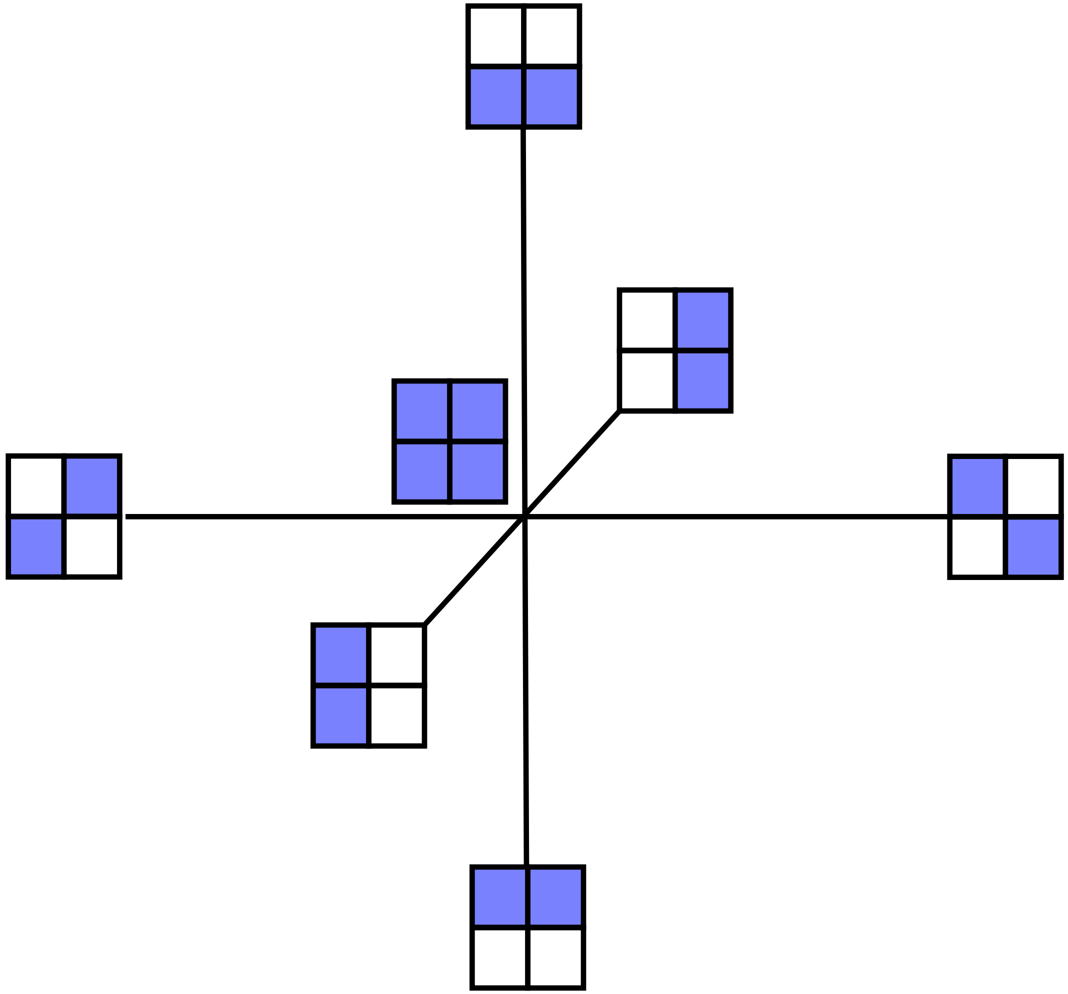


$\{|+\rangle, |-\rangle\}$



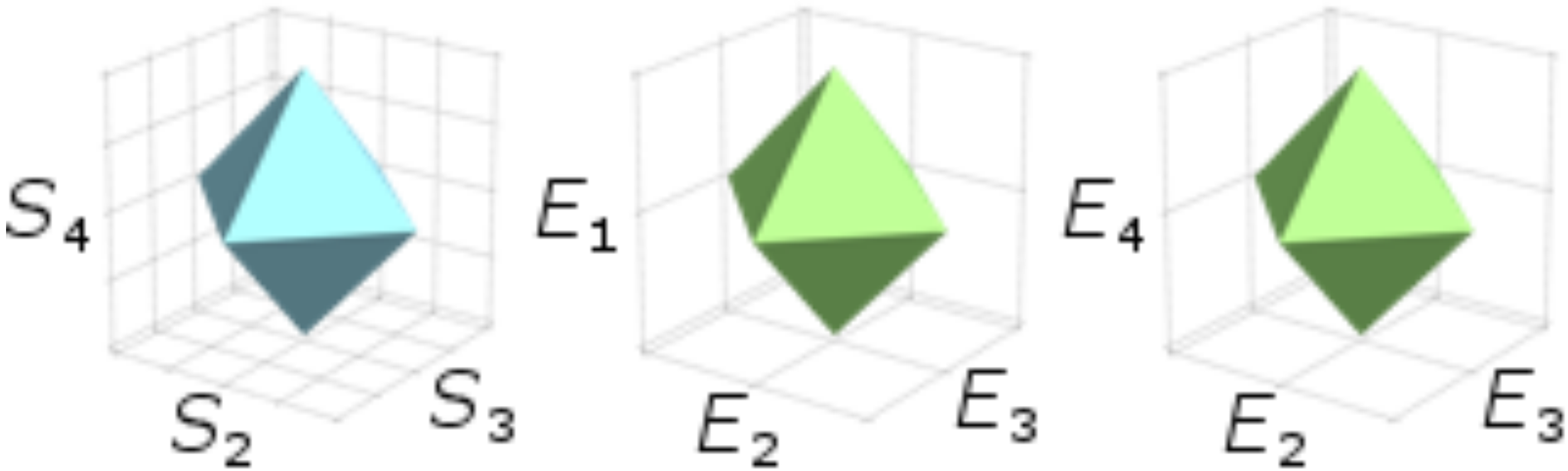
$\{|+i\rangle, |-i\rangle\}$

Valid GPT effects



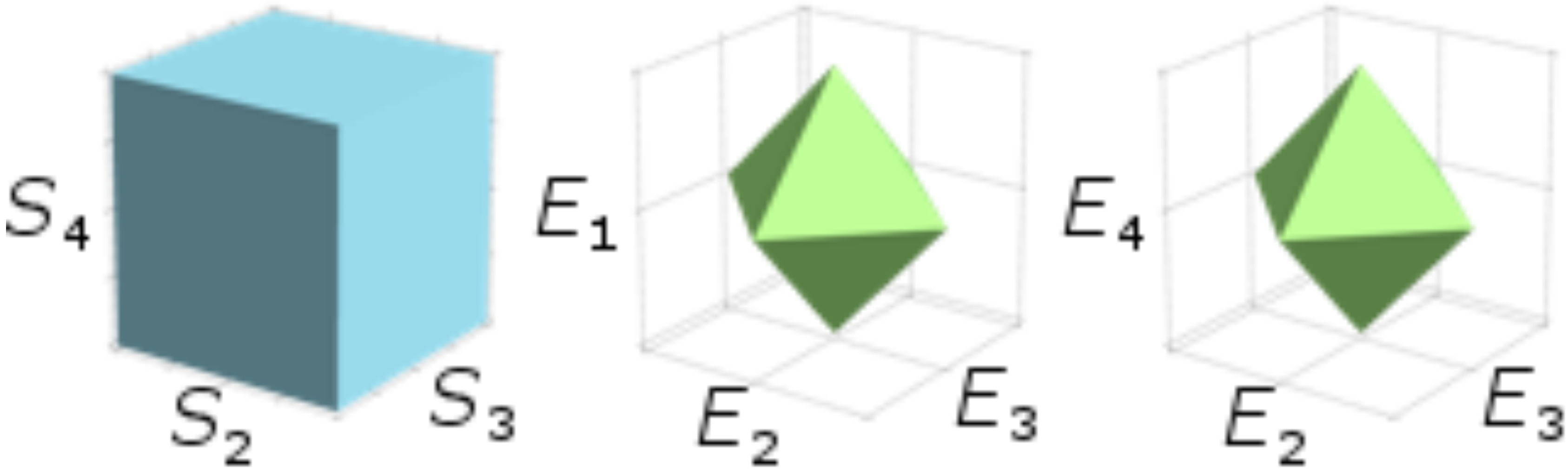
GPT characterization of convex closure of toy theory

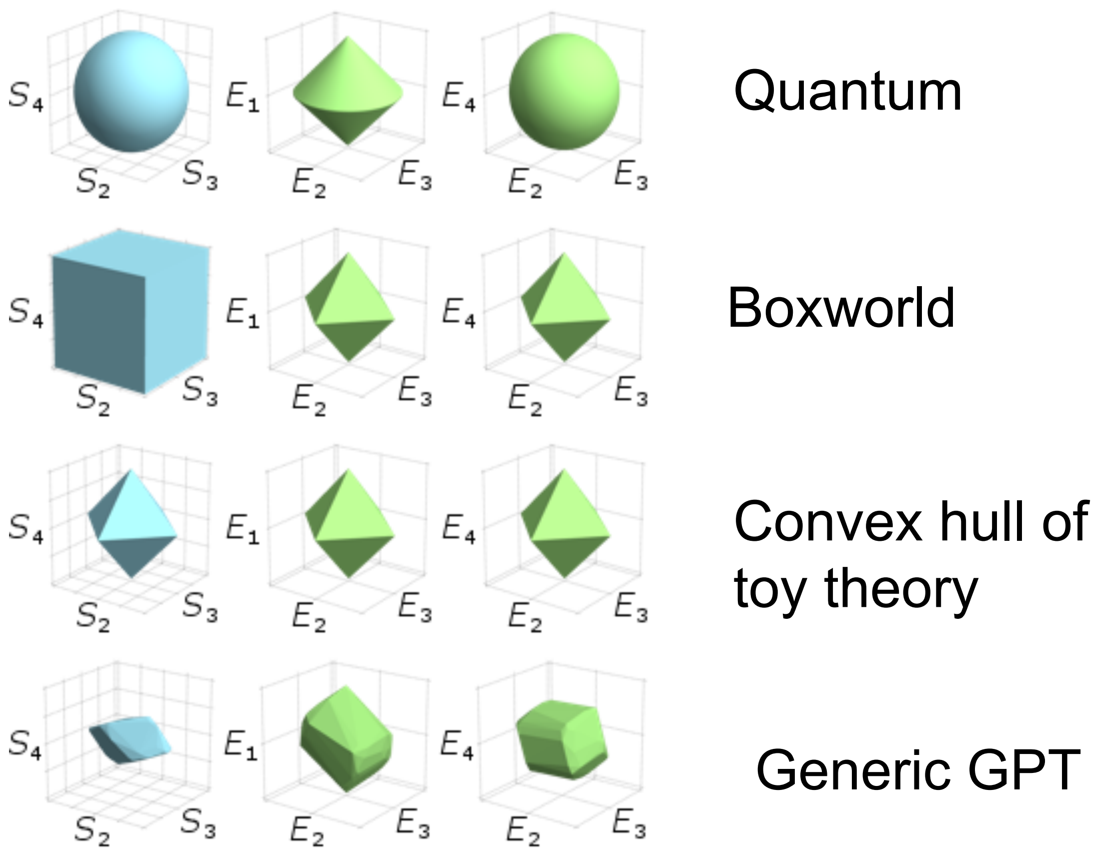
$K = 4$



GPT characterization of boxworld
(Popescu-Rohrlich box correlations)

$K = 4$



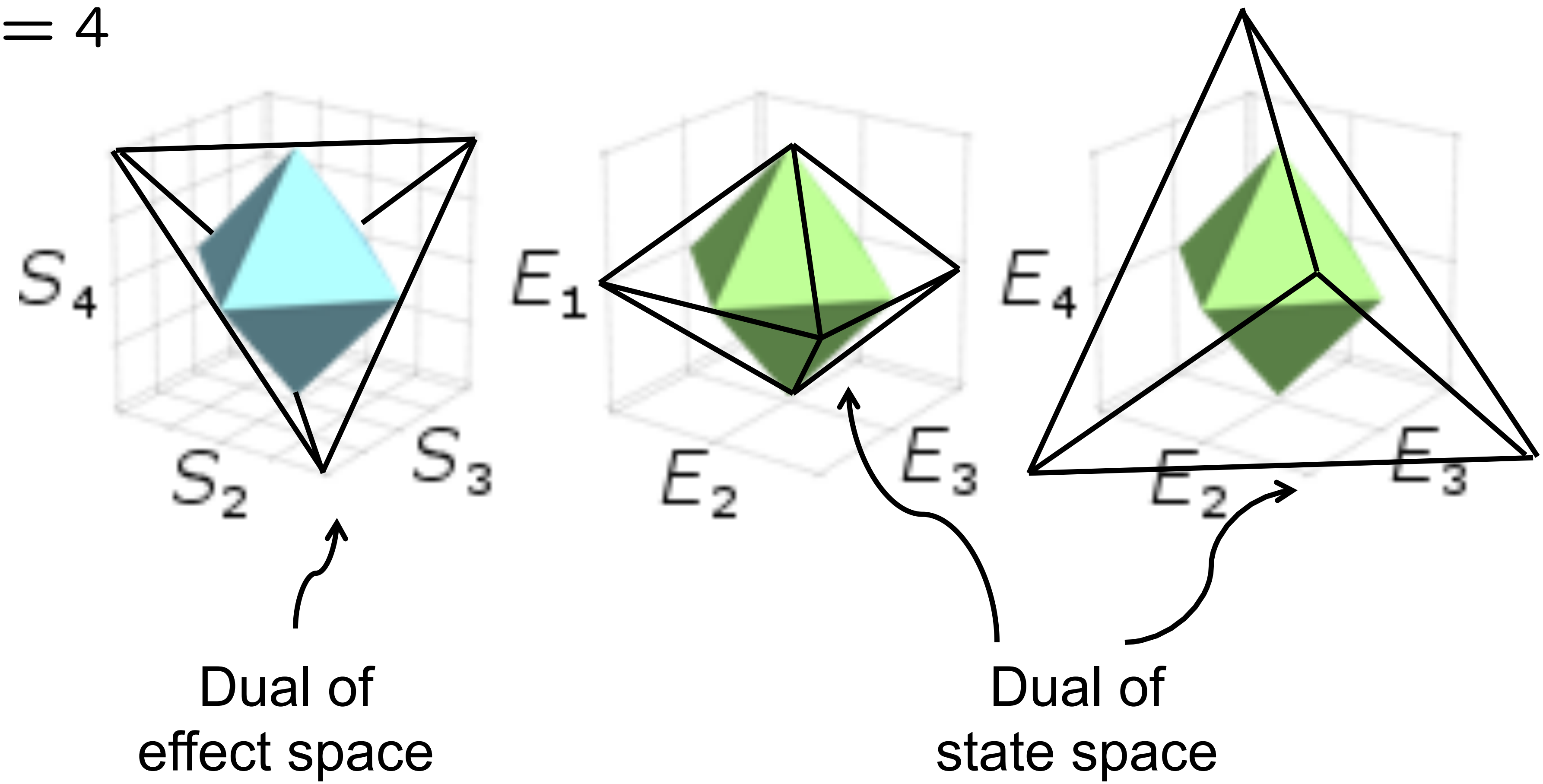

S4
E1
E4
S2
S3
E2
E3
Quantum
Boxworld
Convex hull of
toy theory
Generic GPT

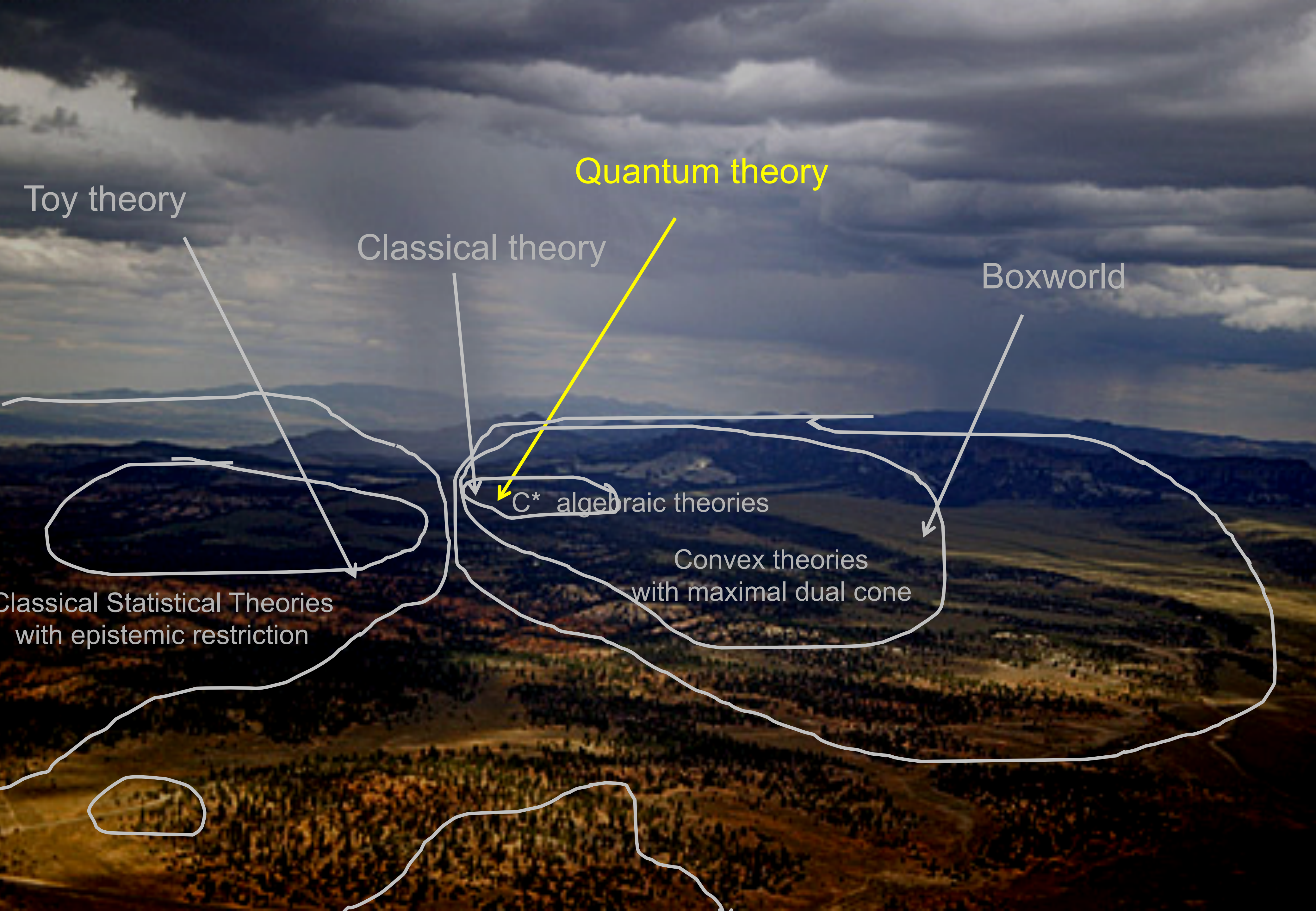
Interesting question about a given GPT:

Does it satisfy
No-restriction hypothesis: the space of GPT effects in a theory include **all** effects that assign positive probabilities to every GPT state in the theory

GPT characterization of convex closure of toy theory

$K = 4$

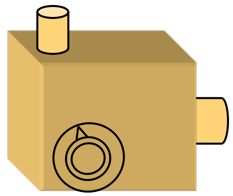


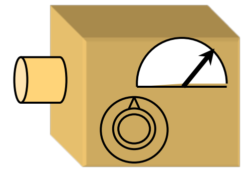

Quantum theory
Toy theory
Classical theory
Boxworld
C* algebraic theories
Convex theories
with maximal dual cone
Classical Statistical Theories
with epistemic restriction

Deviations from quantum theory in the landscape of generalized probabilistic theories: Direct constraints from experimental data

Joint work with:
Matthew Pusey
Mike Mazurek
Kevin Resch

Determining GPT from infinite-run experimental statistics





pass or fail

Preparation

P

Measurement

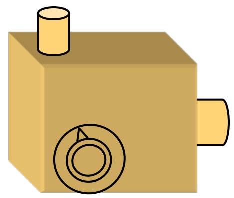
M

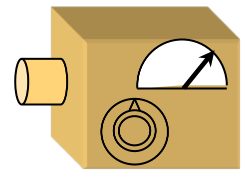
Preparations

$$\begin{pmatrix}
1 & p(0|P_1,M_2) & p(0|P_1,M_3) & p(0|P_1,M_4) & p(0|P_1,M_5) & \cdots \\
1 & p(0|P_2,M_2) & p(0|P_2,M_3) & p(0|P_2,M_4) & p(0|P_2,M_5) & \cdots \\
1 & p(0|P_3,M_2) & p(0|P_3,M_3) & p(0|P_3,M_4) & p(0|P_3,M_5) & \cdots \\
1 & p(0|P_4,M_2) & p(0|P_4,M_3) & p(0|P_4,M_4) & p(0|P_4,M_5) & \cdots \\
1 & p(0|P_5,M_2) & p(0|P_5,M_3) & p(0|P_5,M_4) & p(0|P_5,M_5) & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$

Measurements

Determining GPT from infinite-run experimental statistics





pass or fail

Preparation

P

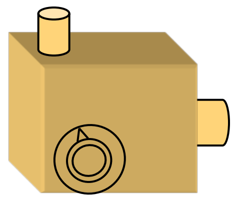
Measurement

M

$$\begin{pmatrix} 1 & P_1^{(2)} & \cdots & P_1^{(k)} \\ 1 & P_2^{(2)} & \cdots & P_2^{(k)} \\ 1 & P_3^{(2)} & \cdots & P_3^{(k)} \\ 1 & P_4^{(2)} & \cdots & P_4^{(k)} \\ 1 & P_5^{(2)} & \cdots & P_5^{(k)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 1 & M_2^{(1)} & M_3^{(1)} & M_4^{(1)} & M_5^{(1)} & \cdots \\ 0 & M_2^{(2)} & M_3^{(2)} & M_4^{(2)} & M_5^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ 0 & M_2^{(k)} & M_3^{(k)} & M_4^{(k)} & M_5^{(k)} & \cdots \end{pmatrix}$$

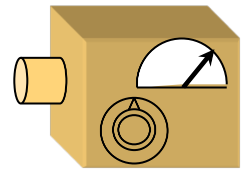
$$p(0|P_2, M_4) = \begin{pmatrix} 1 & P_2^{(2)} & \cdots & P_2^{(k)} \end{pmatrix} \cdot \begin{pmatrix} M_4^{(1)} & \cdots & M_4^{(k)} \end{pmatrix}$$

Determining GPT from infinite-run experimental statistics



Preparation

P



Measurement

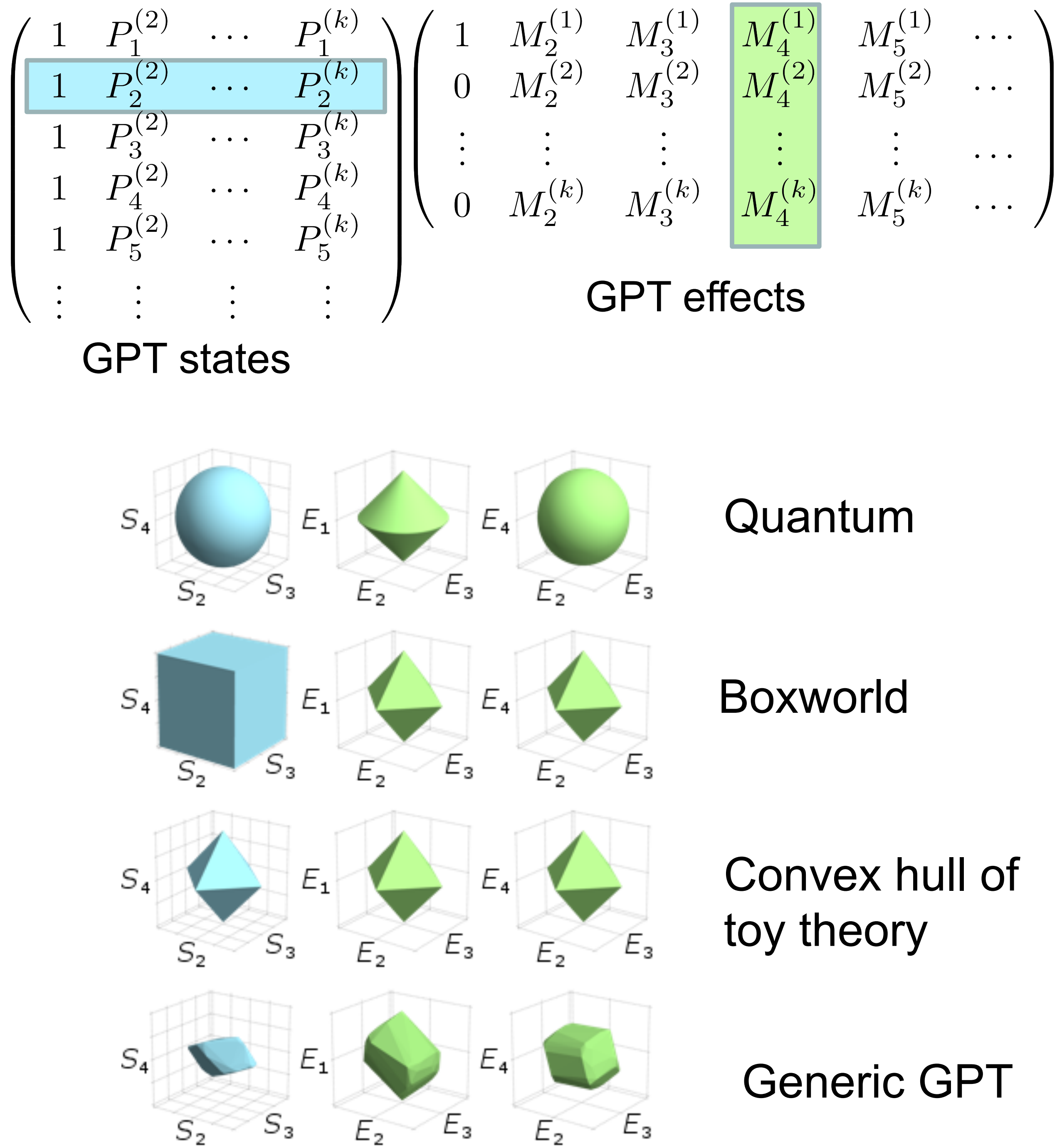
M

$$\begin{pmatrix} 1 & P_1^{(2)} & \cdots & P_1^{(k)} \\ 1 & P_2^{(2)} & \cdots & P_2^{(k)} \\ 1 & P_3^{(2)} & \cdots & P_3^{(k)} \\ 1 & P_4^{(2)} & \cdots & P_4^{(k)} \\ 1 & P_5^{(2)} & \cdots & P_5^{(k)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 1 & M_2^{(1)} & M_3^{(1)} & M_4^{(1)} & M_5^{(1)} & \cdots \\ 0 & M_2^{(2)} & M_3^{(2)} & M_4^{(2)} & M_5^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ 0 & M_2^{(k)} & M_3^{(k)} & M_4^{(k)} & M_5^{(k)} & \cdots \end{pmatrix}$$

GPT states

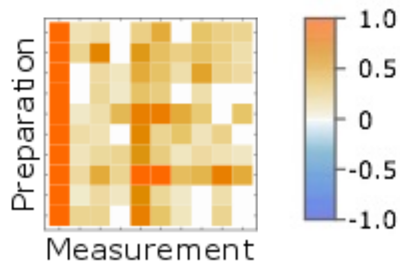
GPT effects

Use singular value decomposition: k = rank of data matrix

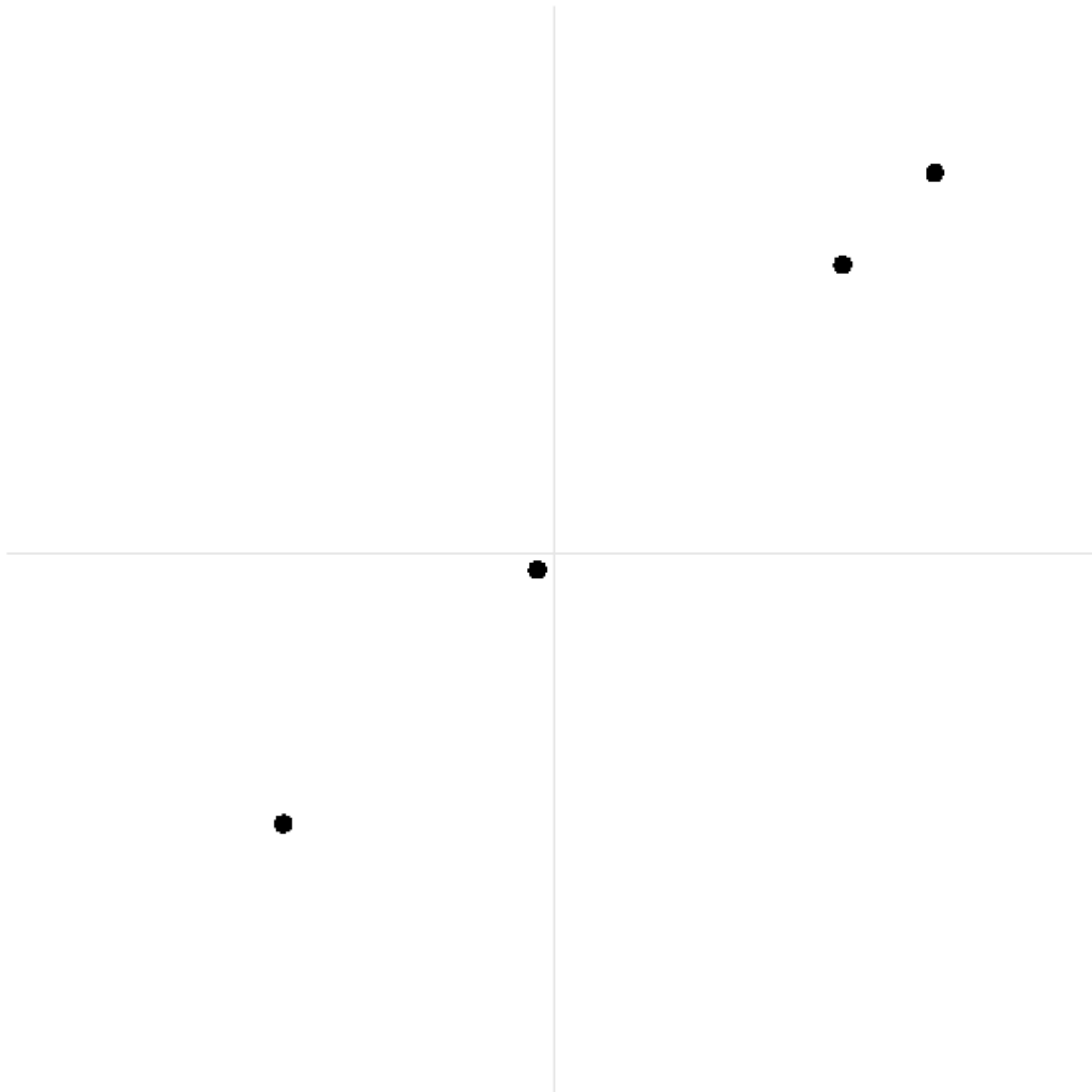

GPT states
GPT effects
S4
S2
S3
E1
E2
E3
E4
Quantum
Boxworld
Convex hull of
toy theory
Generic GPT

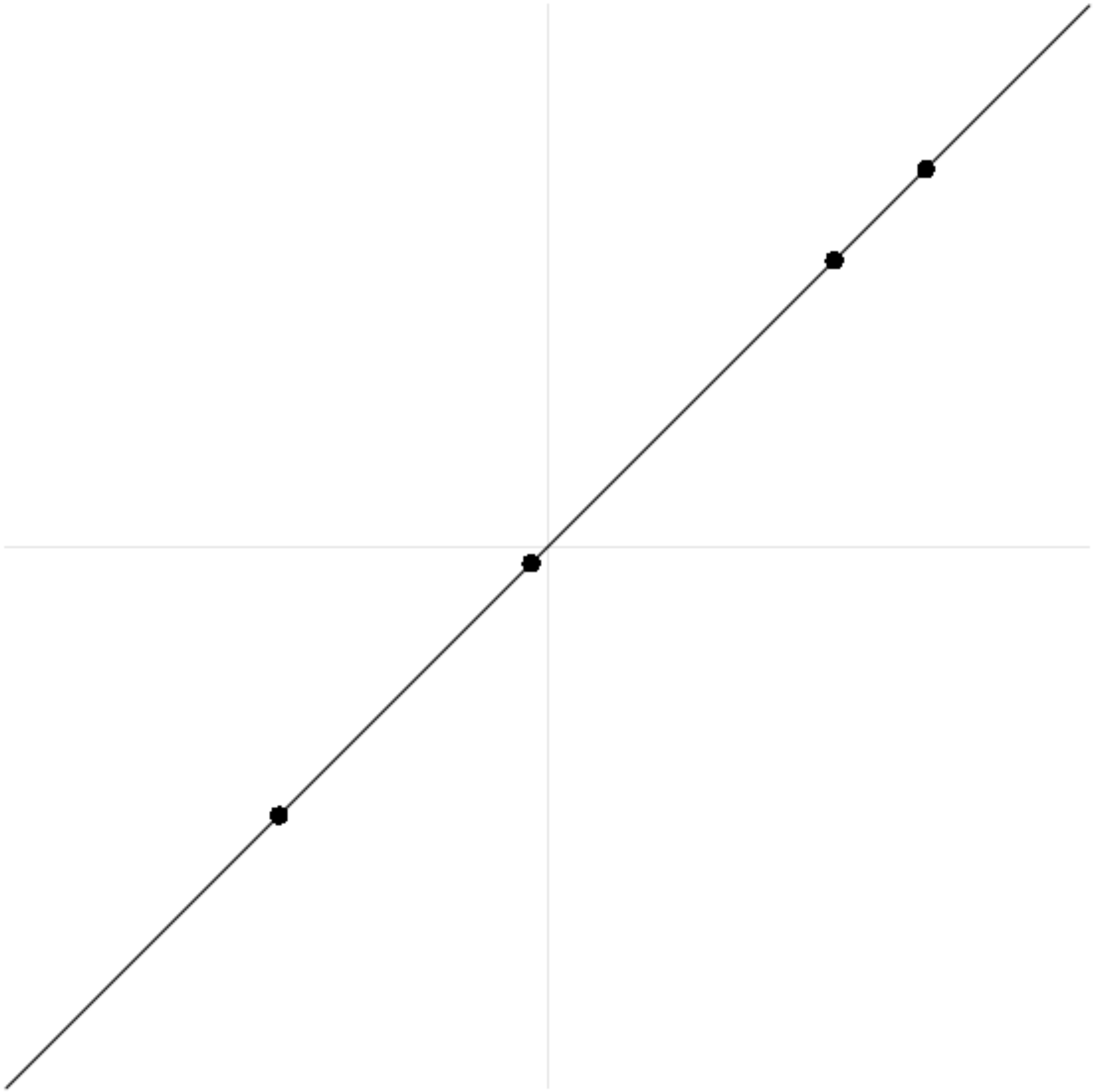
Determining GPT from finite-run experimental statistics

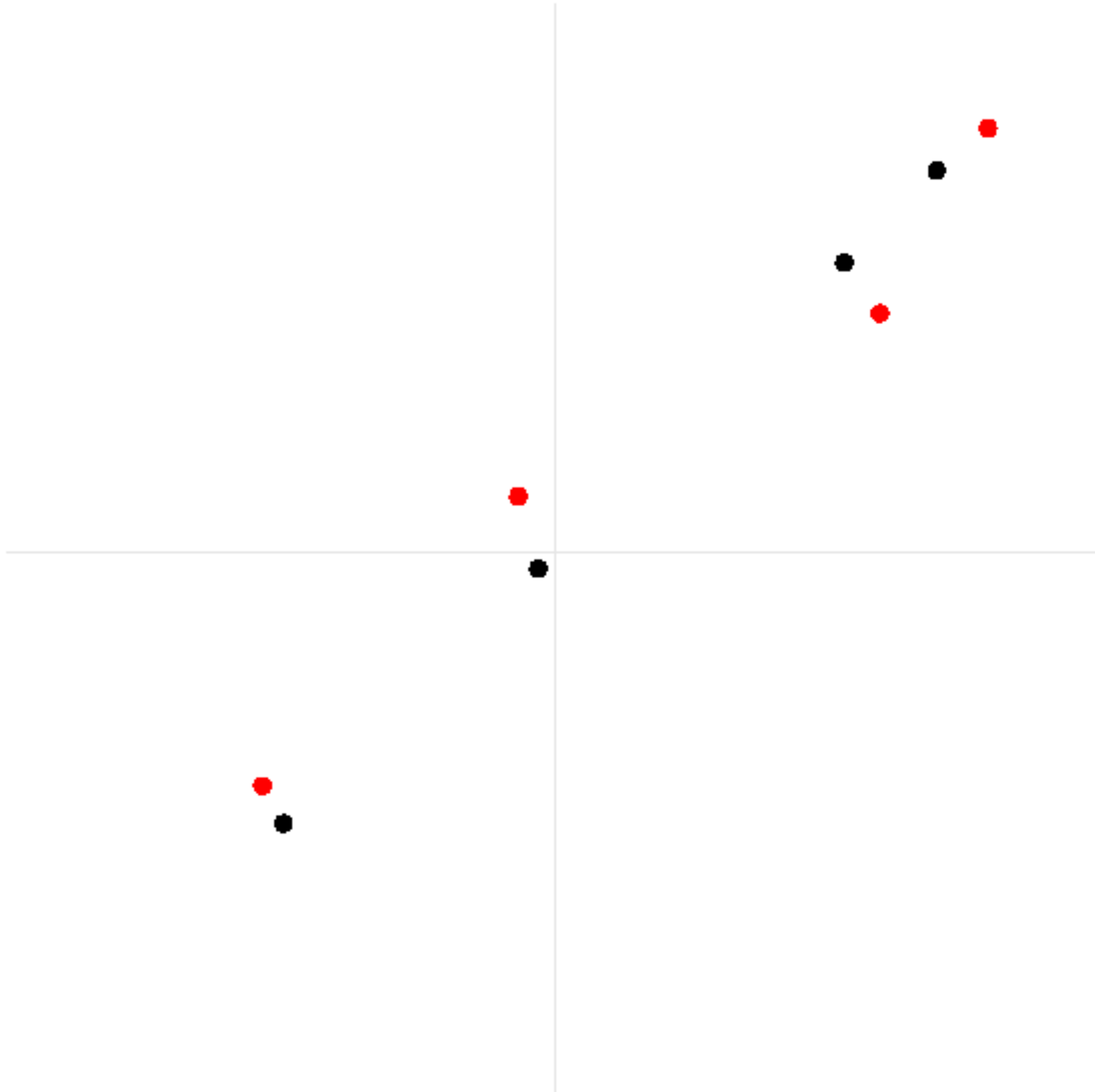
Raw data

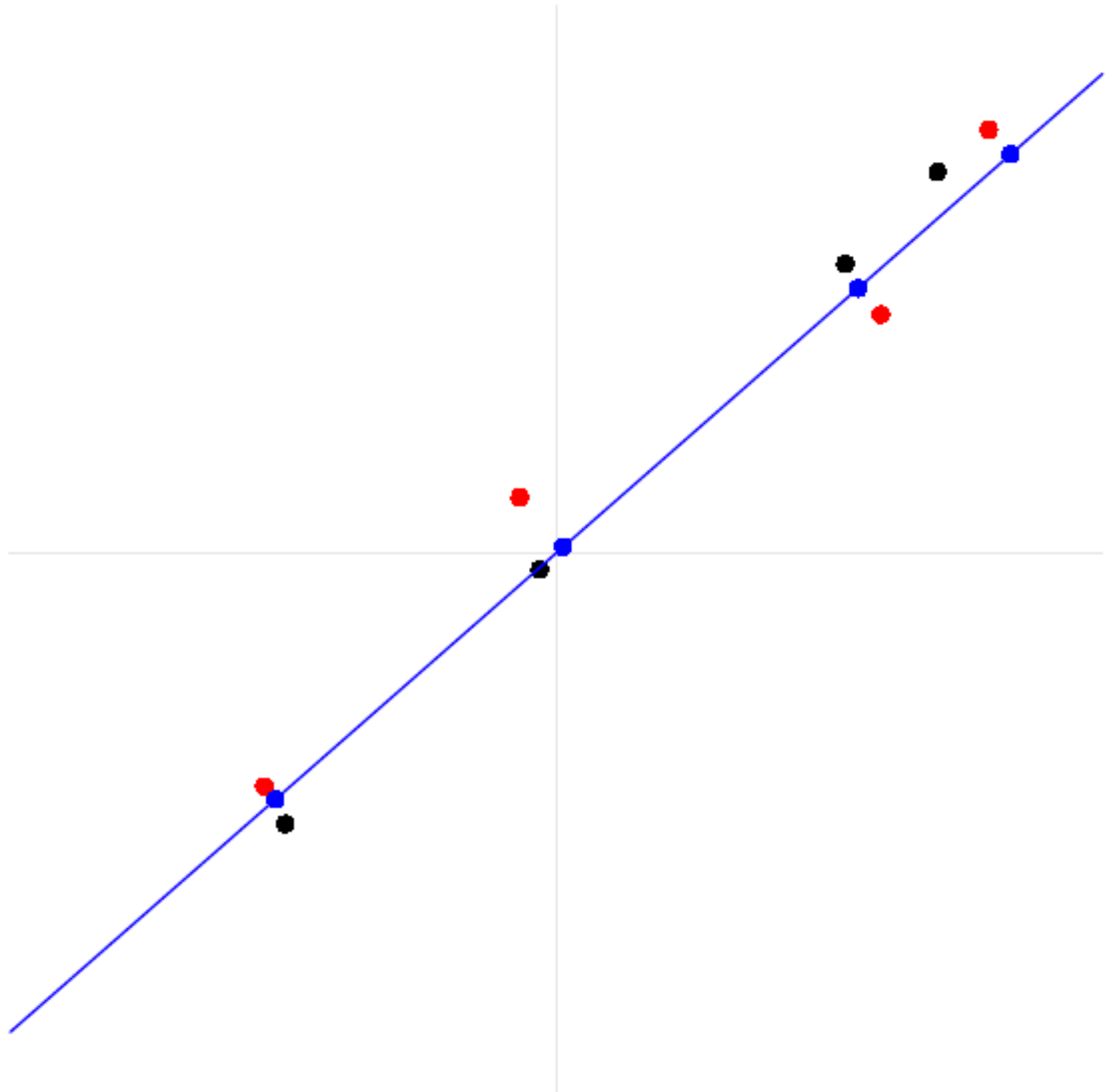


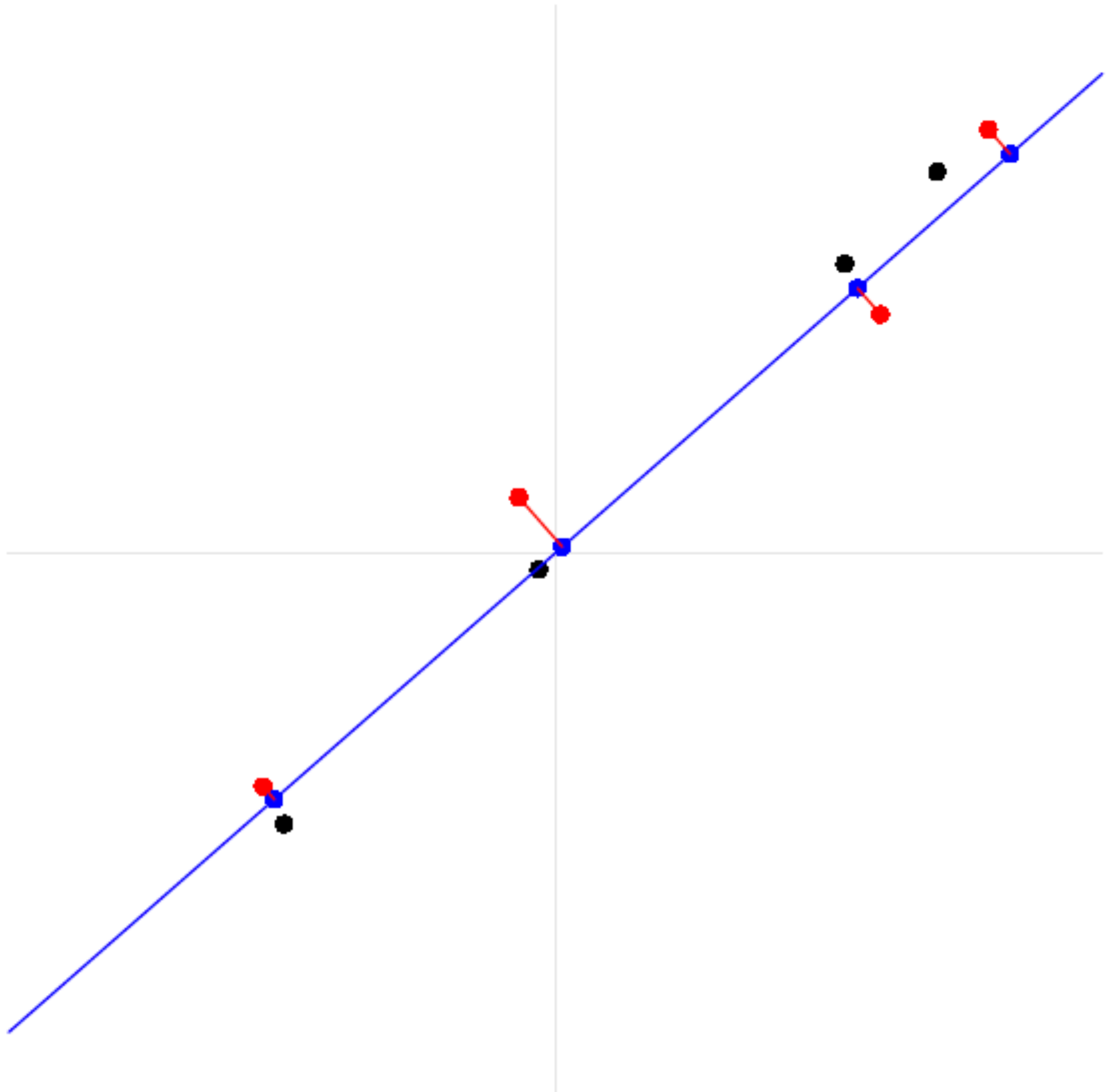
Because of statistical noise, the matrix of raw data is always full rank







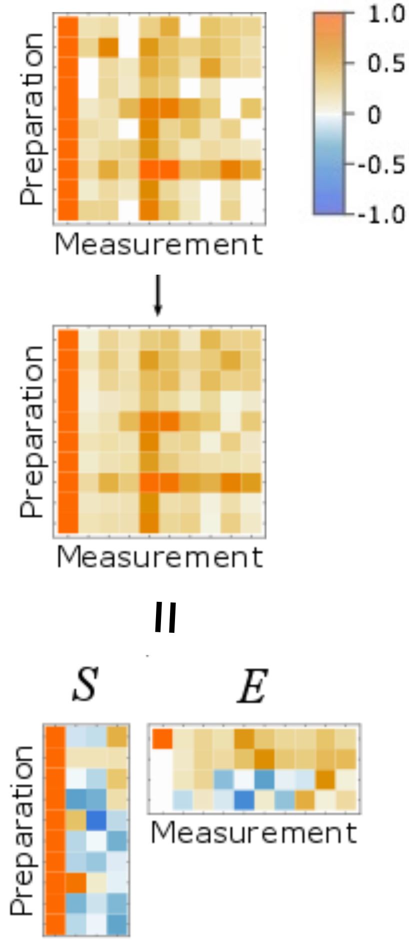




Determining GPT from finite-run experimental statistics

Raw data

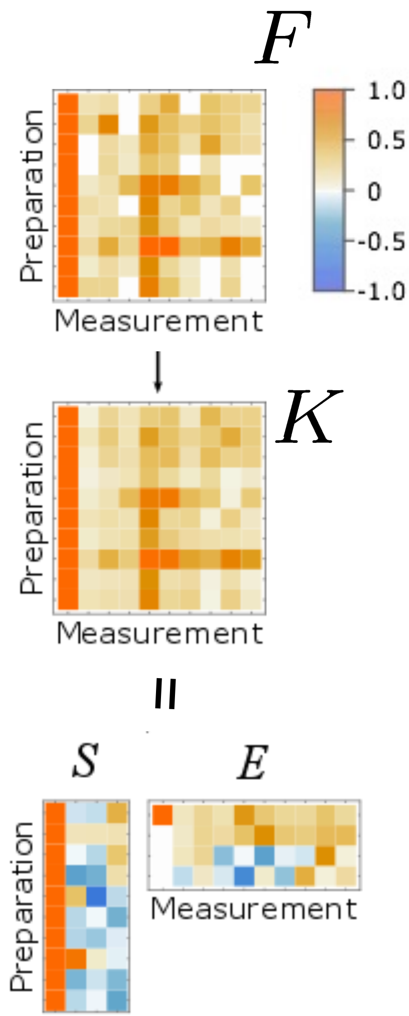
Find GPT model of best fit of rank k



Determining GPT from finite-run experimental statistics

Raw data

Find GPT model of best fit of rank k



In variation over K_{ij} satisfying

$$\mathrm{rank}(K) = k$$

and factorizing appropriately, minimize

$$\chi^2 = \sum_i \sum_j \left(\frac{K_{ij} - F_{ij}}{\Delta F_{ij}} \right)^2$$

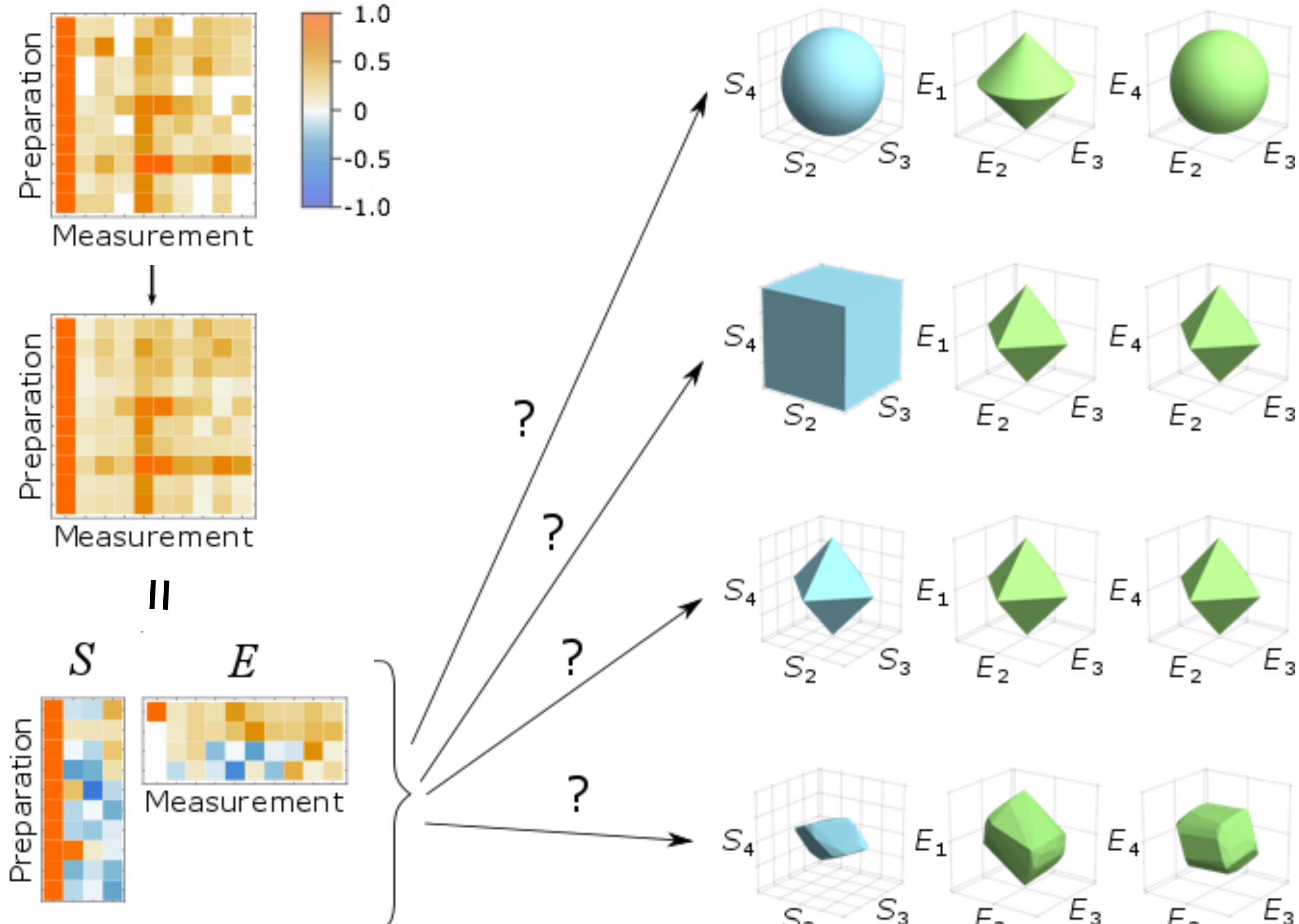
for Poissonian noise

This is the “weighted low-rank approximation problem”

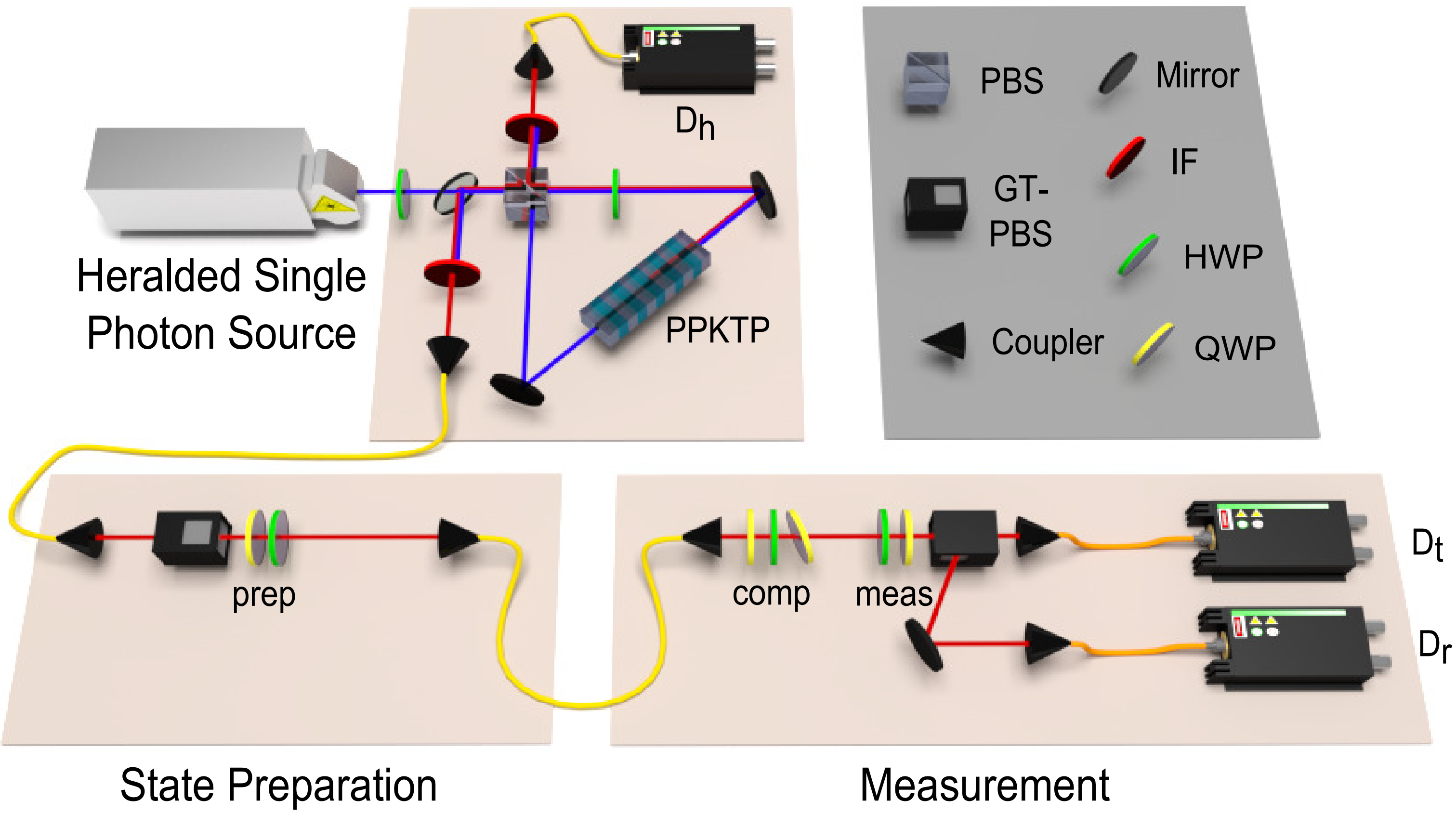
Determining GPT from finite-run experimental statistics

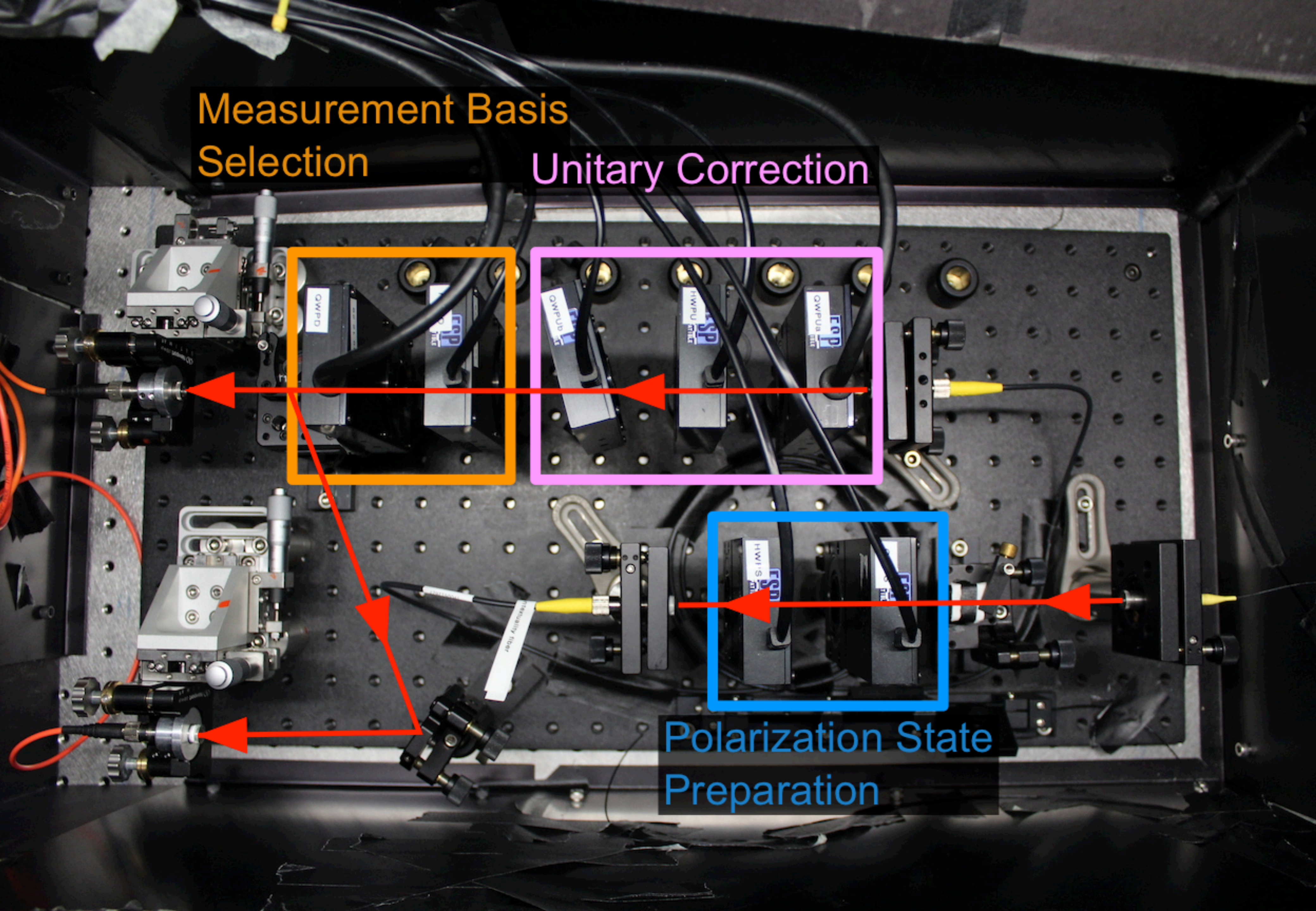
Raw data

Find GPT model of best fit of rank k



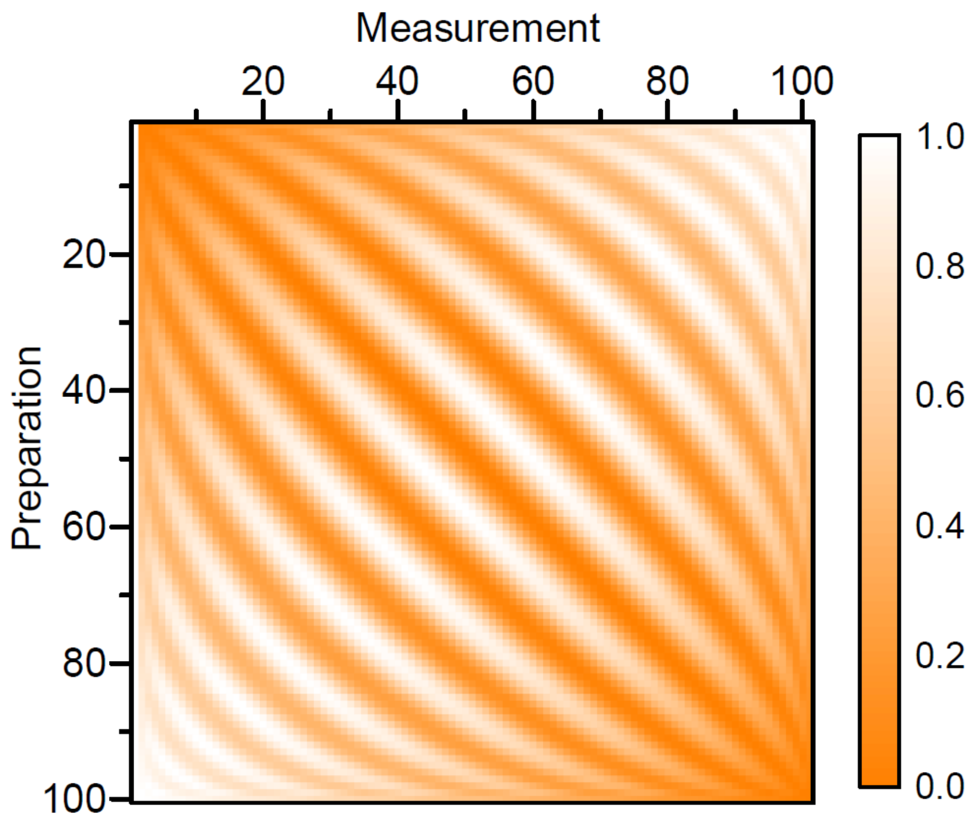
Experimental set-up



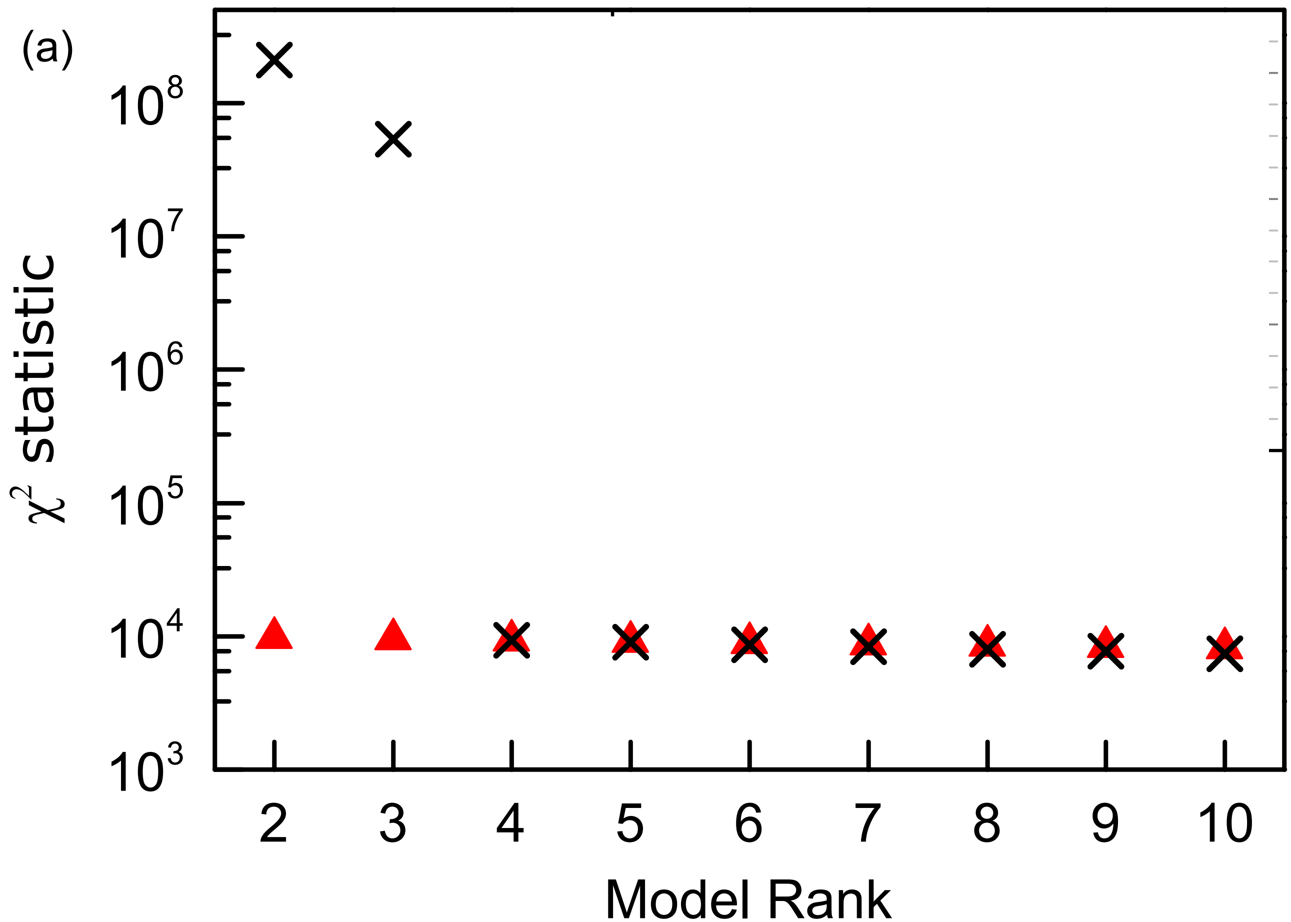

Measurement Basis
Selection
Unitary Correction
Polarization State
Preparation

Experimental data

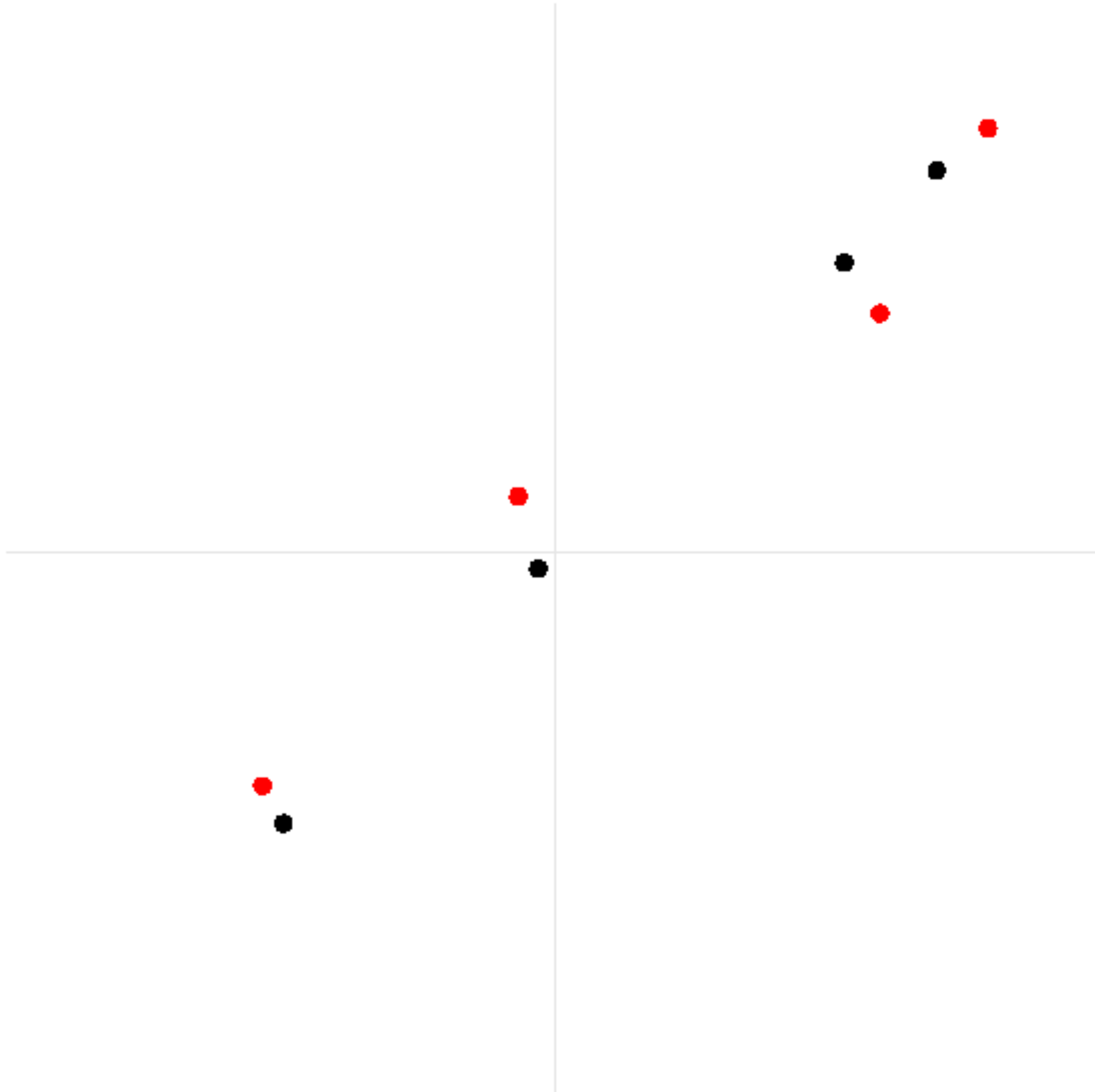
100 measurements on 100 preparations



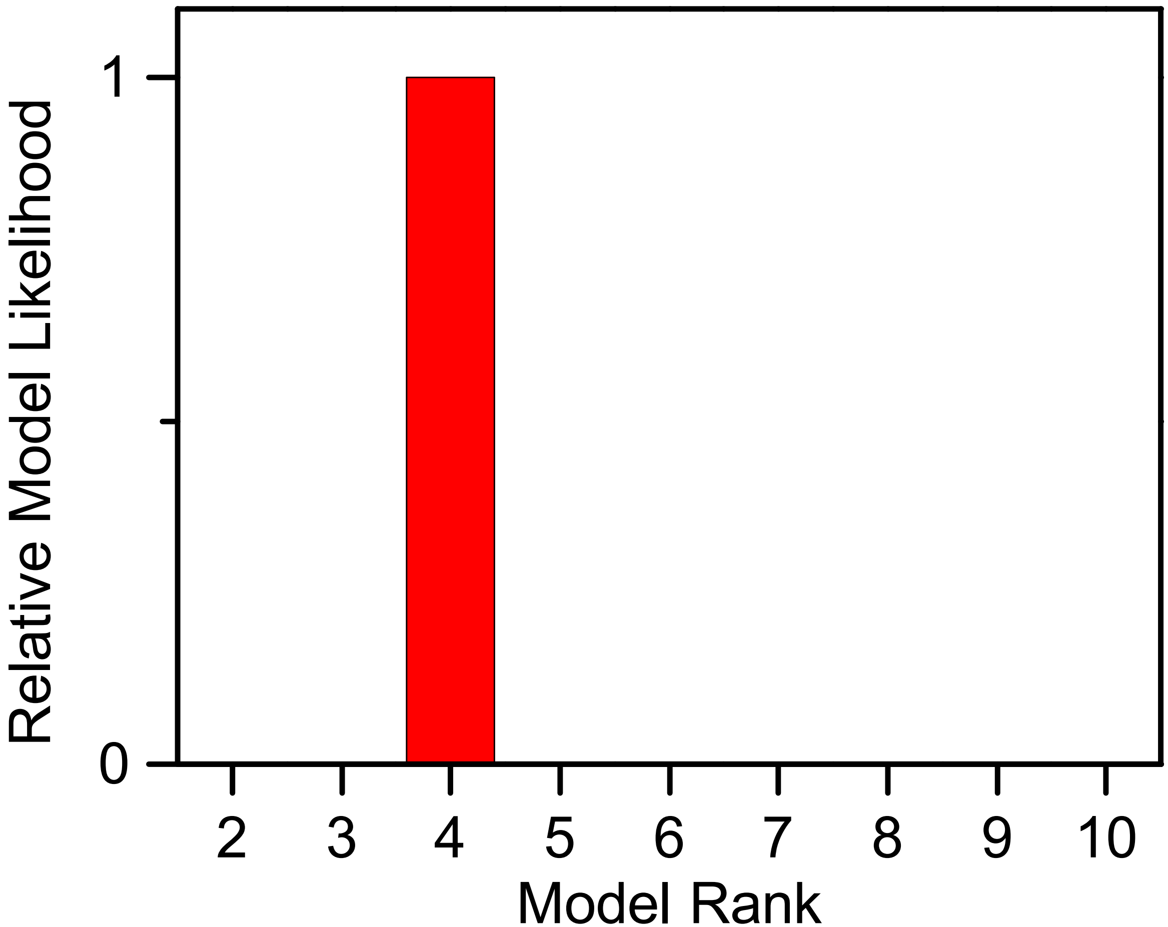
Characterize quality of fit by χ^2 statistic



Characterize tradeoff between quality of fit and overfitting by Akaike information criterion



Characterize tradeoff between quality of fit and overfitting by Akaike information criterion

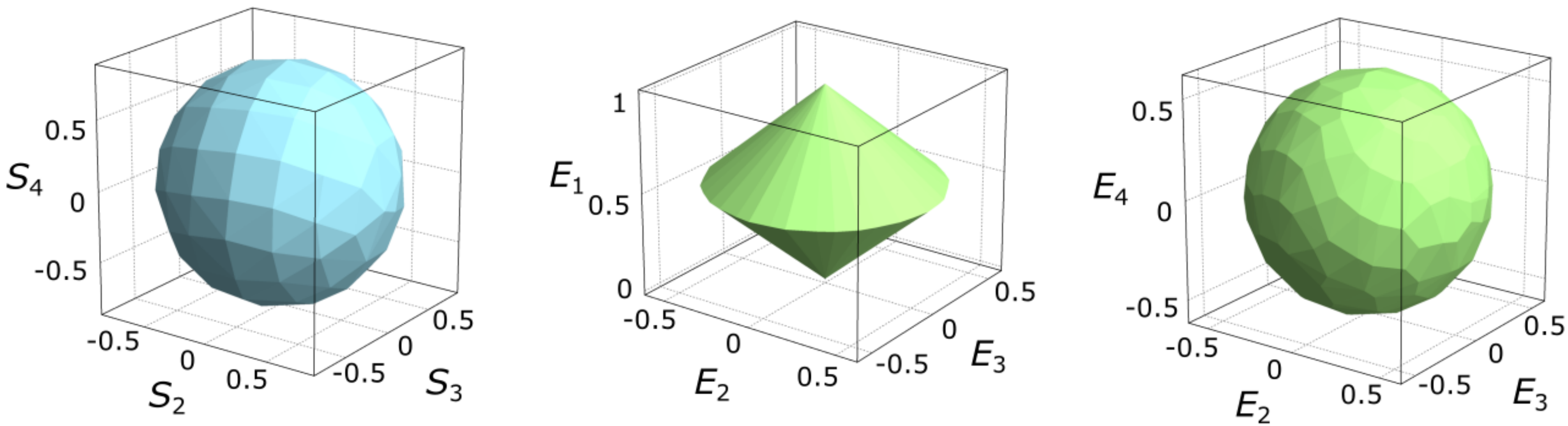


rank 4: 0.9998
rank 5: 1.99×10^{-4}
rank 6: 1.6×10^{-13}
all others: $<10^{-25}$

Use a GPT of rank 4!

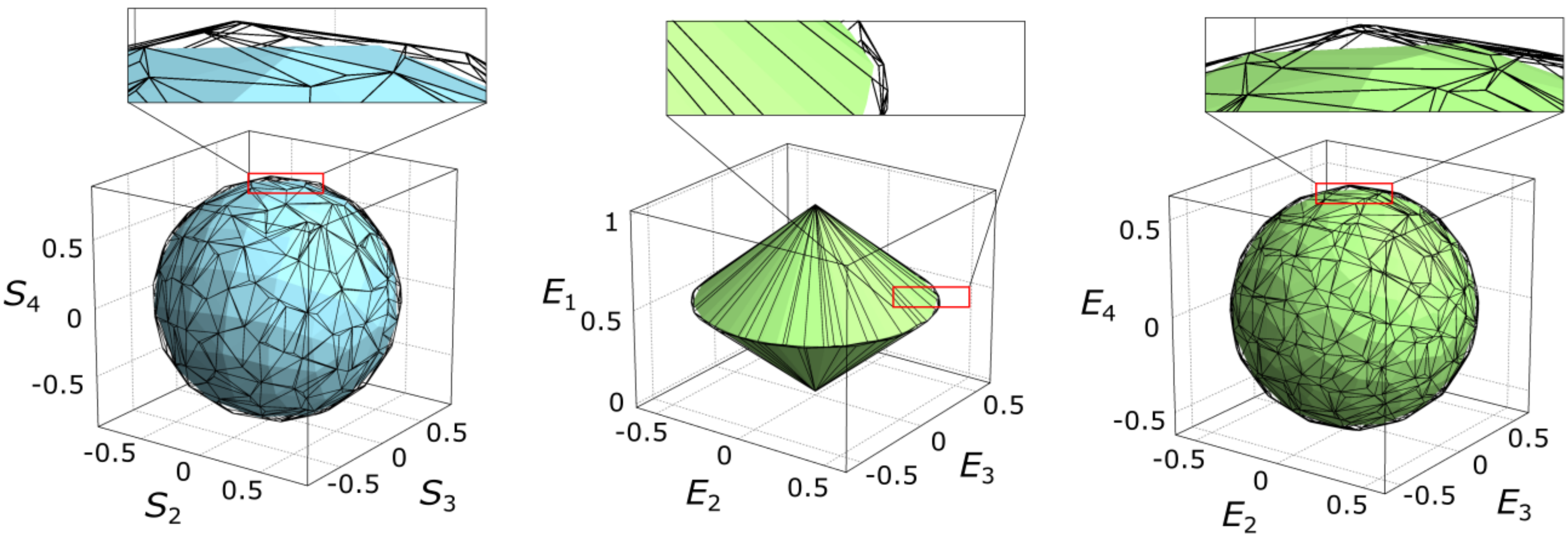
Rank-4 GPT of best fit for the experimental data

100 measurements on 100 preparations

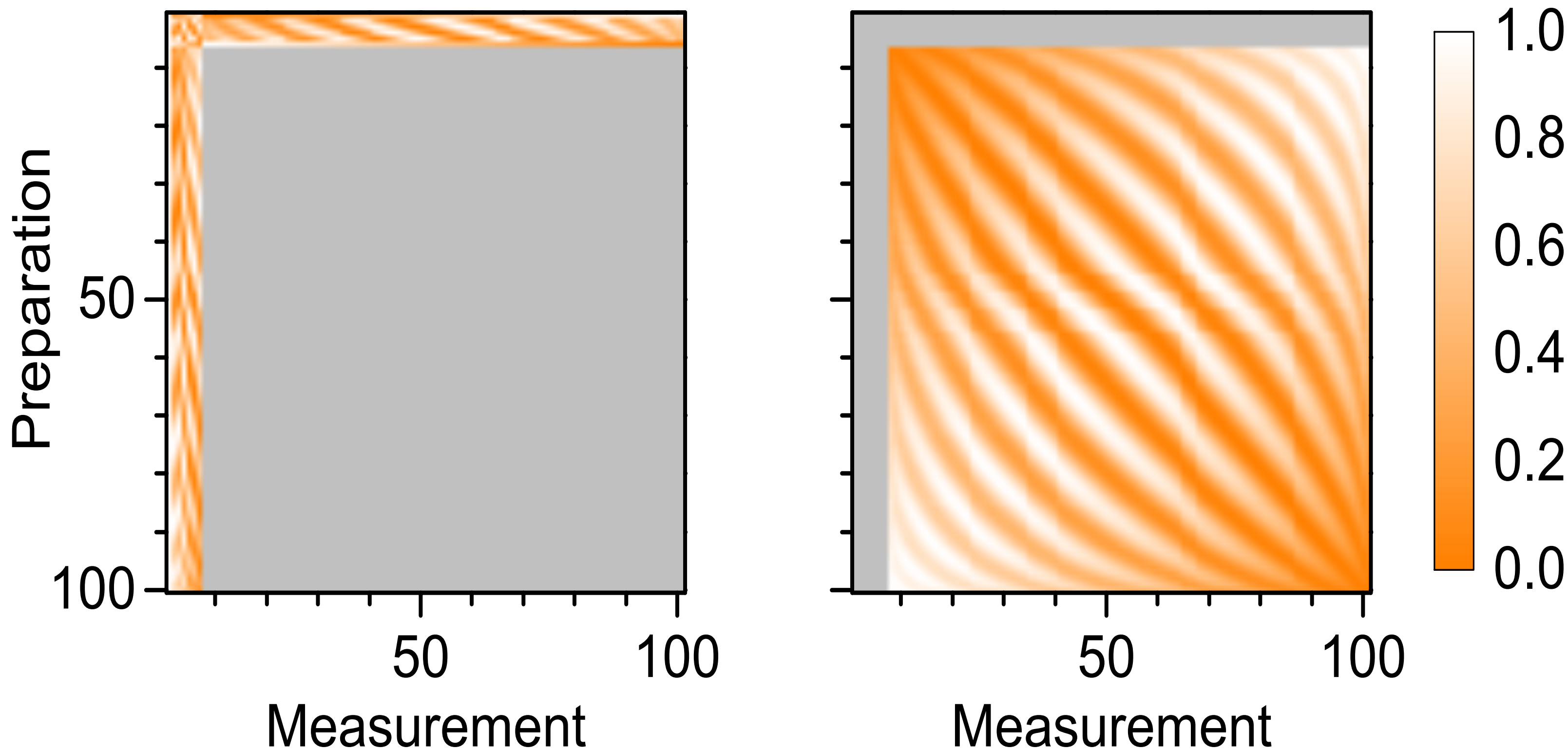


Rank-4 GPT of best fit for the experimental data

100 measurements on 100 preparations



$$V_{S_{min}}/V_{S_{max}} = 0.91267 \pm 0.00001$$

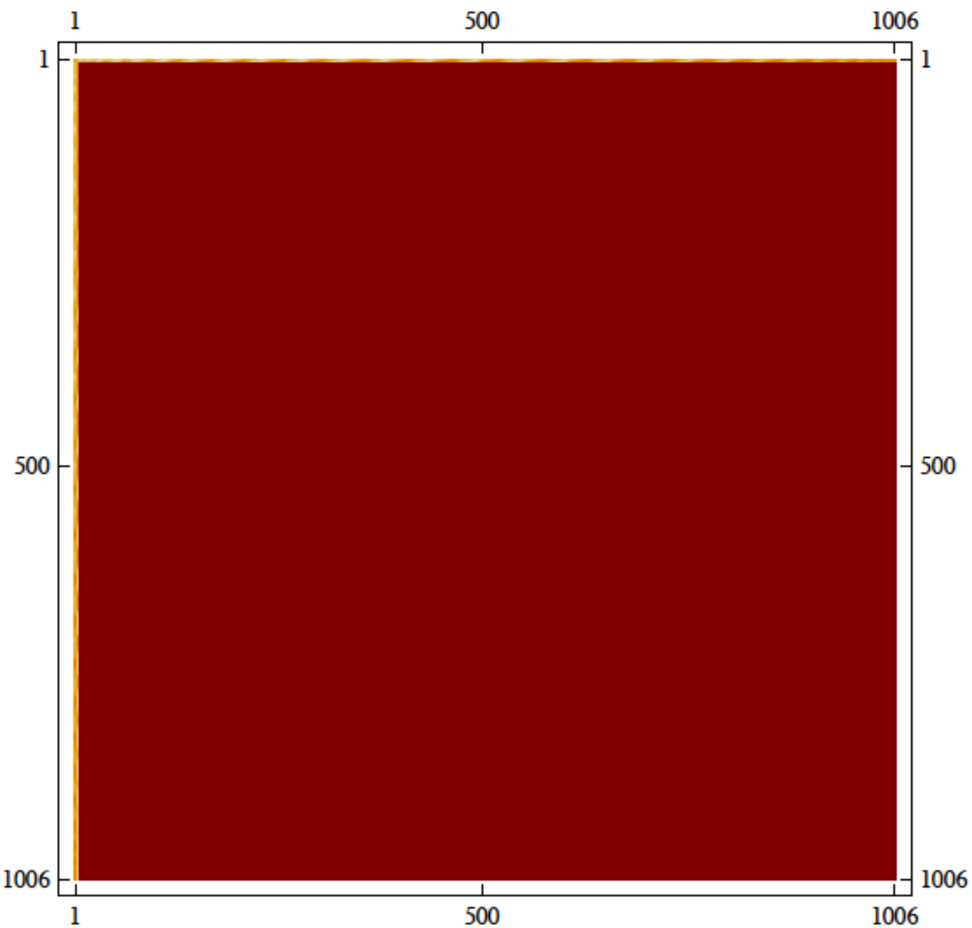


Preparation
50
100
50
100
Measurement
50
100
Measurement
1.0
0.8
0.6
0.4
0.2
0.0

More experimental data

1006 measurements on 6 preparations

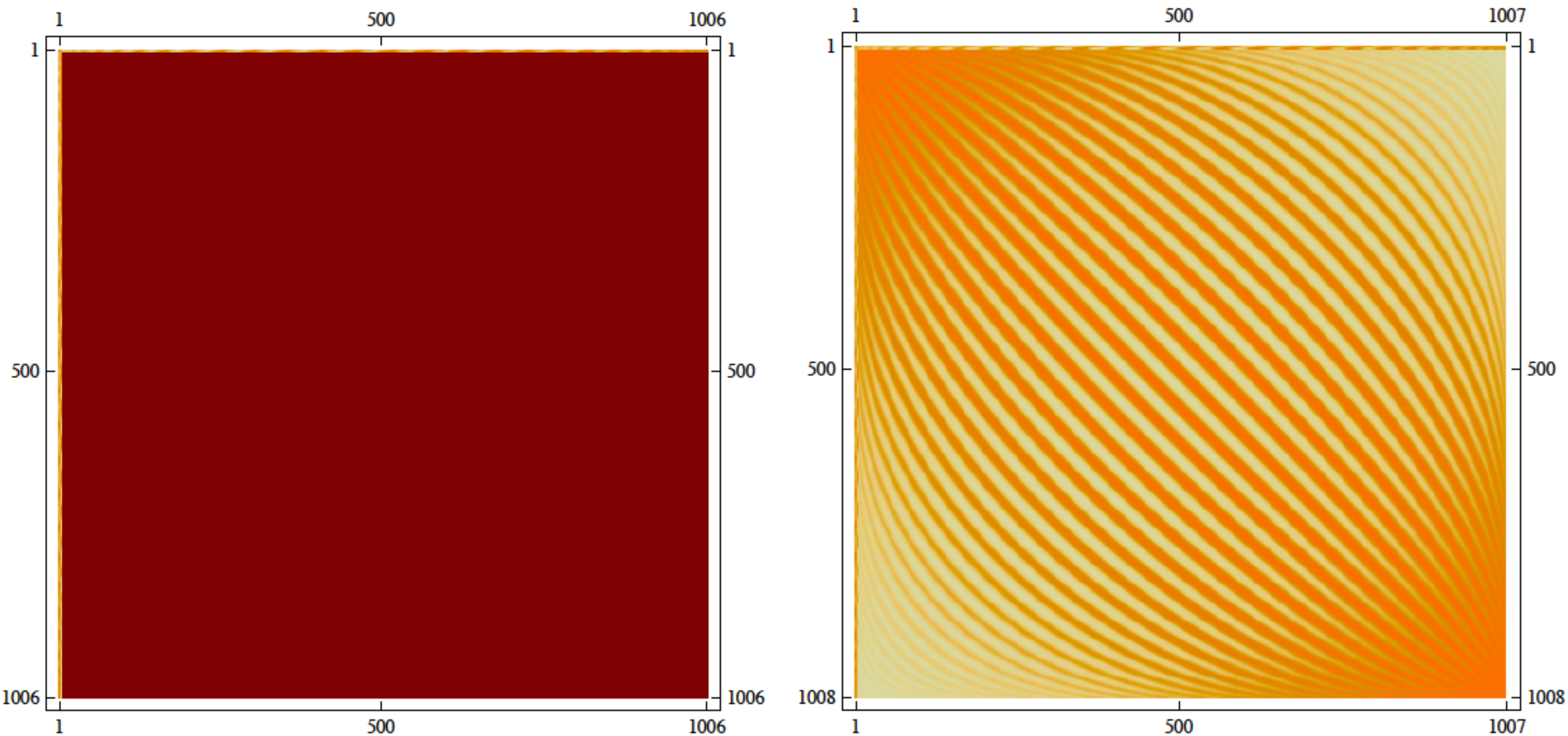
6 measurements on 1006 preparations



More experimental data

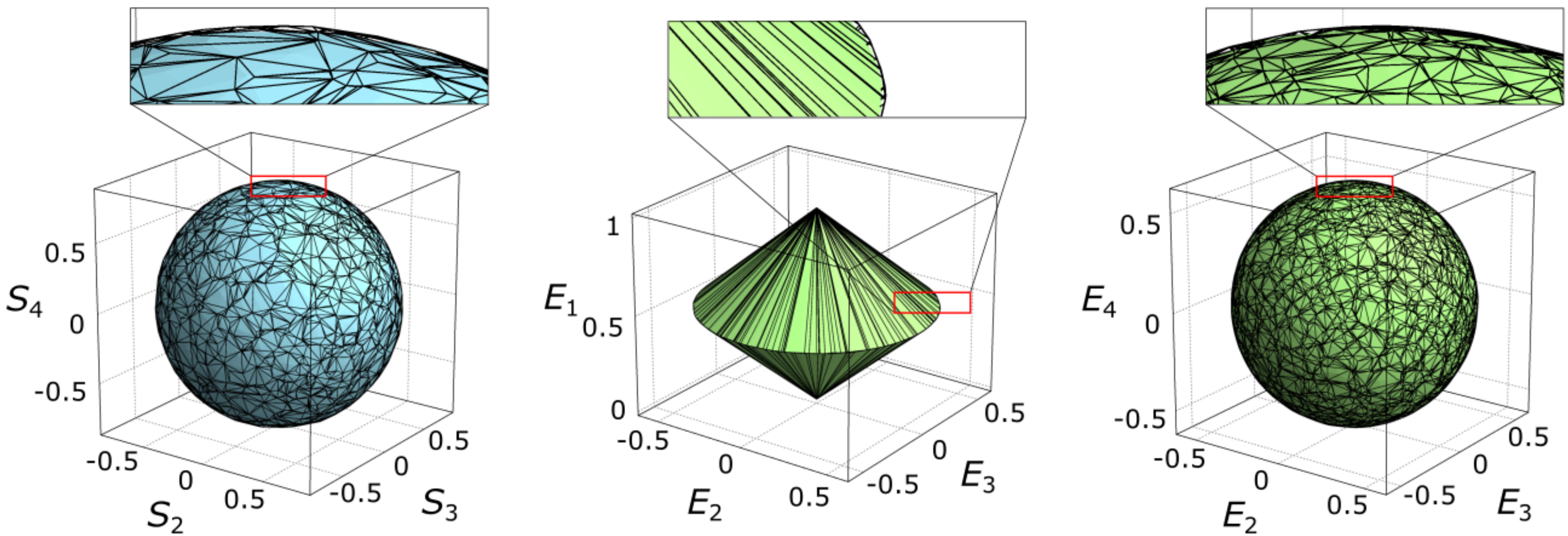
1006 measurements on 6 preparations

6 measurements on 1006 preparations



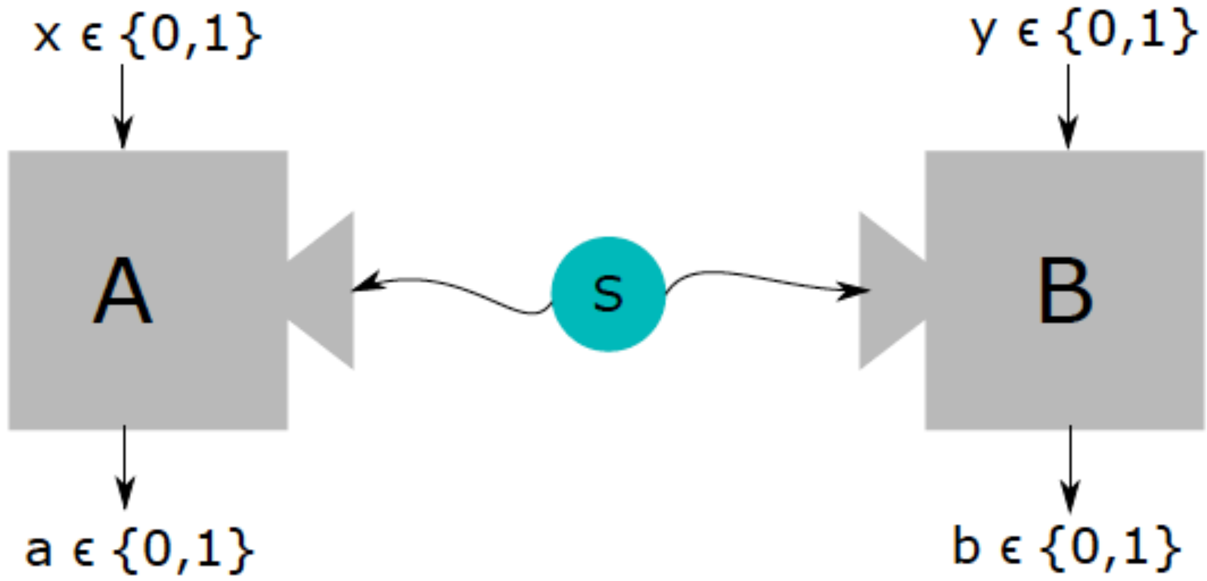
Rank-4 GPT of best fit for the experimental data

1006 measurements on 1006 preparations

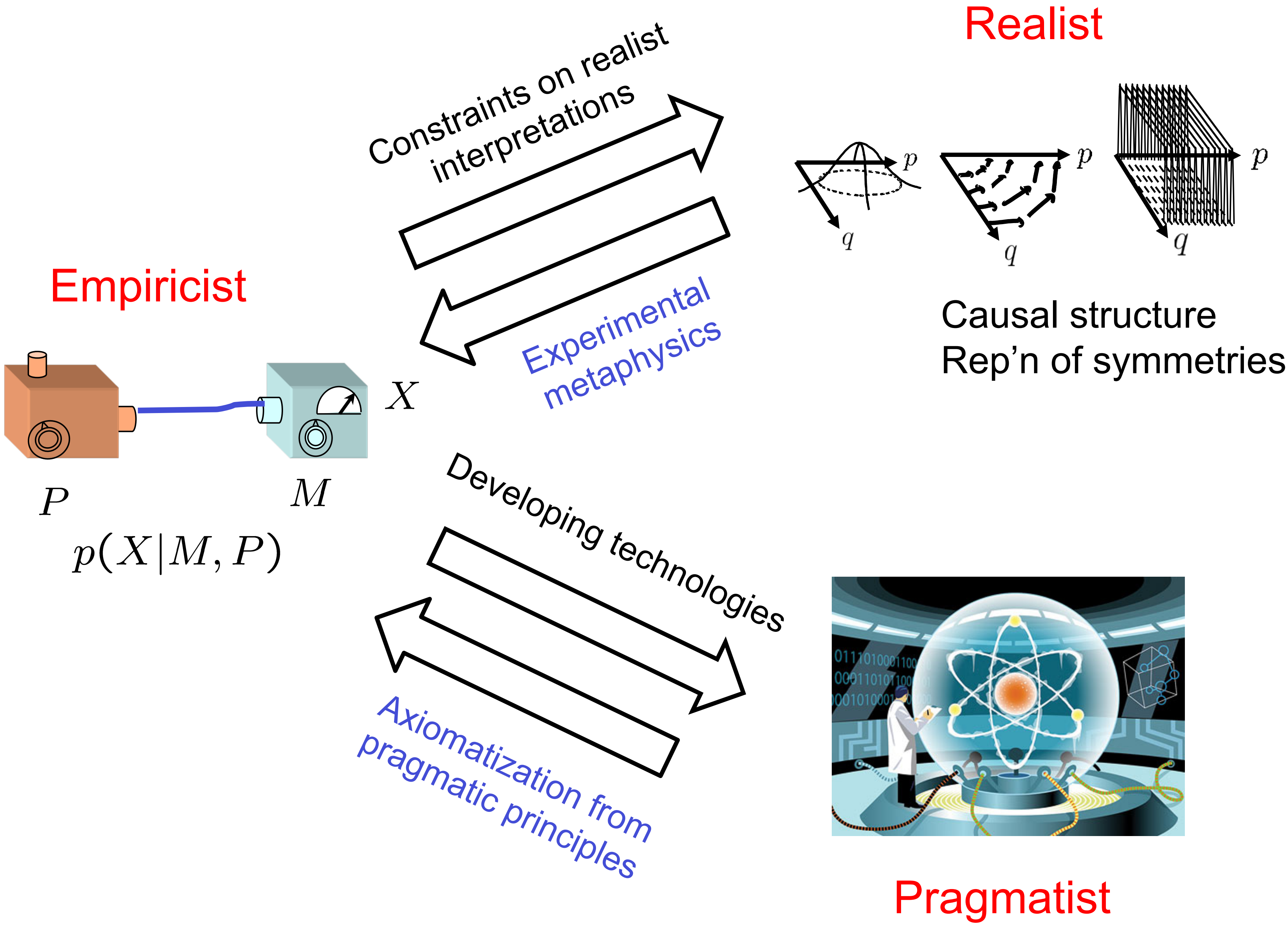


$$V_{S_{min}}/V_{S_{max}} = 0.968 \pm 0.001$$

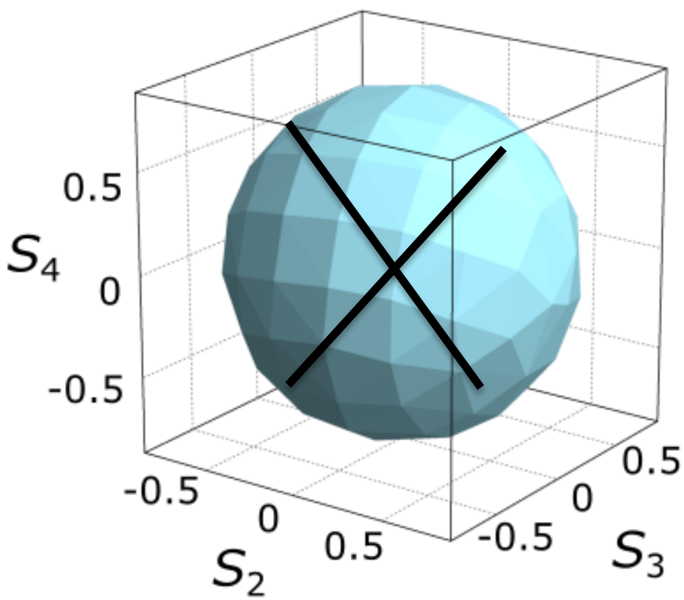
Experimental constraints on violations of Tsirelson bound

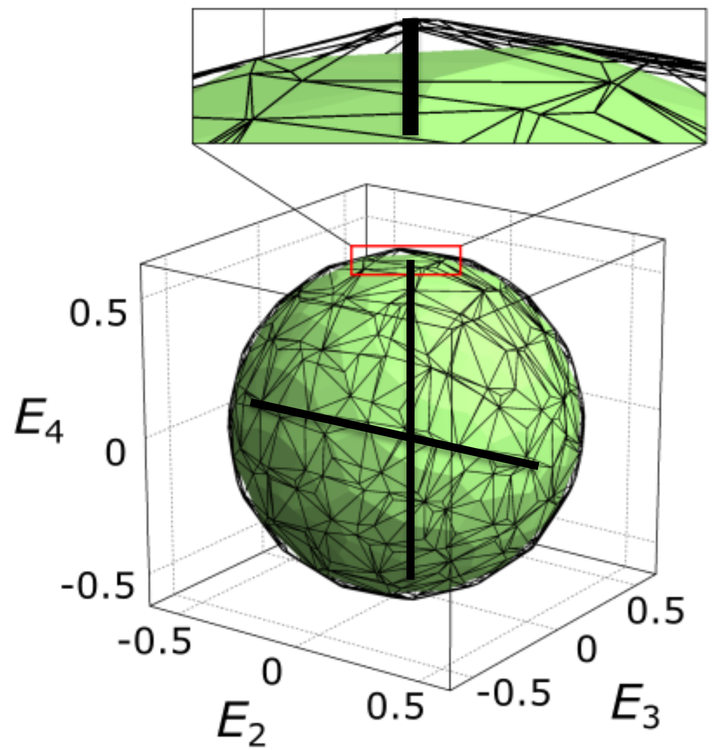


$$\frac{1}{4}\sum_{x,y} p(a \oplus b = xy | x, y) \underset{LHV}{\leq} \frac{3}{4} \underset{QM}{\leq} \frac{2+\sqrt{2}}{4}$$


Realist
Constraints on realist interpretations
p
q
Causal structure
Rep'n of symmetries
Empiricist
Experimental metaphysics
X
P
M
$p(X|M,P)$
Developing technologies
Axiomatization from pragmatic principles
Pragmatist

Experimental constraints on violations of Tsirelson bound





For our GPT-of-best-fit we find

$$\frac{1}{4}\sum_{x,y} p(a\oplus b = xy|x, y) \leq 0.87196 \pm 0.00006$$

Maximal QM value:

$$\frac{2+\sqrt{2}}{4} \approx 0.85355$$

Some morals of the story

The GPT framework provides a means of analyzing experimental data that does not presume the correctness of quantum theory. Use it for any experiment that seeks to look for deviations from QT!

Tomography for states and measurements can be achieved in a bootstrap manner

Don’t worry only about underfitting. Worry also about overfitting.