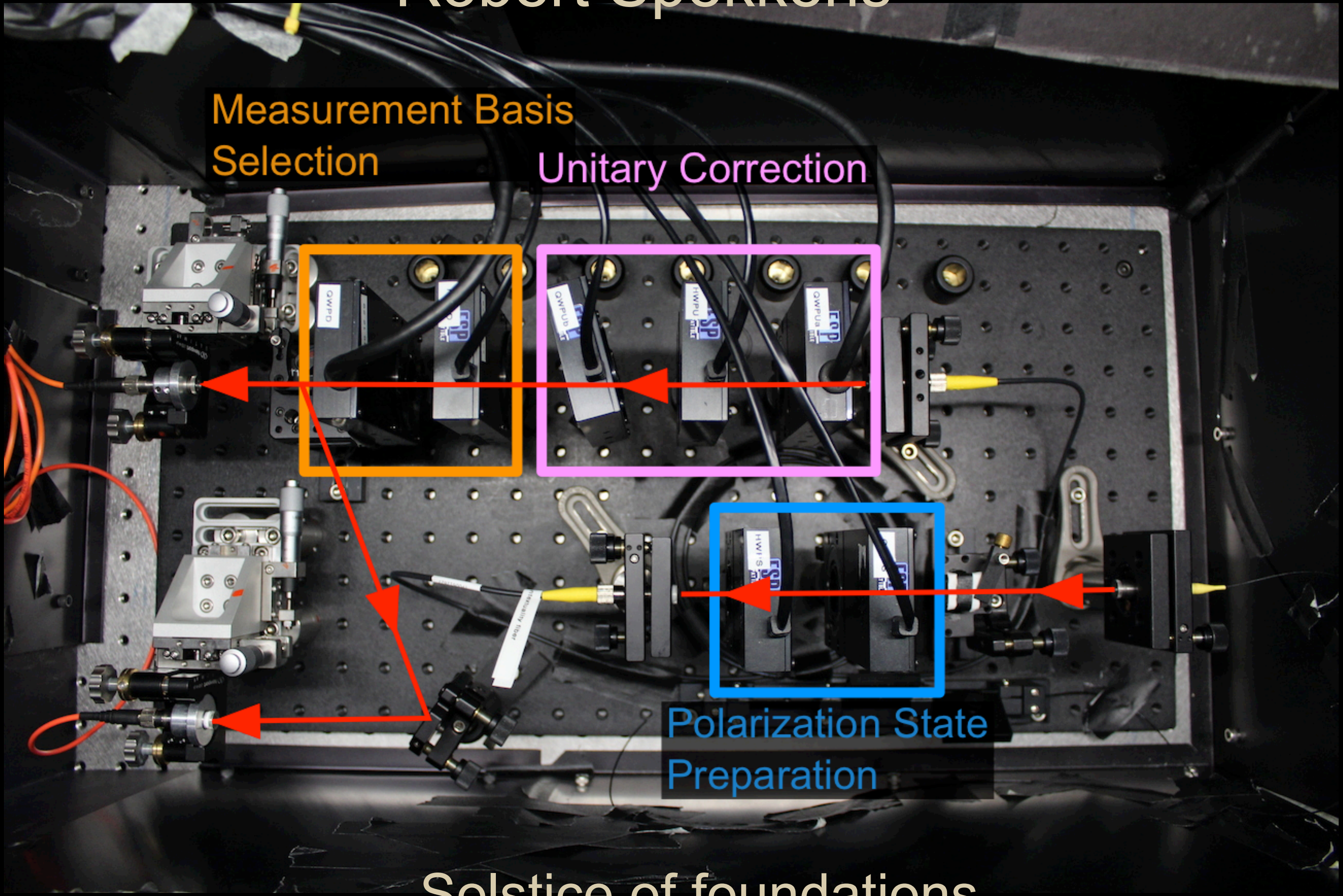
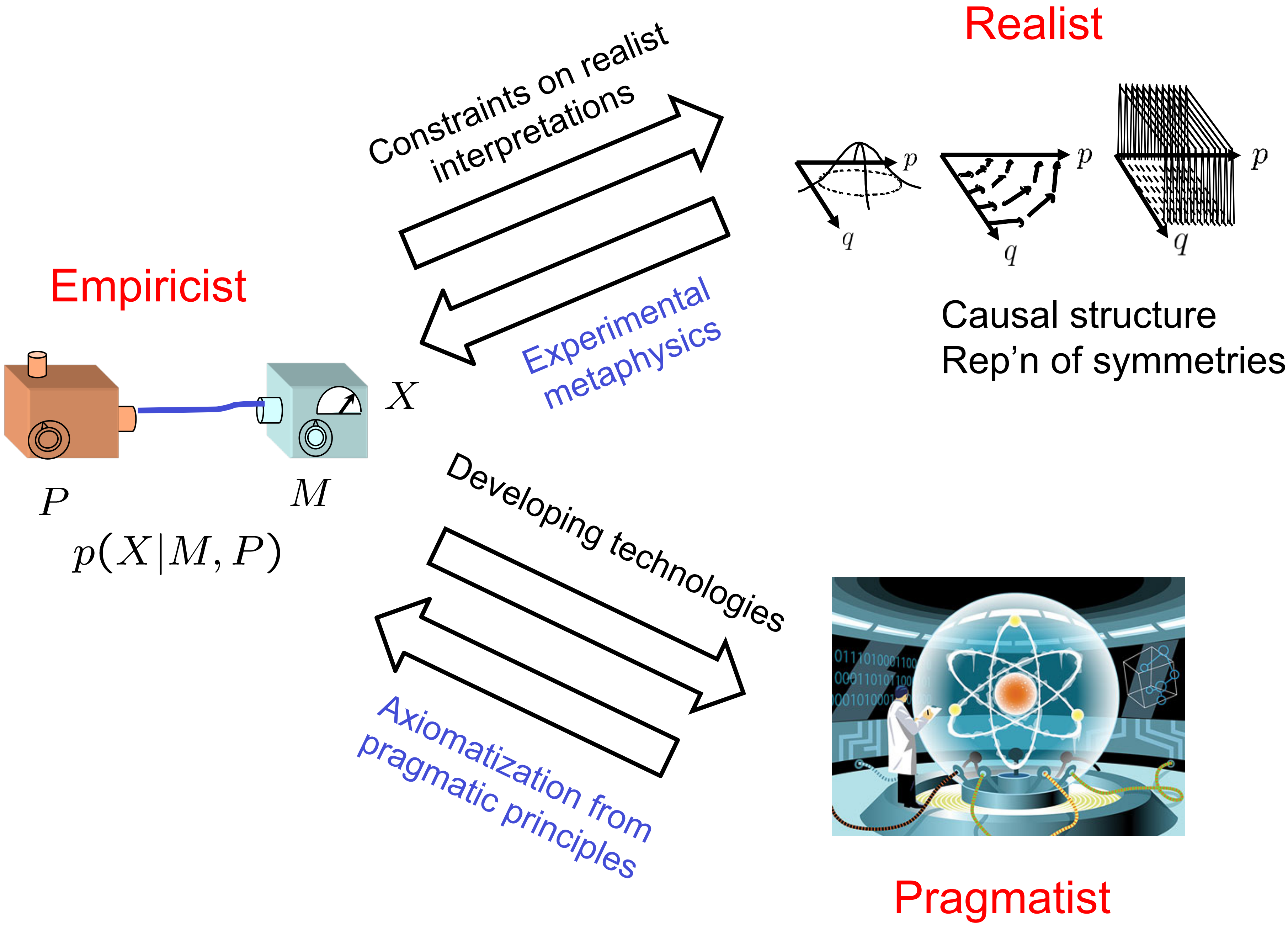
Experimental quantum foundations II

Robert Spekkens



Solstice of foundations
June 21, 2017



Realist
Constraints on realist interpretations
p
q
Causal structure
Rep'n of symmetries
Empiricist
Experimental metaphysics
X
P
M
$p(X|M,P)$
Developing technologies
Axiomatization from pragmatic principles
Pragmatist

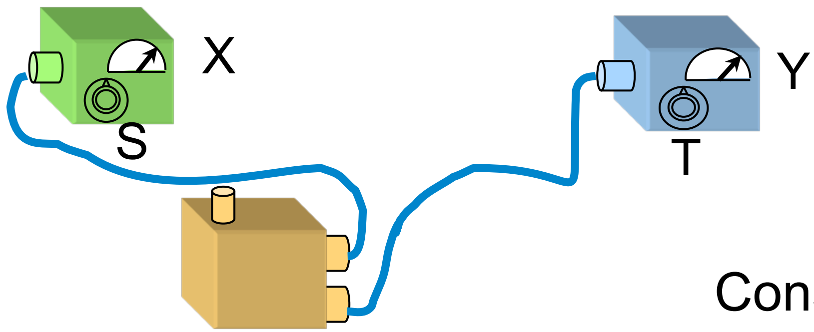
What we want in a notion of nonclassicality

Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios

What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure of local causality	✓	✓	✗

What is needed to witness the failure of local causality

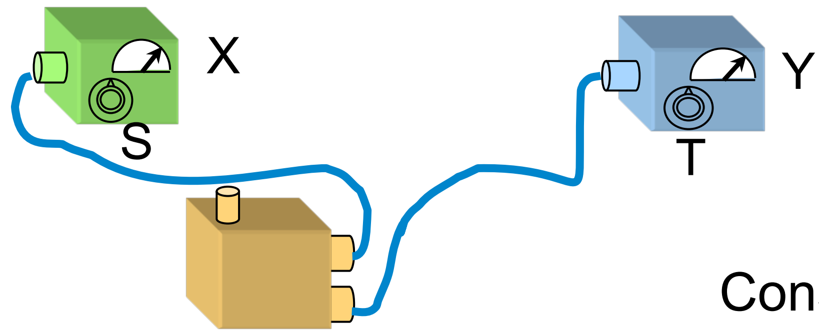


Consider P(XY|ST)

What we want in a notion of nonclassicality

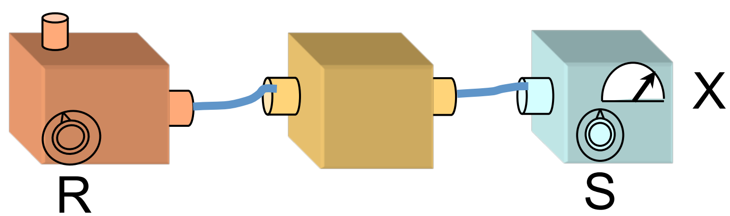
	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure of local causality	✓	✓	✗
Failure of noncontextuality			✓

What is needed to witness the failure of local causality



Consider P(XY|ST)

What is needed to witness the failure of noncontextuality



Consider P(X|RS)

What we want in a notion of nonclassicality

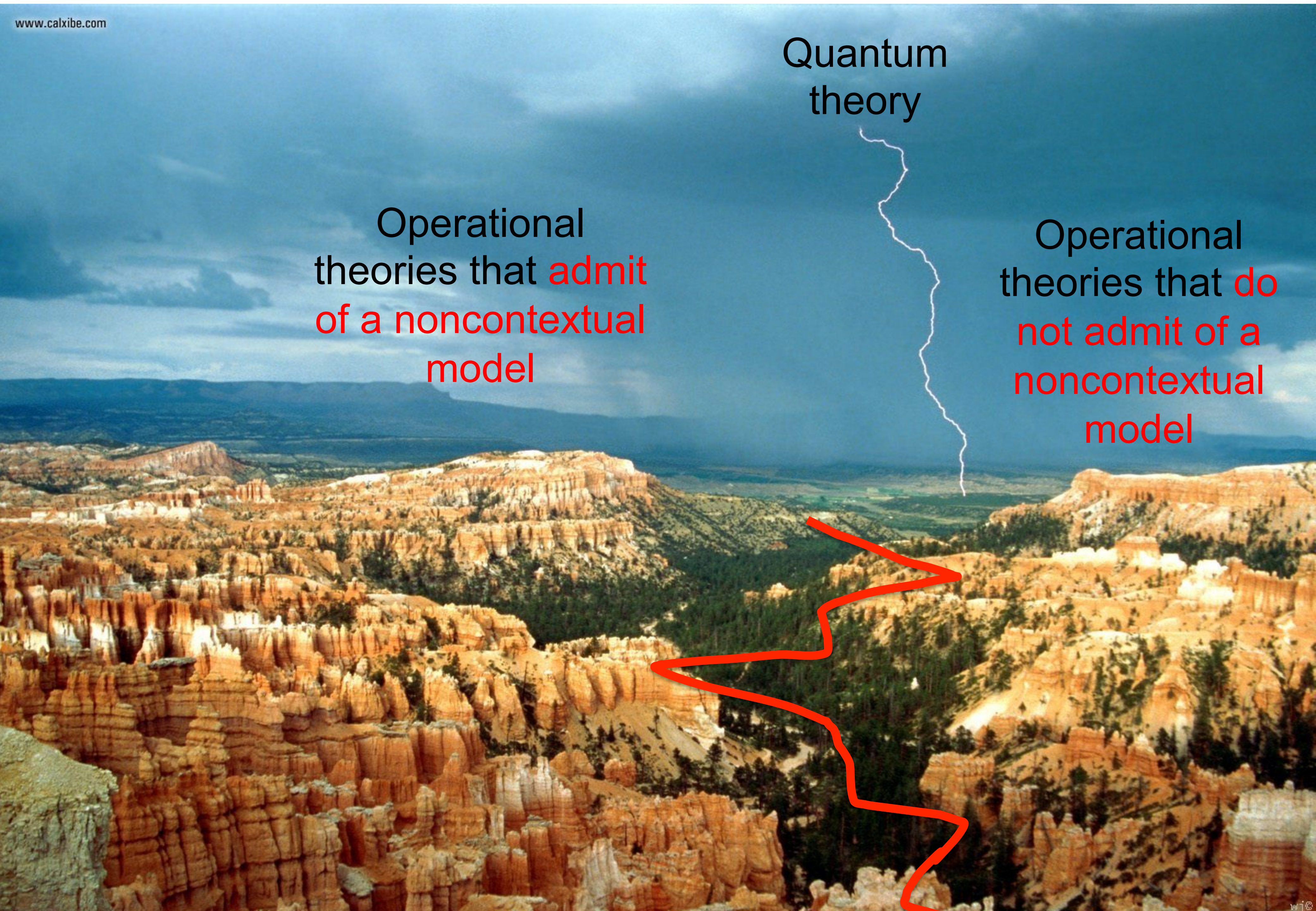
	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure of local causality	✓	✓	✗
Failure of noncontextuality	?	?	✓

What we want in a notion of nonclassicality

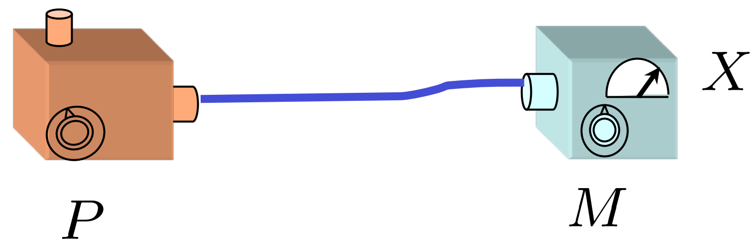
	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure of local causality	✓	✓	✗
Failure of noncontextuality	?	✓	✓

What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure of local causality	✓	✓	✗
Failure of noncontextuality	✓	✓	✓

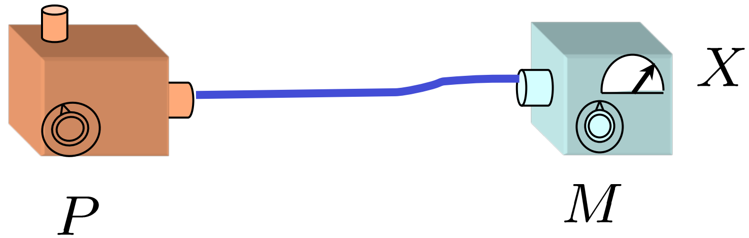

www.calxibe.com
Quantum theory
Operational theories that admit of a noncontextual model
Operational theories that do not admit of a noncontextual model

Operational theory



$$p(X|M,P)$$

Operational theory



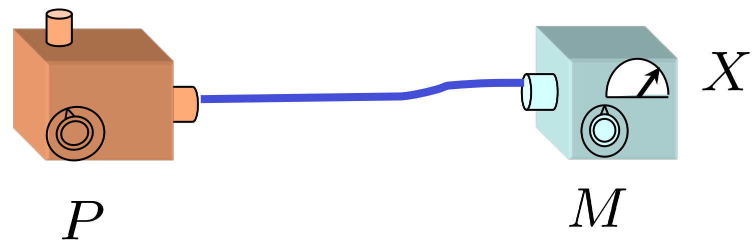
$$p(X|M,P)$$

Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates
between P and M

Operational theory



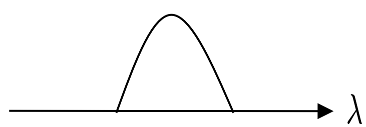
$$p(X|M,P)$$

Ontological model of an operational theory

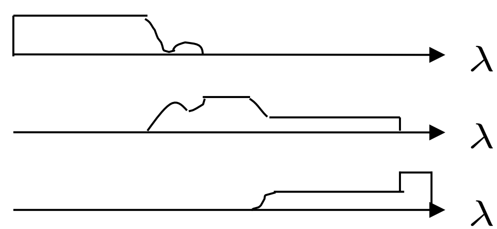
$\lambda \in \Lambda$ Ontic state space

causally mediates between P and M

$\mathrm{P} \leftrightarrow \mu(\lambda|\mathrm{P})$



$\mathrm{M} \leftrightarrow \xi(X|\mathrm{M},\lambda)$



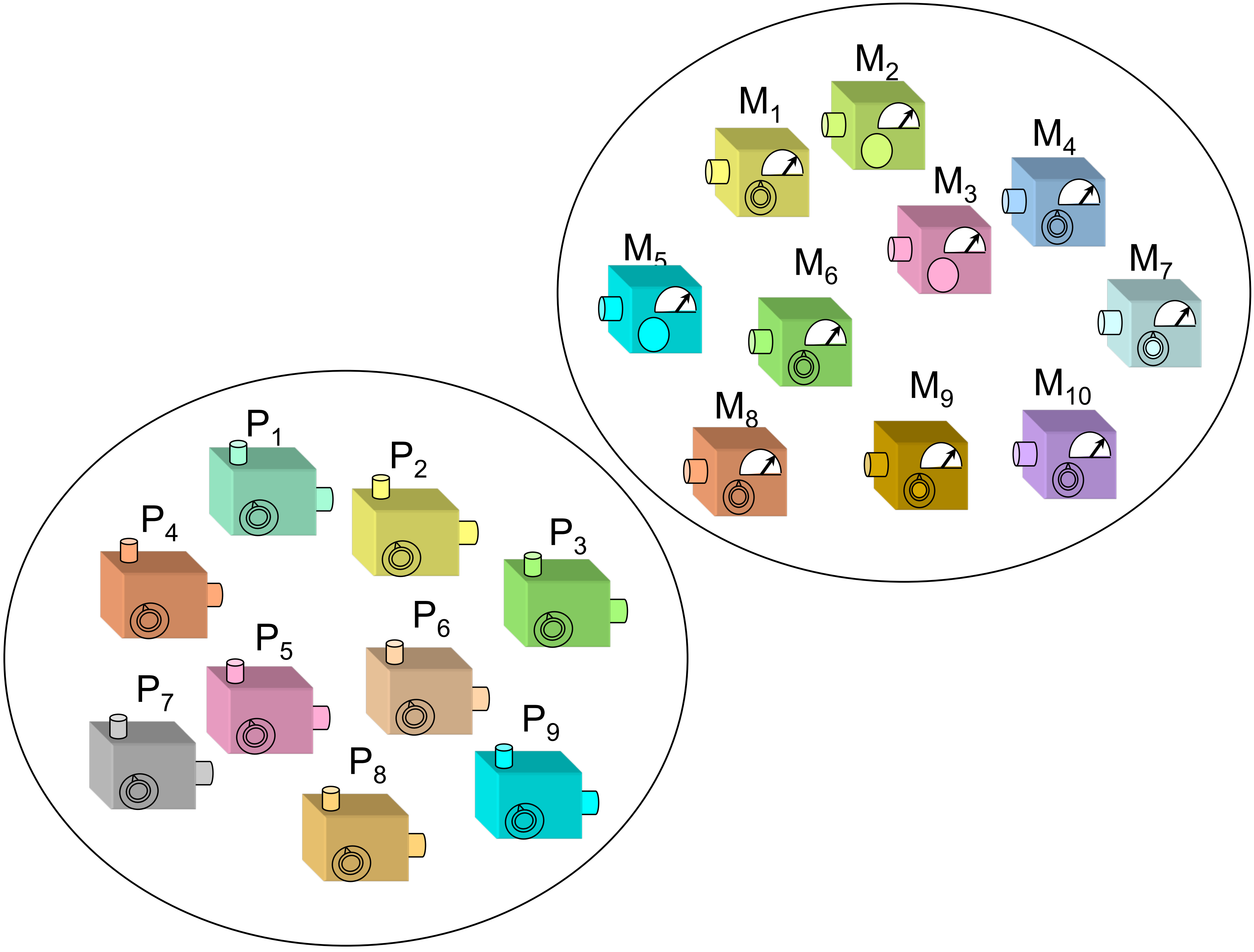
$$p(X|M,P) = \sum_\lambda \xi(X|M,\lambda)\,\mu(\lambda|P)$$

An ontological model of an operational theory is noncontextual if

Operational equivalence of two experimental procedures $\longrightarrow$ Equivalent representations in the ontological model

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation Noncontextuality



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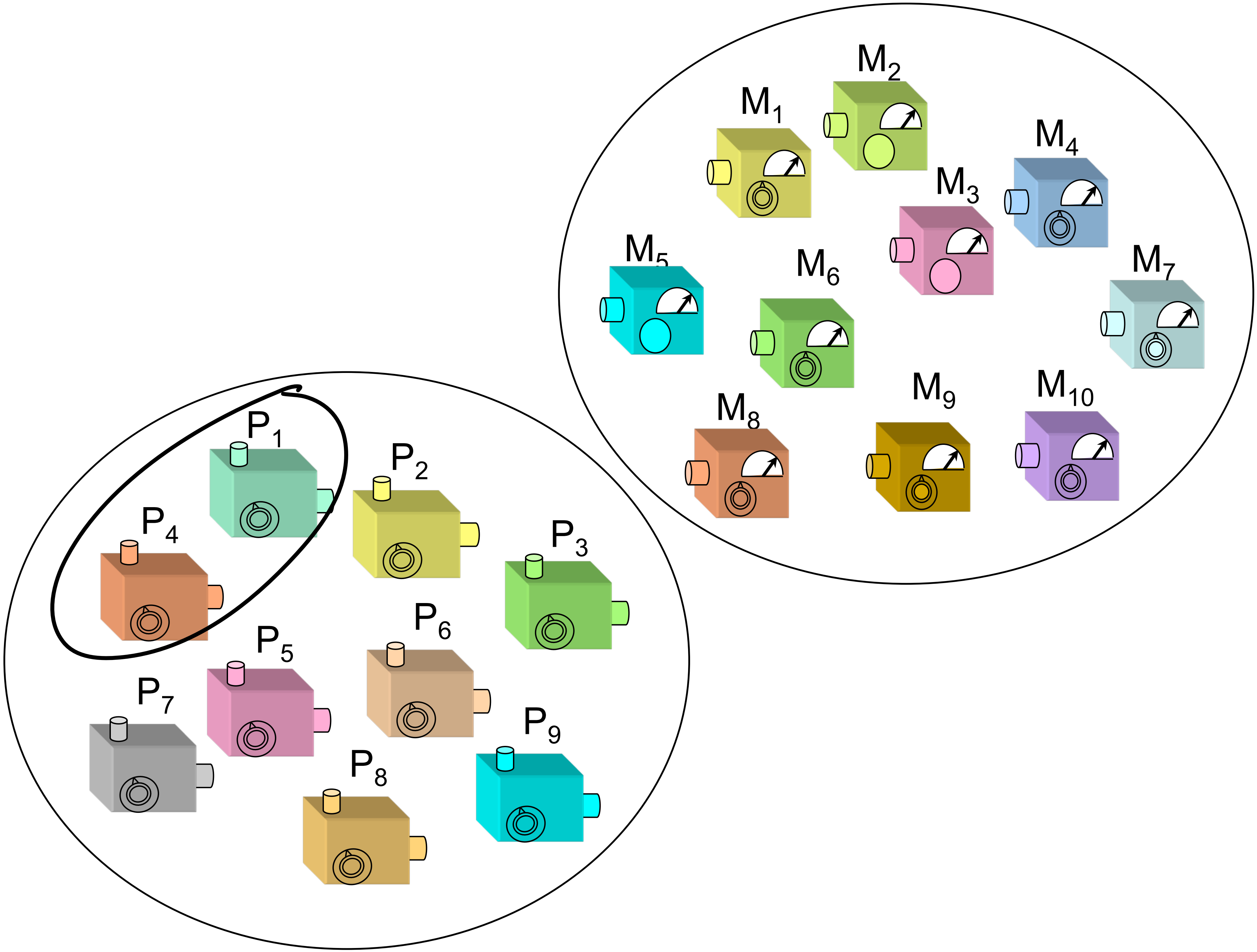

M_1
M_2
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M_5
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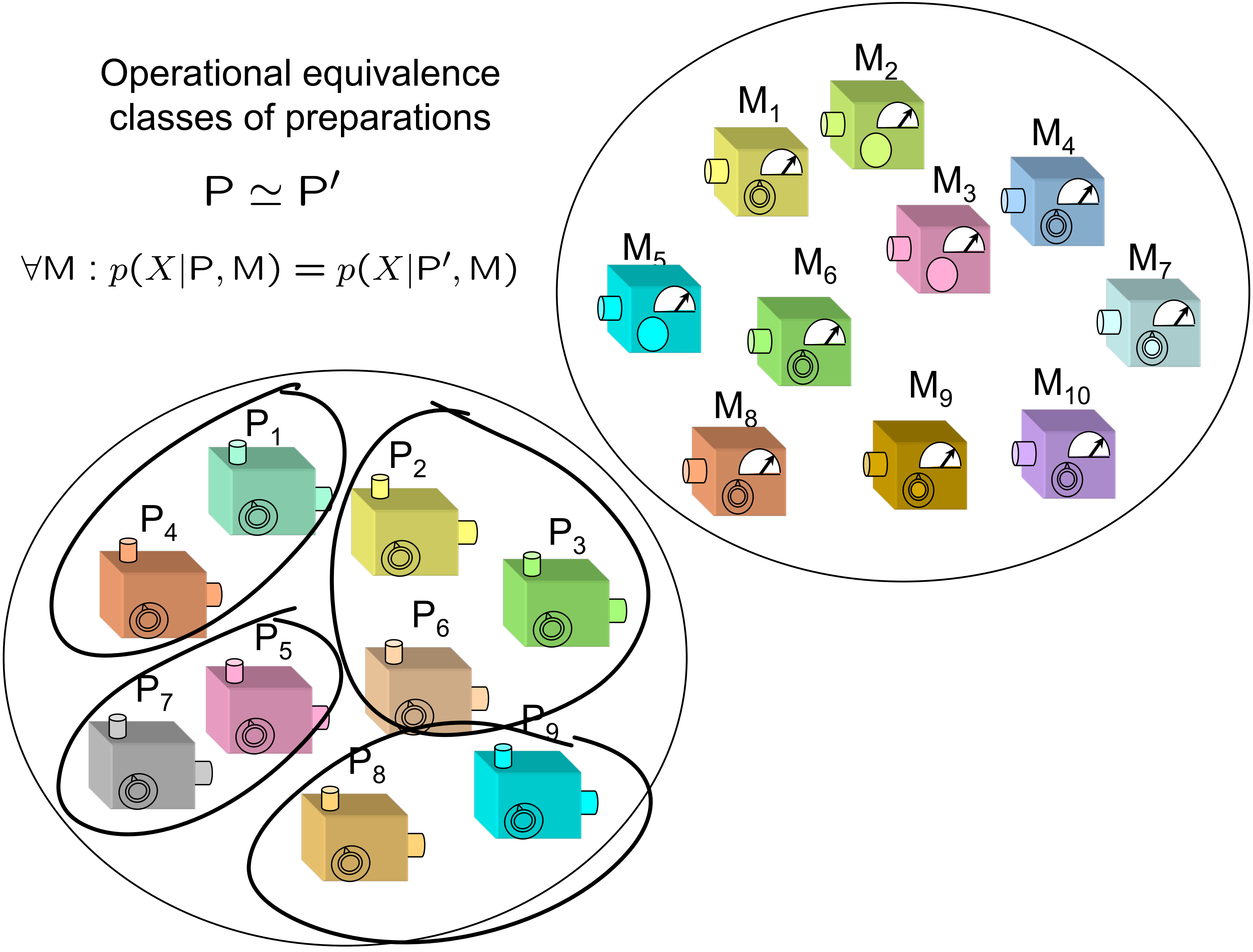
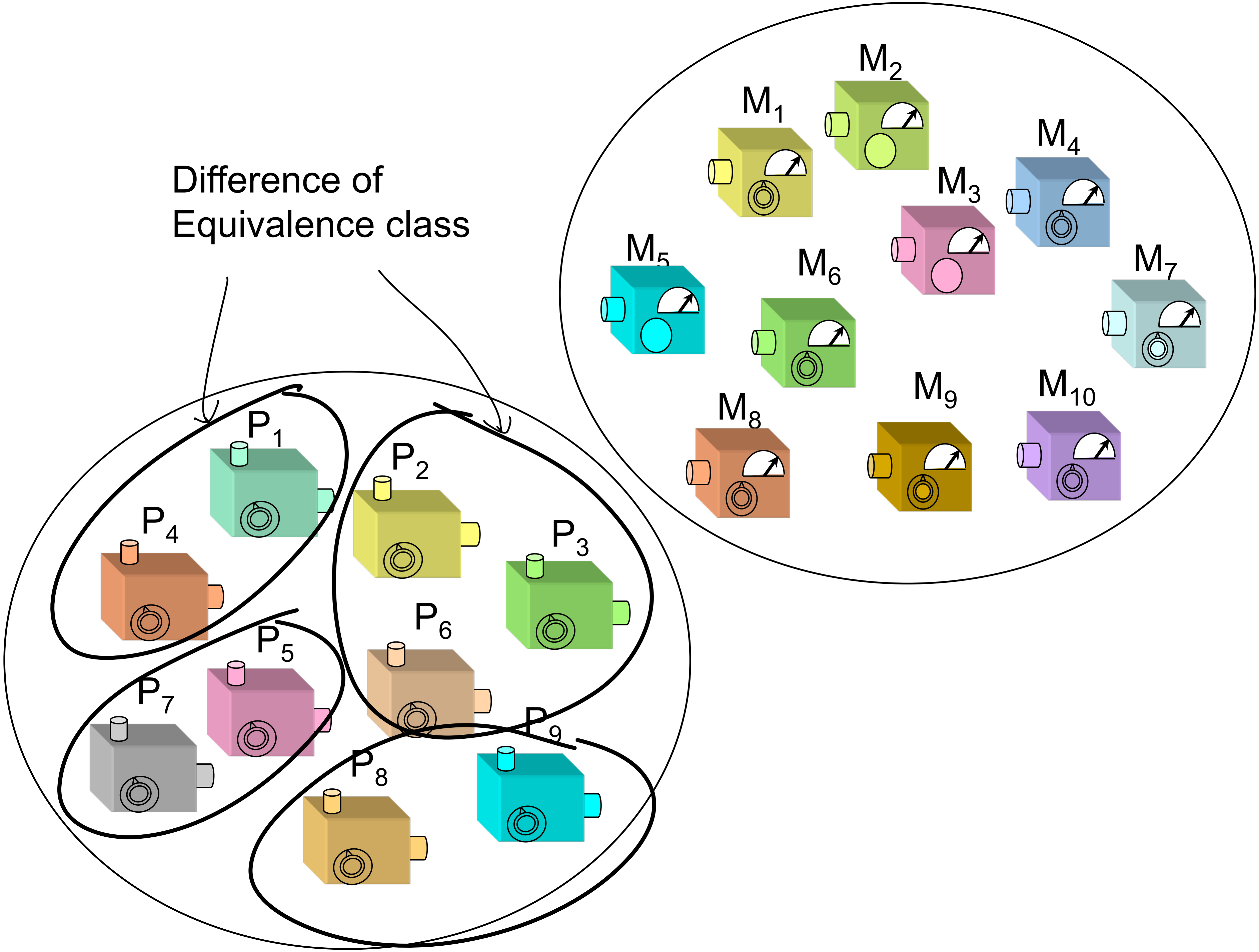
M_8
M_9
M_{10}
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9

Operational equivalence
classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$




Difference of
Equivalence class
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
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P_1
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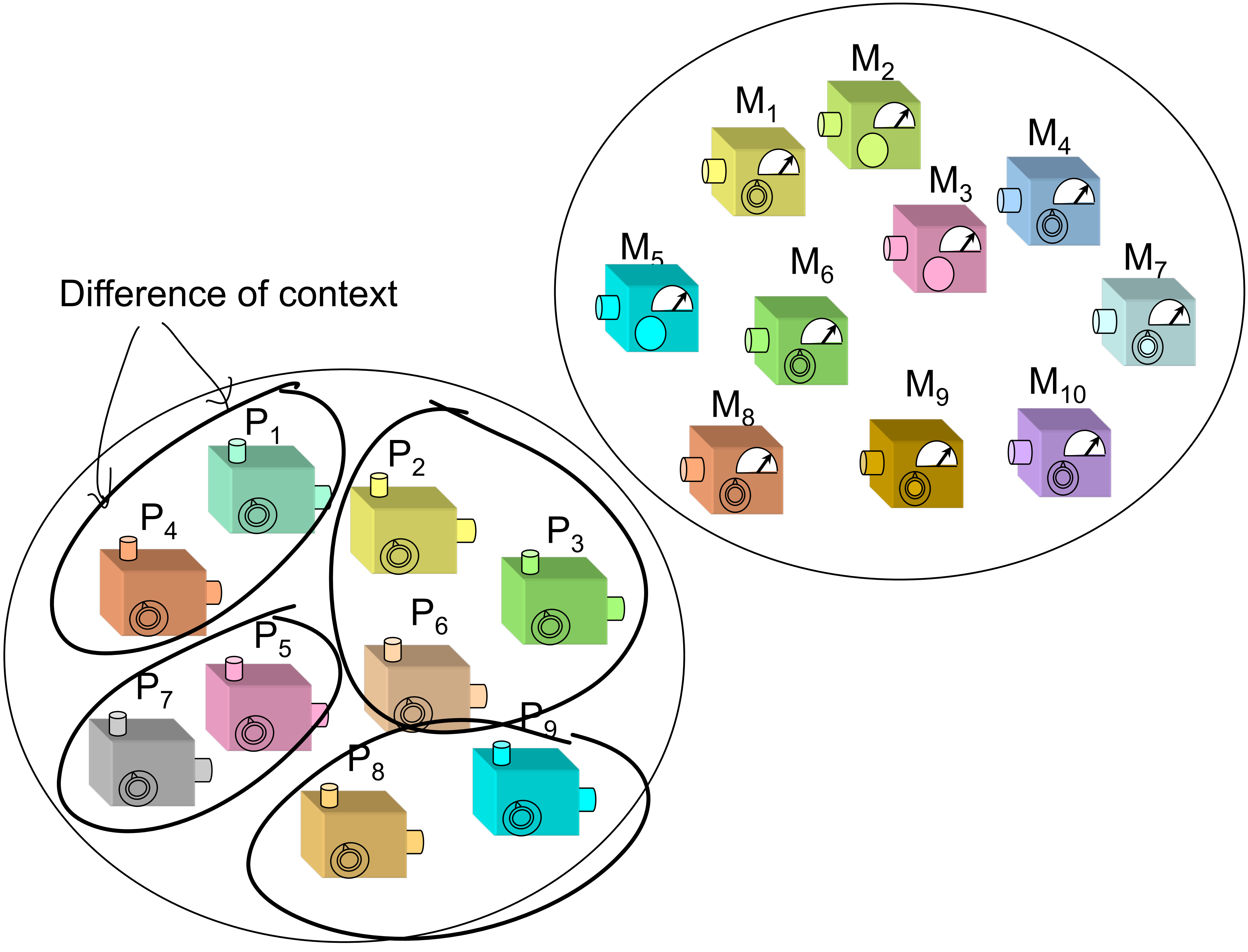
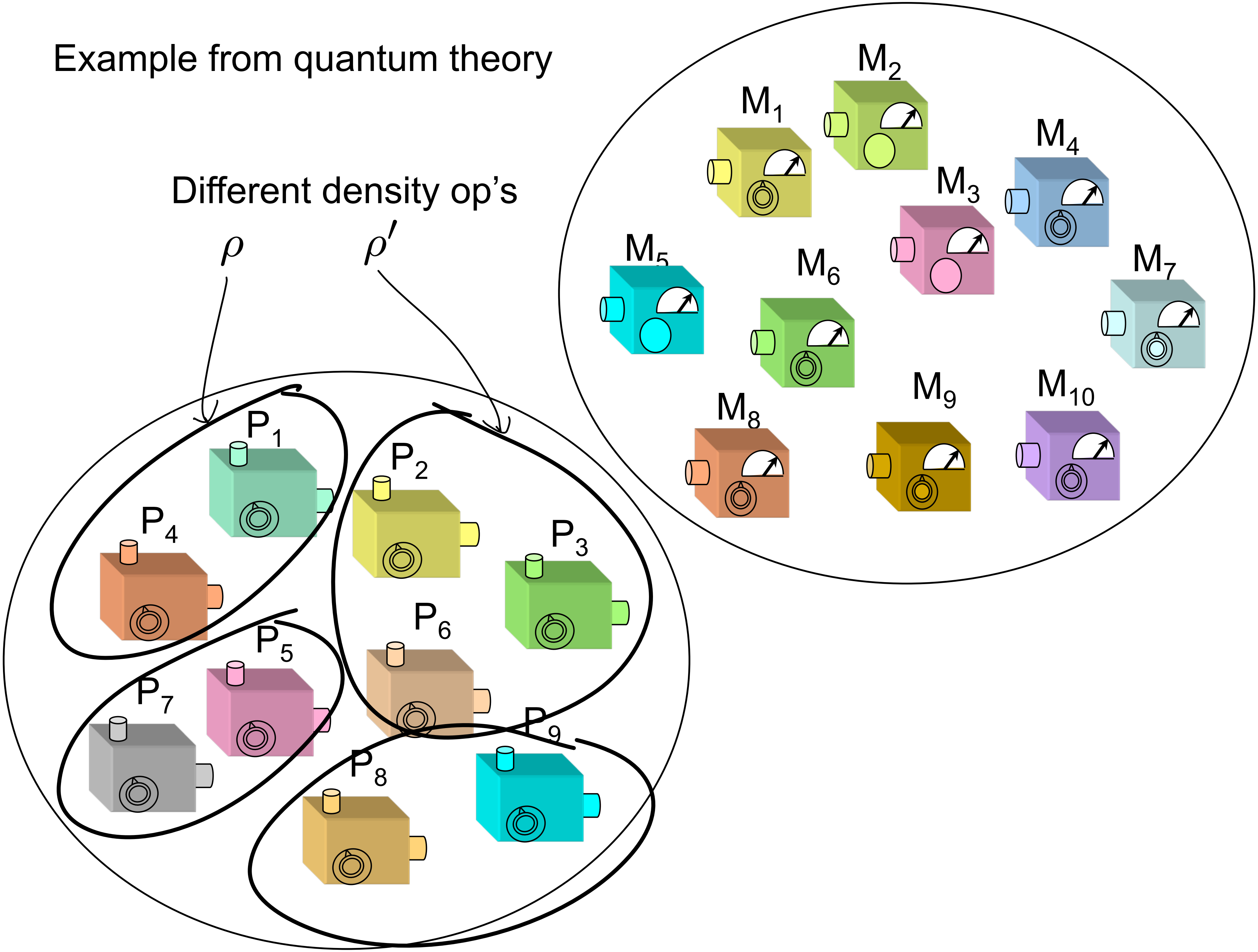


Difference of context
P1
P2
P3
P4
P5
P6
P7
P8
P9
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10

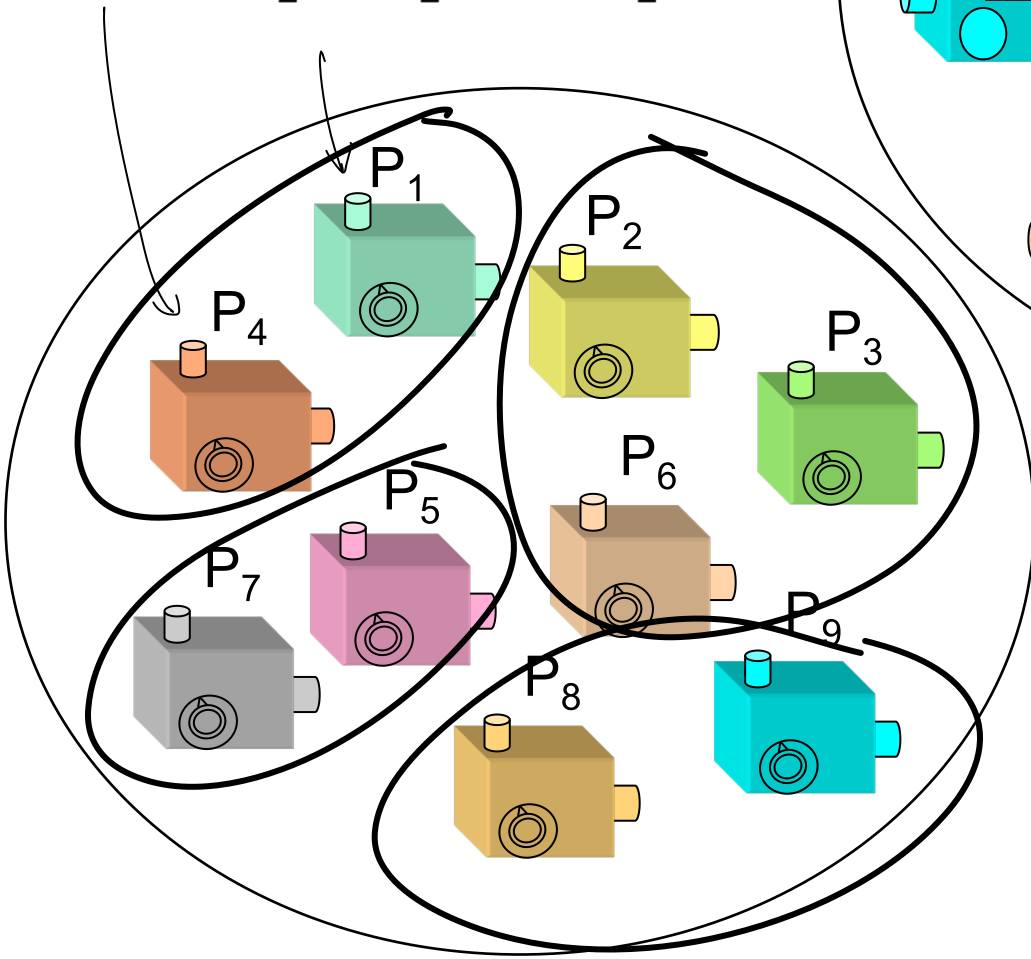
Example from quantum theory

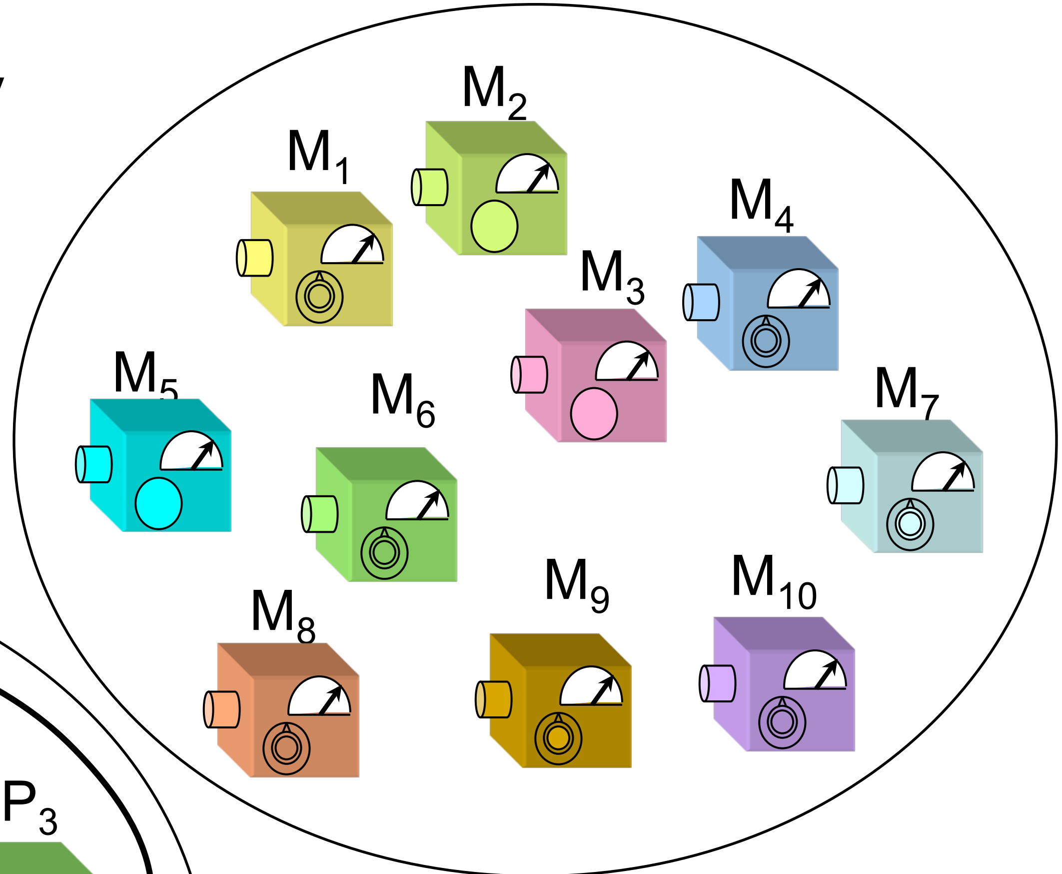


Example from quantum theory

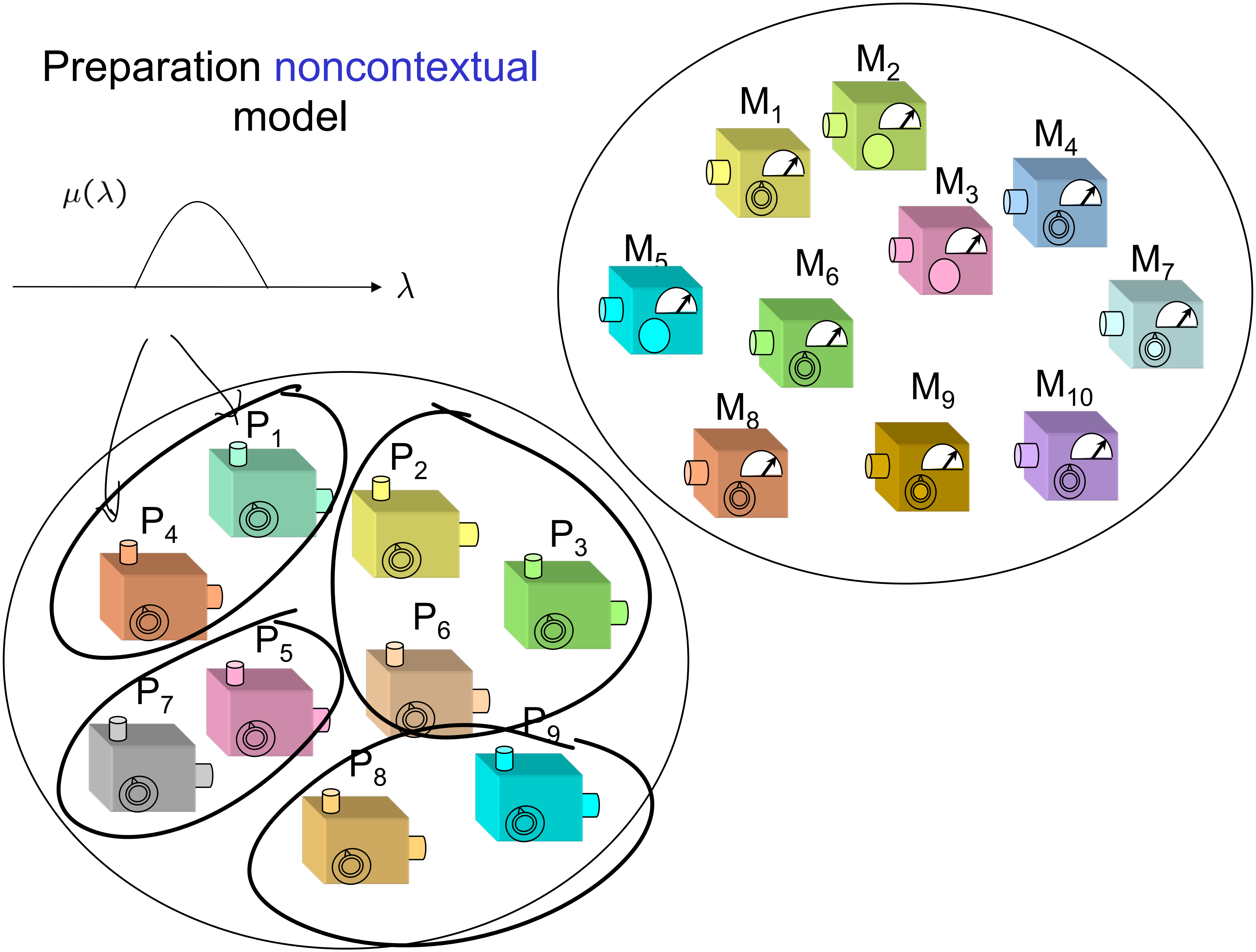
$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

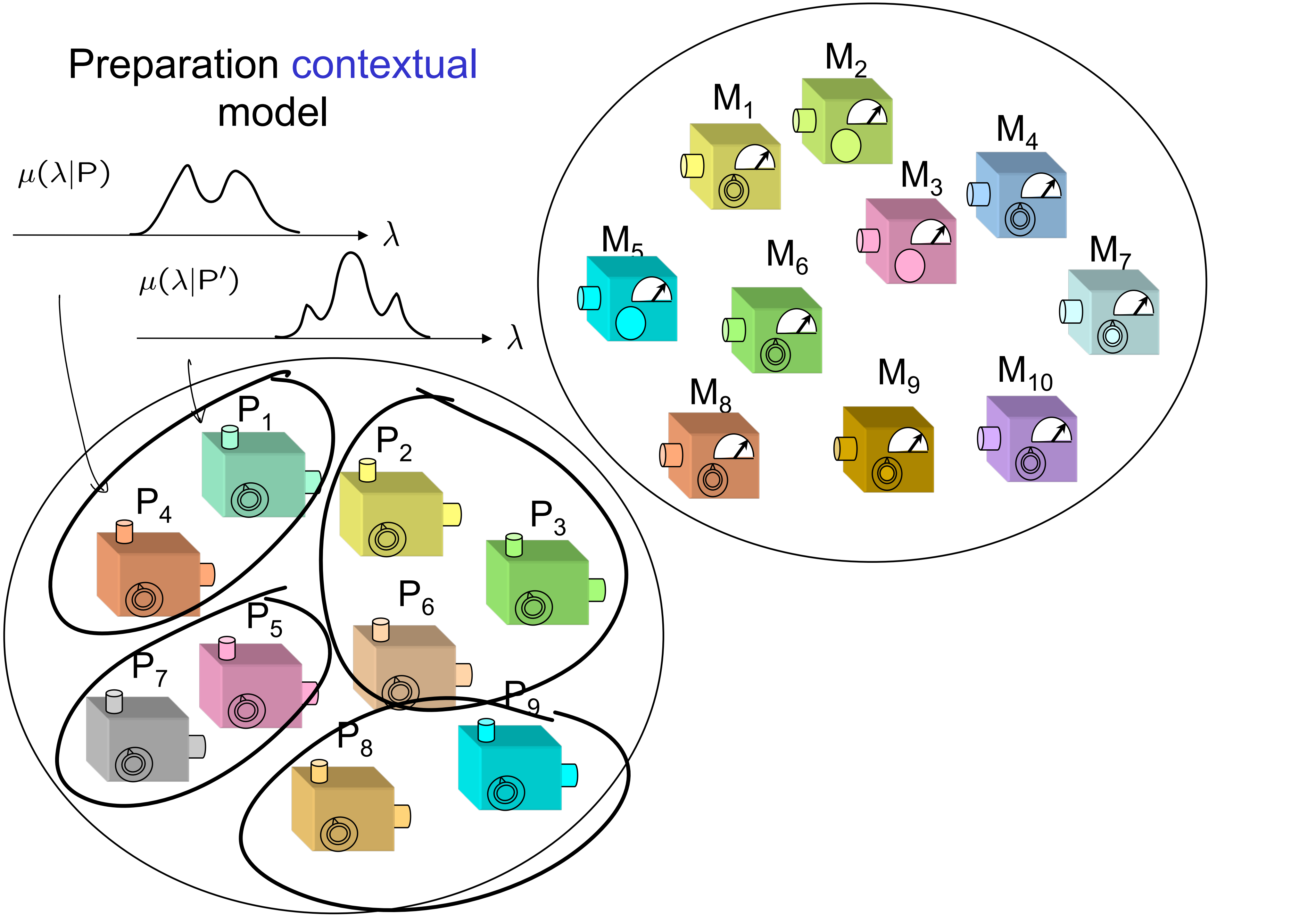
$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$





Preparation noncontextual model





Preparation contextual model
$\mu(\lambda|\mathrm{P})$
λ
$\mu(\lambda|\mathrm{P}')$
λ
P_1
P_2
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P_7
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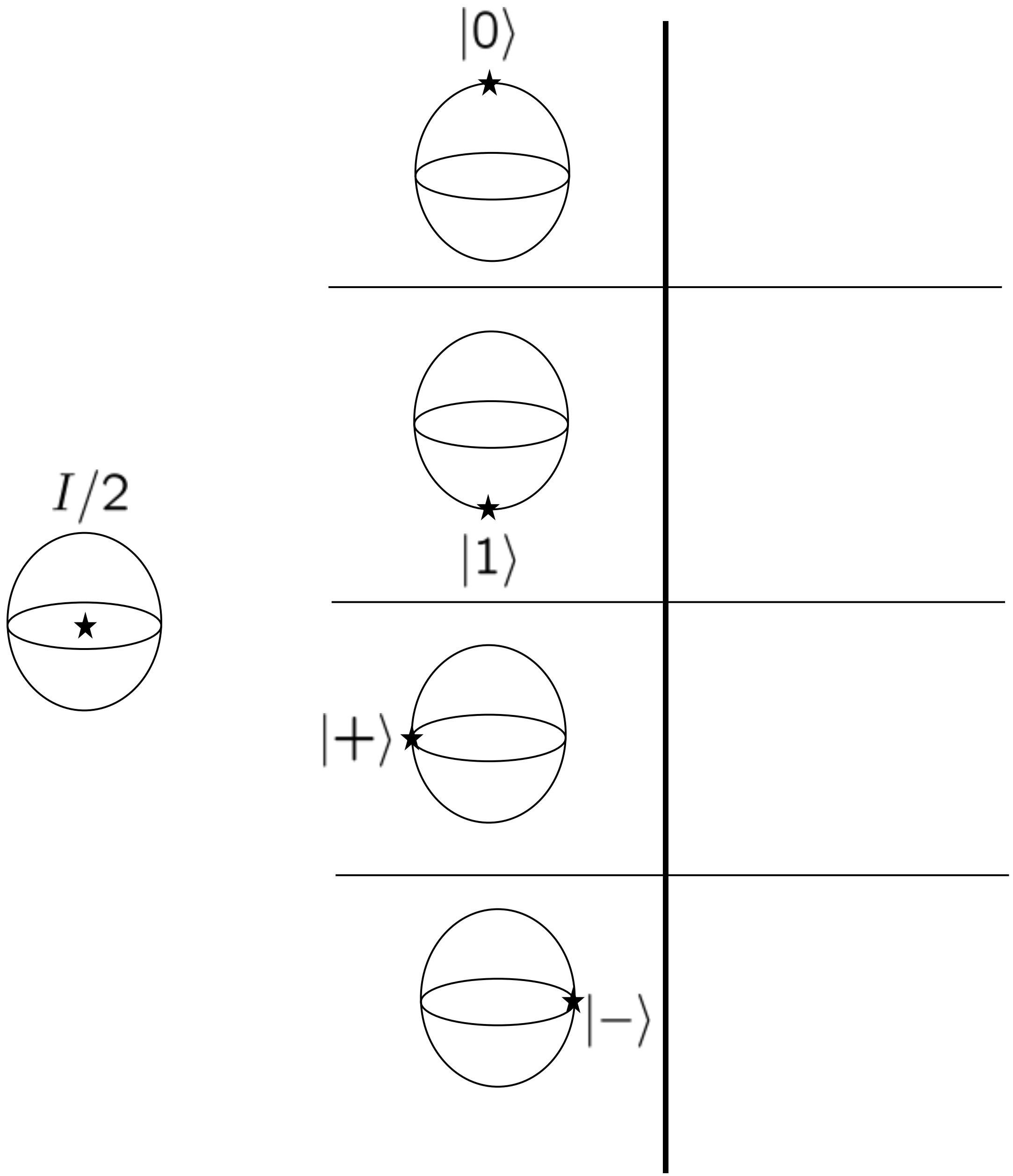
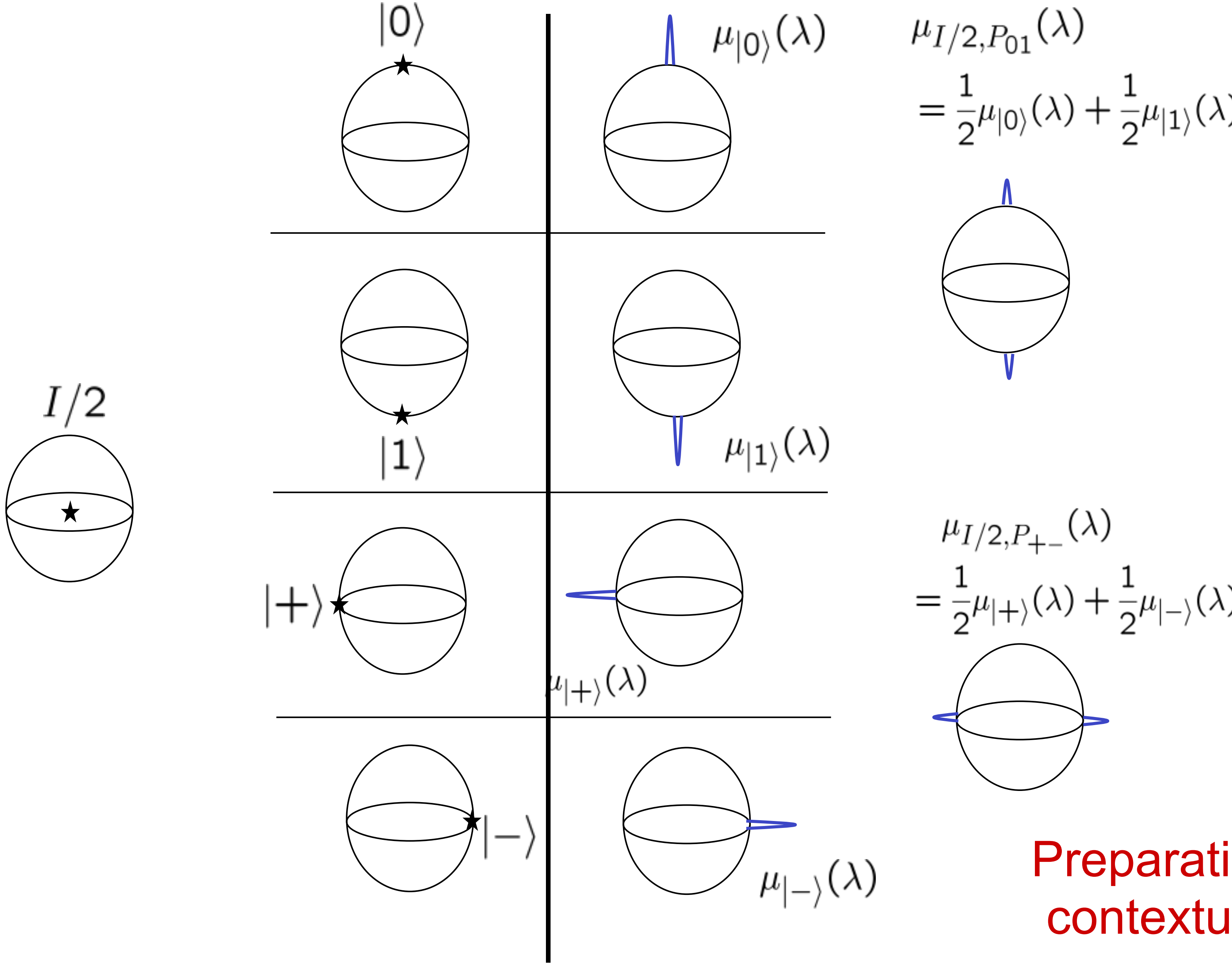
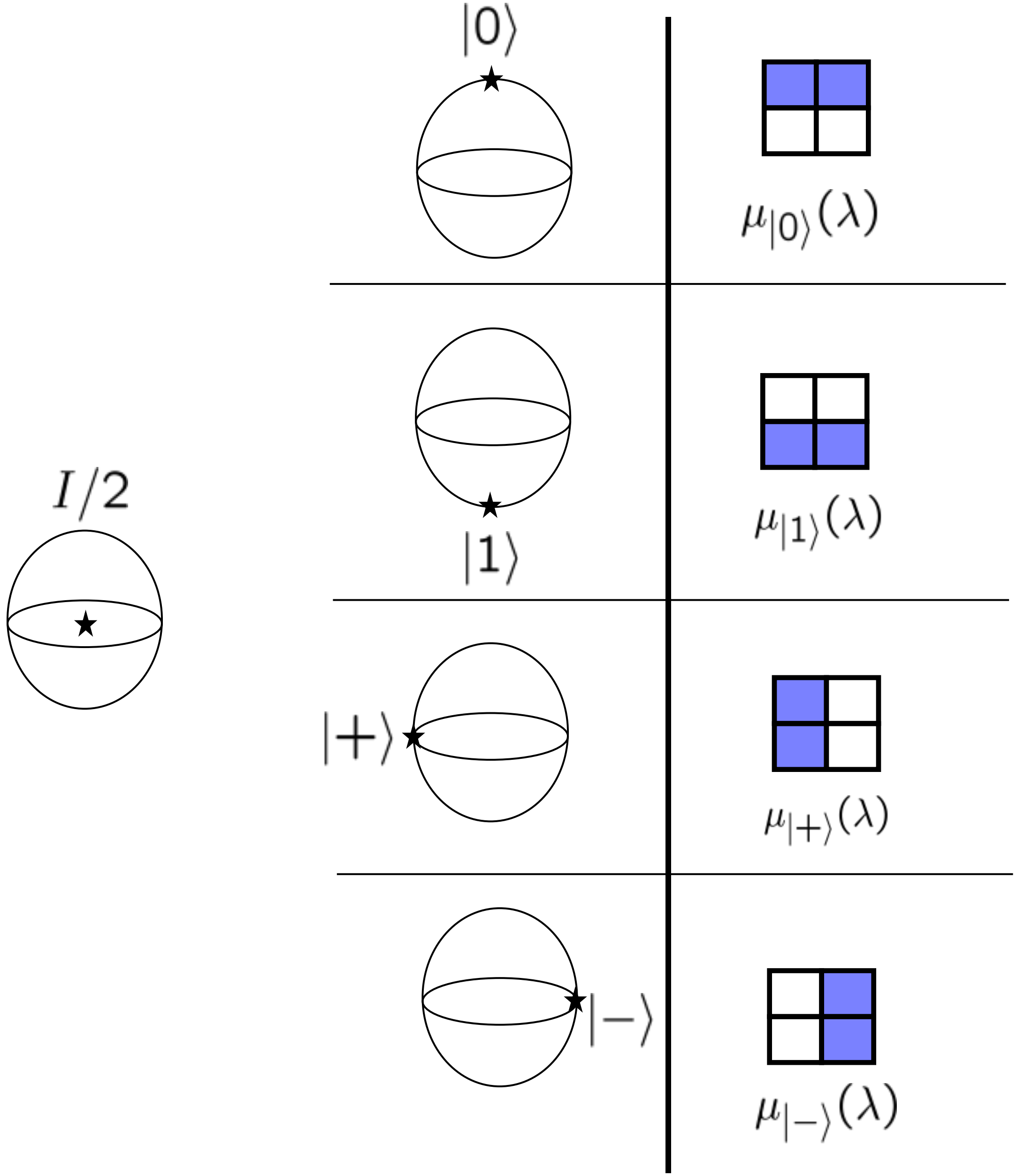
M_{10}

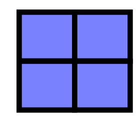

$I/2$
$|0\rangle$
$|1\rangle$
$|+\rangle$
$|-\rangle$



Preparation contextual

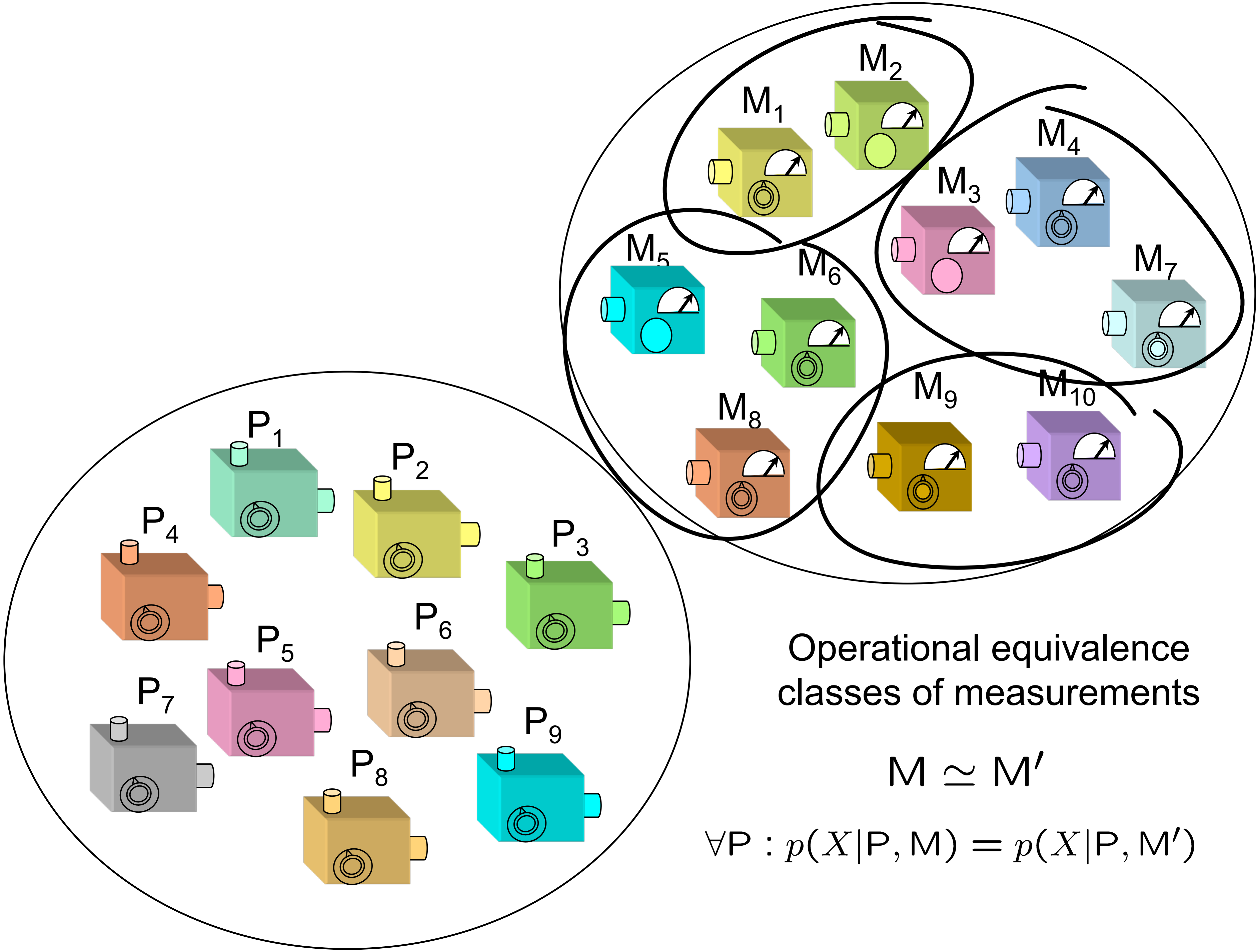


$$\begin{aligned}&\mu_{I/2}(\lambda)\\ &= \frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda)\\ &= \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)\end{aligned}$$



Preparation noncontextual

Measurement Noncontextuality



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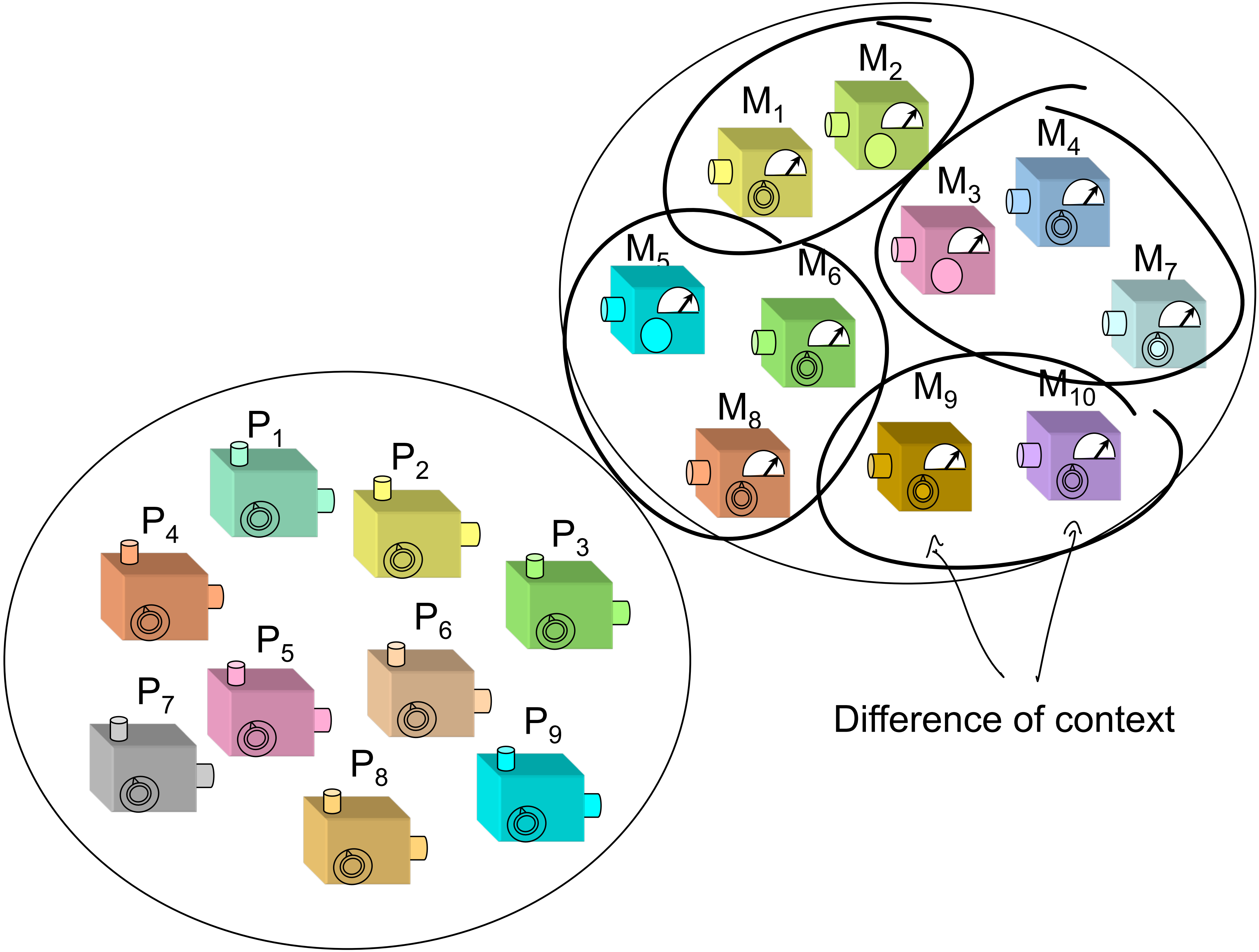
P9
Operational equivalence classes of measurements
$\mathsf{M} \simeq \mathsf{M}'$
$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$



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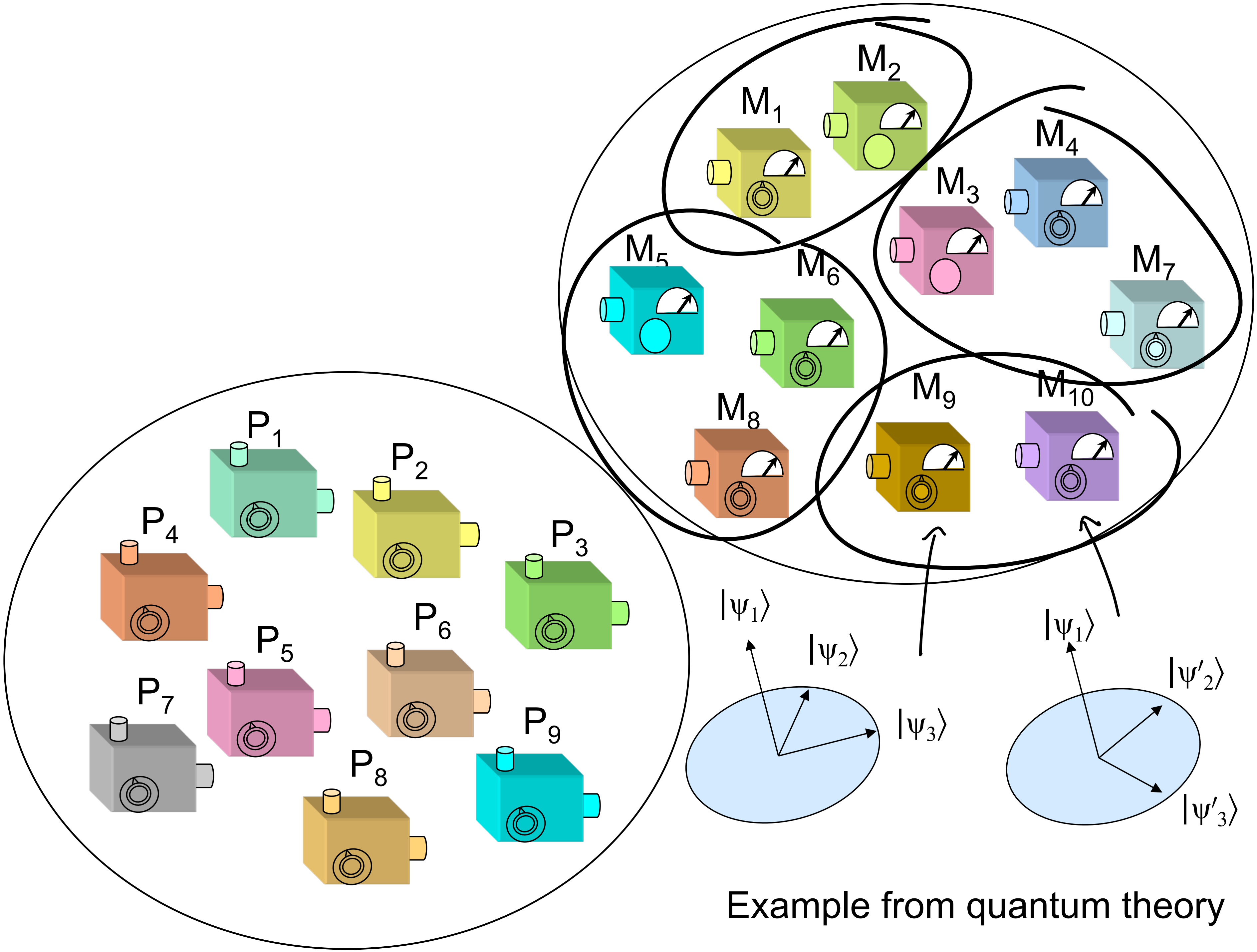
Difference of context



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|ψ1⟩
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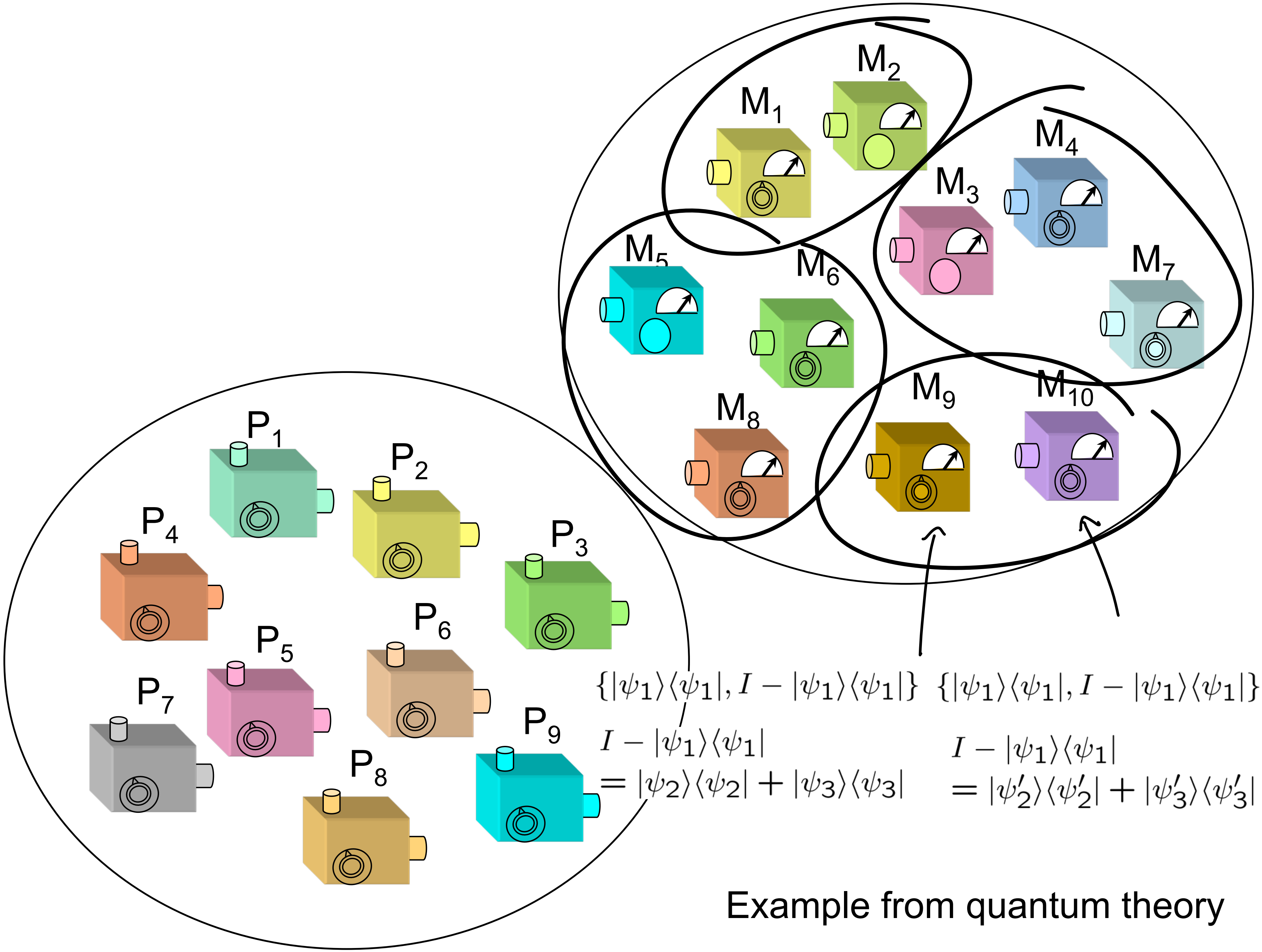
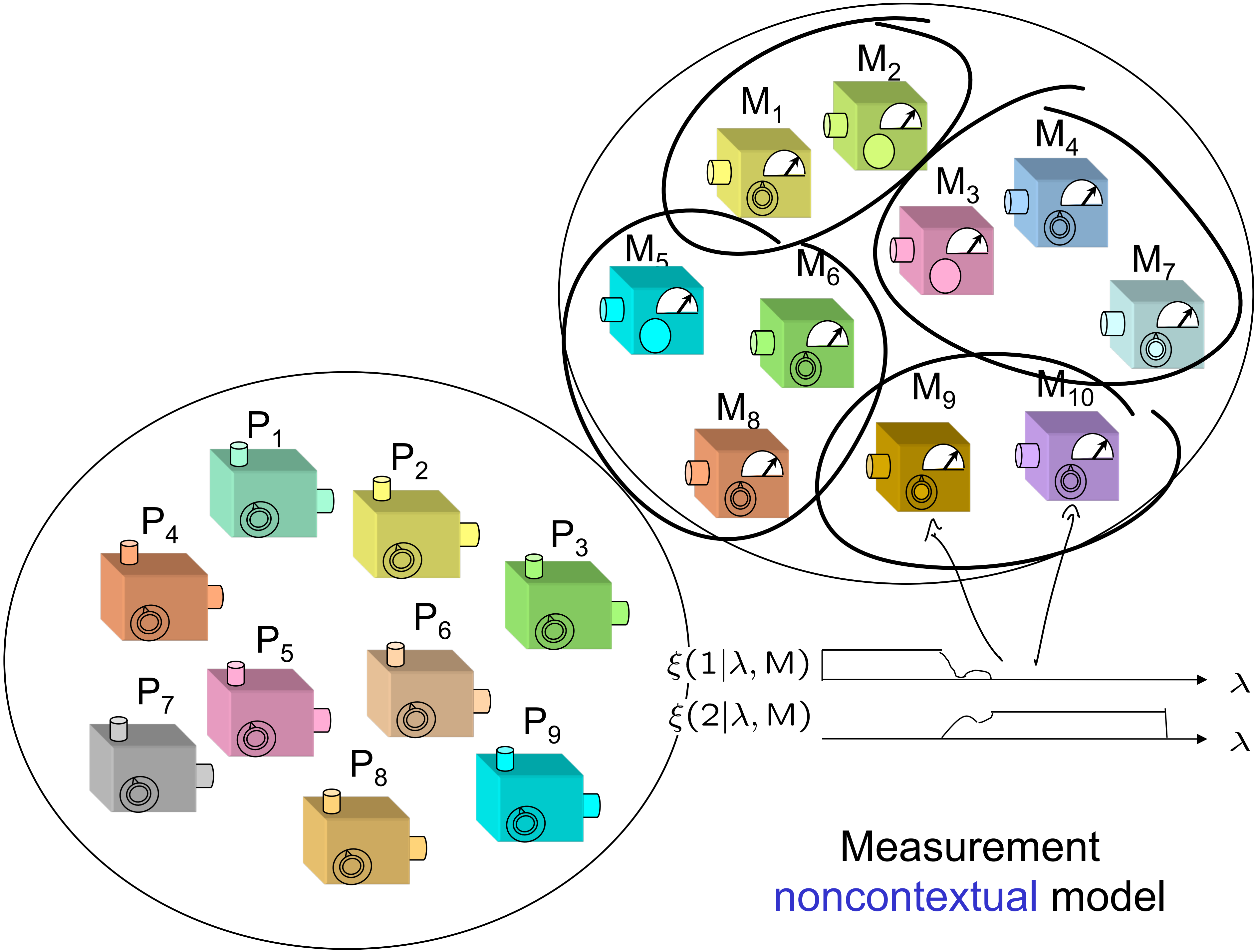
|ψ1⟩
|ψ′2⟩
|ψ′3⟩
Example from quantum theory



Example from quantum theory



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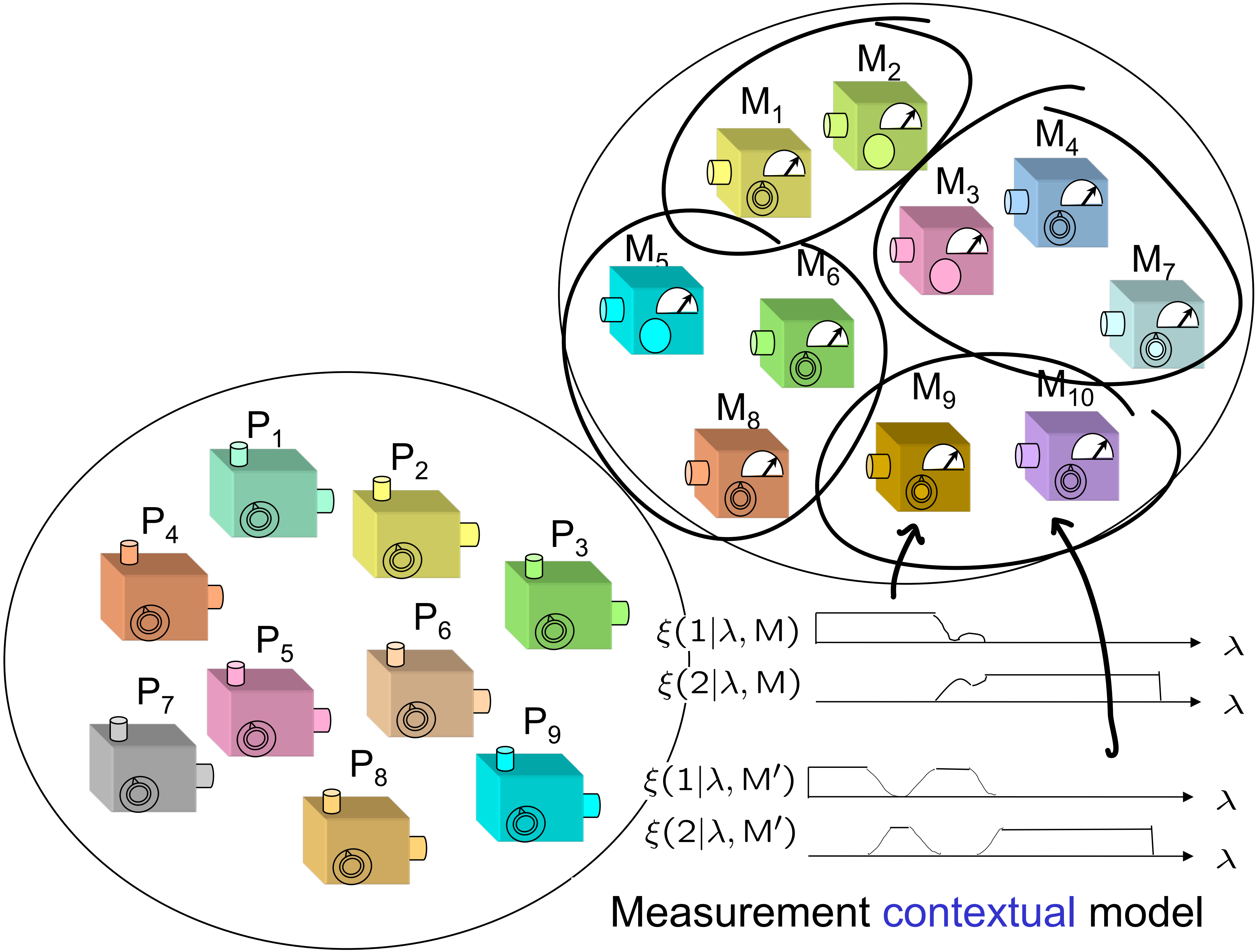
P9
ξ(1|λ, M)
λ
ξ(2|λ, M)
λ
Measurement
noncontextual model



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P9
ξ(1|λ, M)
λ
ξ(2|λ, M)
λ
ξ(1|λ, M′)
λ
ξ(2|λ, M′)
λ
Measurement contextual model

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality

$$\longrightarrow$$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality

$$\longrightarrow$$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

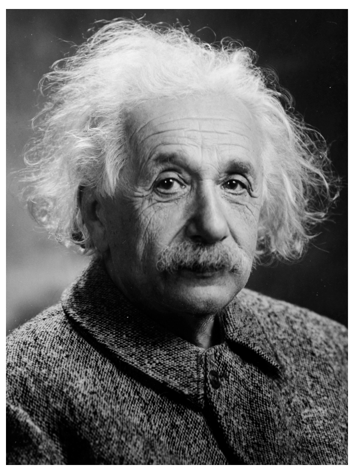
The best explanation of context-independence at the operational level is context-independence at the ontological level

Noncontextuality is a special case of

Leibniz's principle of the identity of indiscernibles

The principle's credentials:

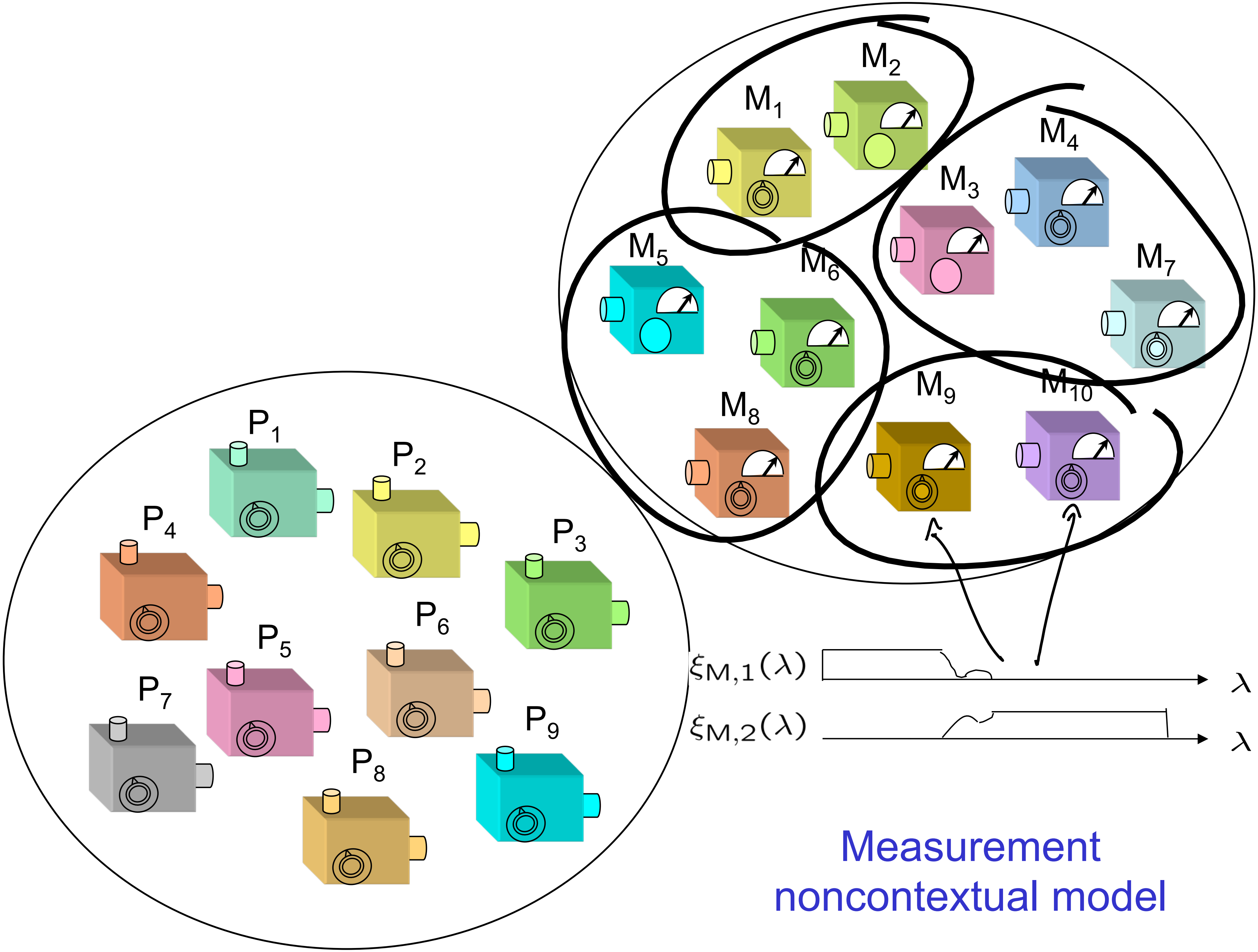
- Einstein's evidence against a preferred rest frame for electrodynamics
- Einstein's strong equivalence principle
- Einstein's hole argument
- Motivation for Bell's notion of local causality
- No fine-tuning in causal inference

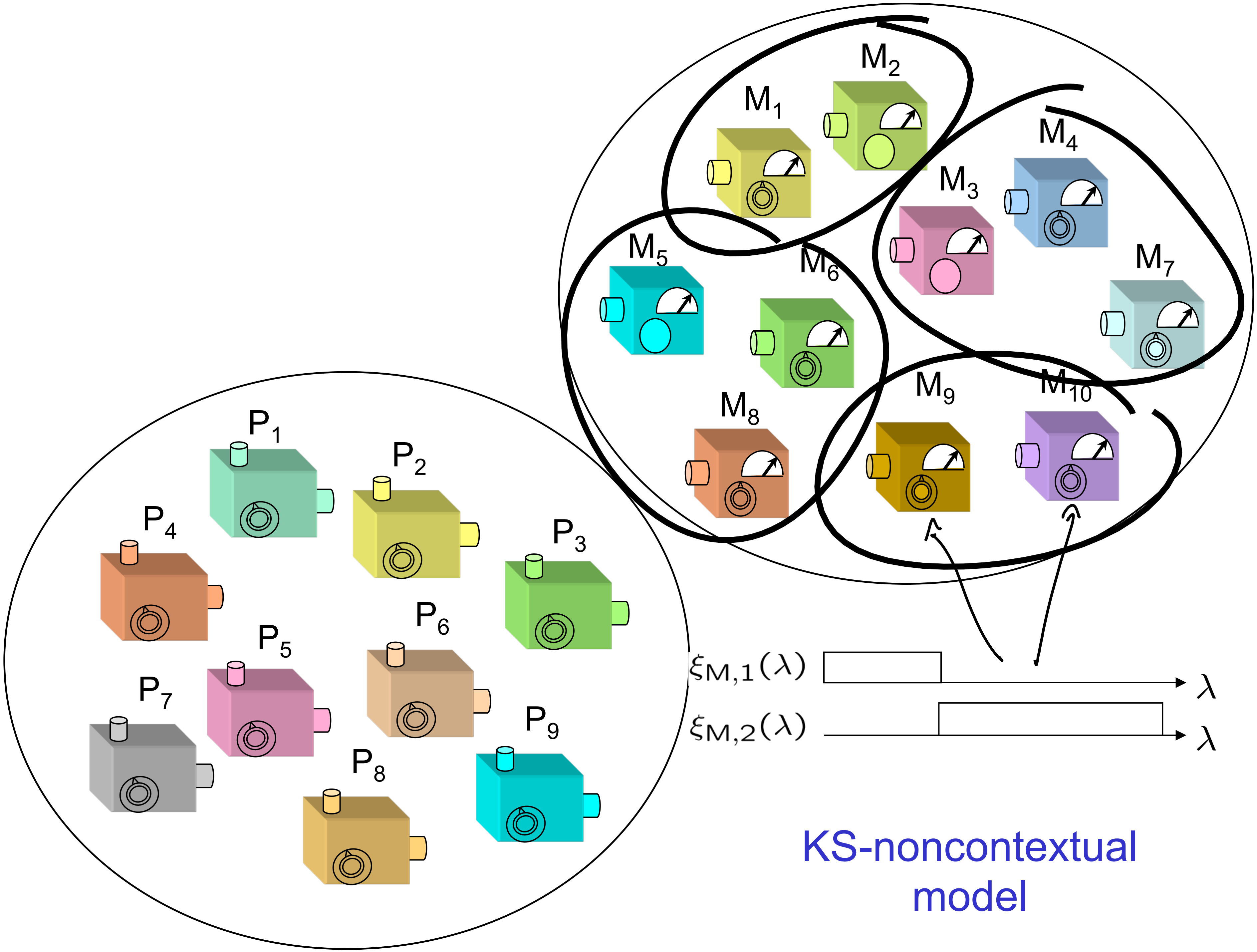
From this perspective, the only natural assumption is universal noncontextuality

Obstacles to a direct experimental test of universal noncontextuality

Obstacle #1: How to contend with noisy measurements?



Measurement noncontextual model



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P_8
P_9
$\xi_{M,1}(\lambda)$
λ
$\xi_{M,2}(\lambda)$
λ
KS-noncontextual model

measurement noncontextuality

and = KS-noncontextuality

outcome determinism

But, in face of violation of inequality for KS-noncontextuality,
we could give up outcome determinism

There is no analogue of Fine's theorem for noncontextual models

That is

Noisy measurements must be assigned outcome indeterministically by the ontic state,

See: RWS, The status of determinism in proofs of the impossibility of a noncontextual model of quantum theory, Found. Phys. 44, 1125 (2014)

Inequalities for KS-noncontextuality presume outcome determinism, therefore not applicable to noisy measurements

Measurement noncontextuality is applicable to noisy measurements. How to derive inequalities that make nontrivial use of measurement noncontextuality?

Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noisy measurements?

Obstacle #2: How to contend with inexactness of operational equivalences?

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality

$$\longrightarrow$$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality

$$\longrightarrow$$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noisy measurements?

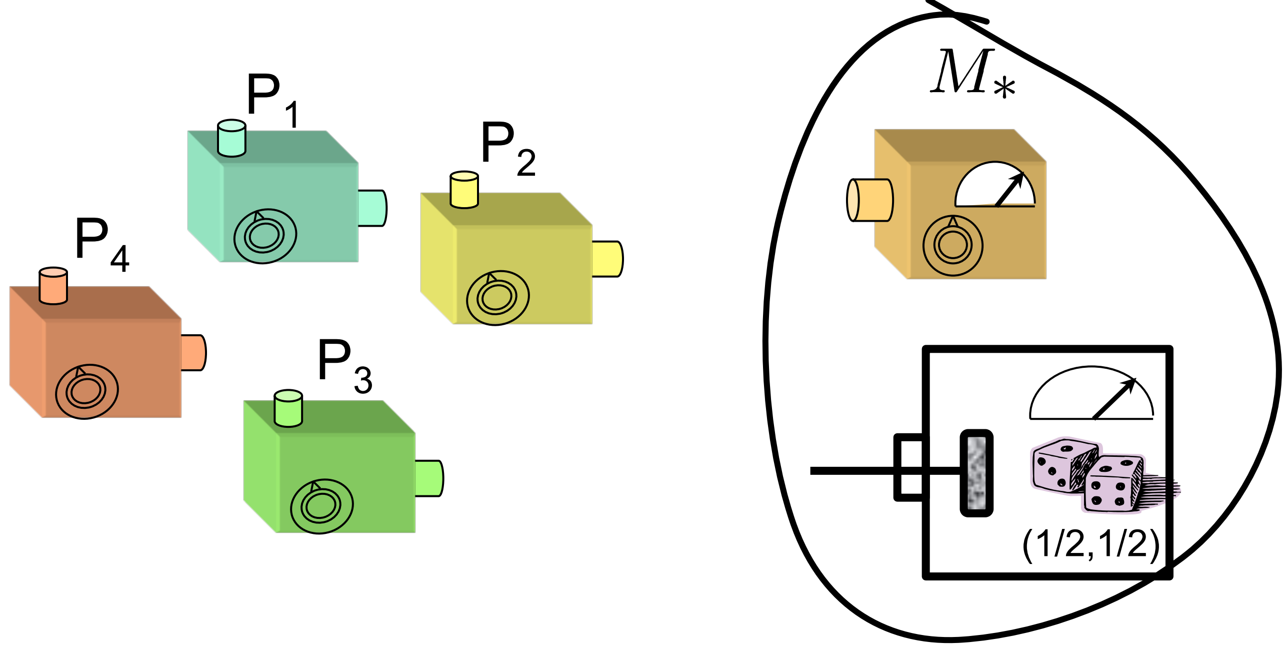
Obstacle #2: How to contend with inexactness of operational equivalences?

An experimental test of noncontextuality without unphysical idealizations

Joint work with:
Ravi Kunjwal, Matt Pusey (theory)
Mike Mazurek, Kevin Resch (experiment)
Nat. Commun. 7, 11780 (2016)

Deriving a noncontextuality inequality that is robust to experimental noise

See also:
R. Kunjwal and RWS, PRL 115, 110403 (2015)
And my talk at the conference

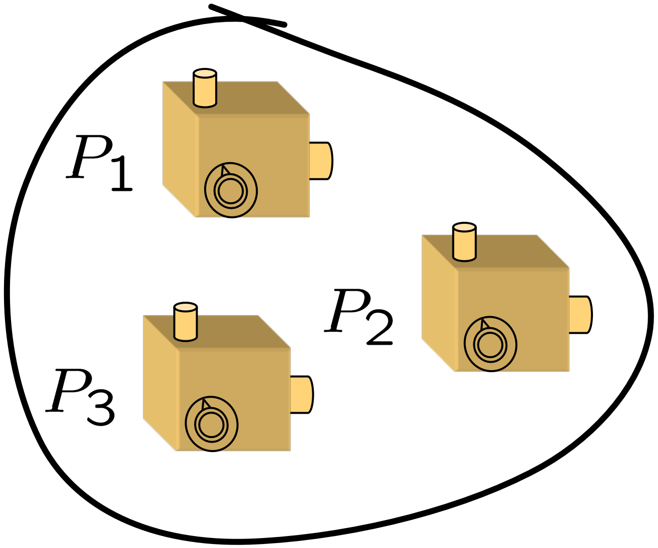


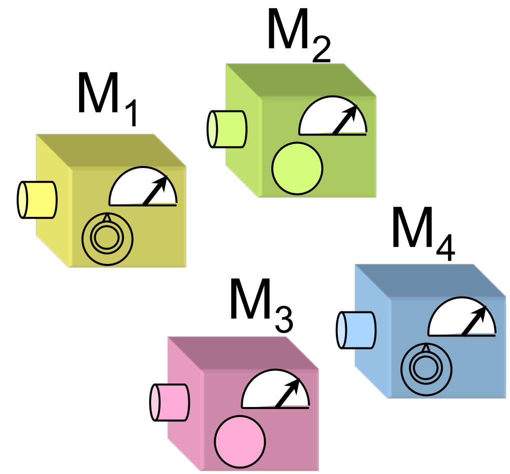
$M_* \simeq$ coin flip

$$p(X=0,1|M_*,P)=\frac{1}{2},\ \forall P \in \mathcal{P}.$$

Measurement noncontextuality

$$\boxed{\xi(X=0,1|M_*,\lambda)=\frac{1}{2},\ \forall \lambda \in \Lambda}$$





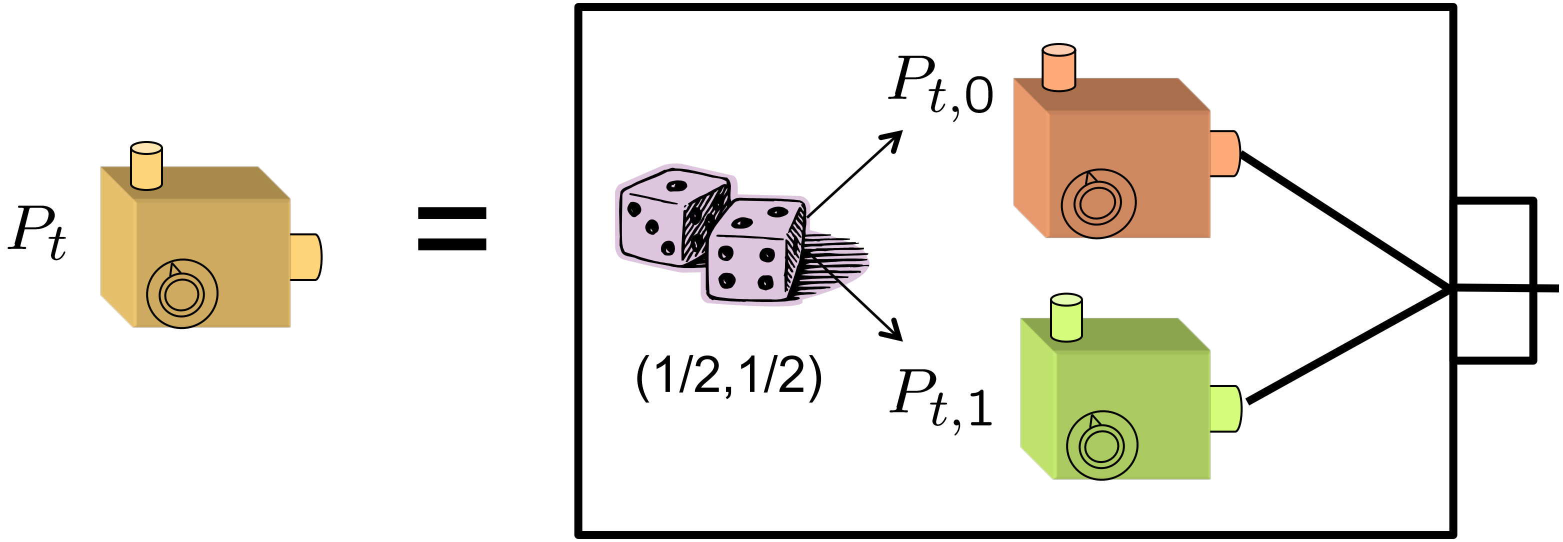
$$P_1 \simeq P_2 \simeq P_3$$

$$p(X|M,P_1) = p(X|M,P_2) = p(X|M,P_3) \;\; \forall M \in \mathcal{M}$$

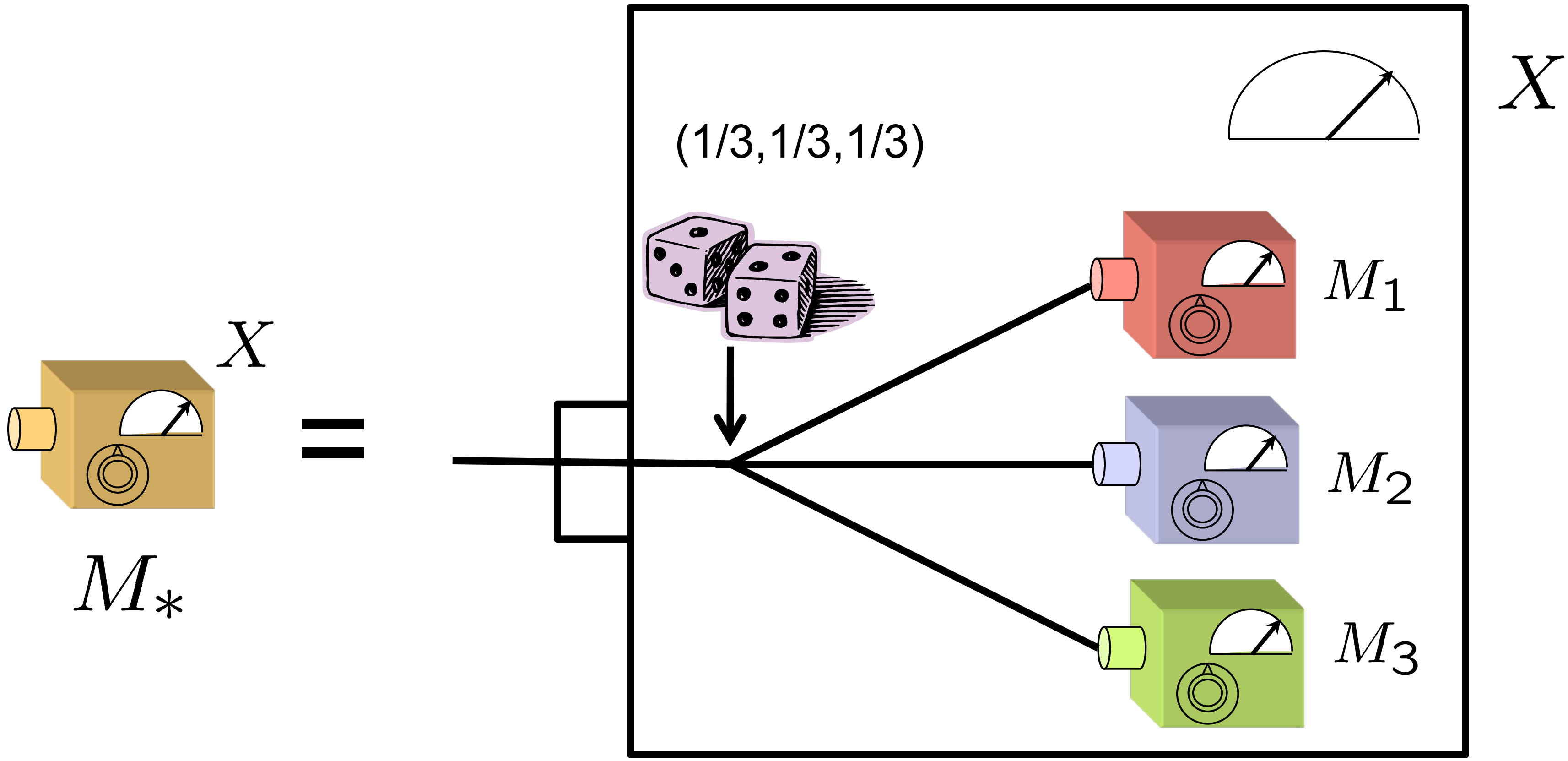
$$\Big\downarrow \text{ Preparation noncontextuality}$$

$$\boxed{\mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3) \;\; \forall \lambda \in \Lambda}$$

$$t \in \{1,2,3\}$$

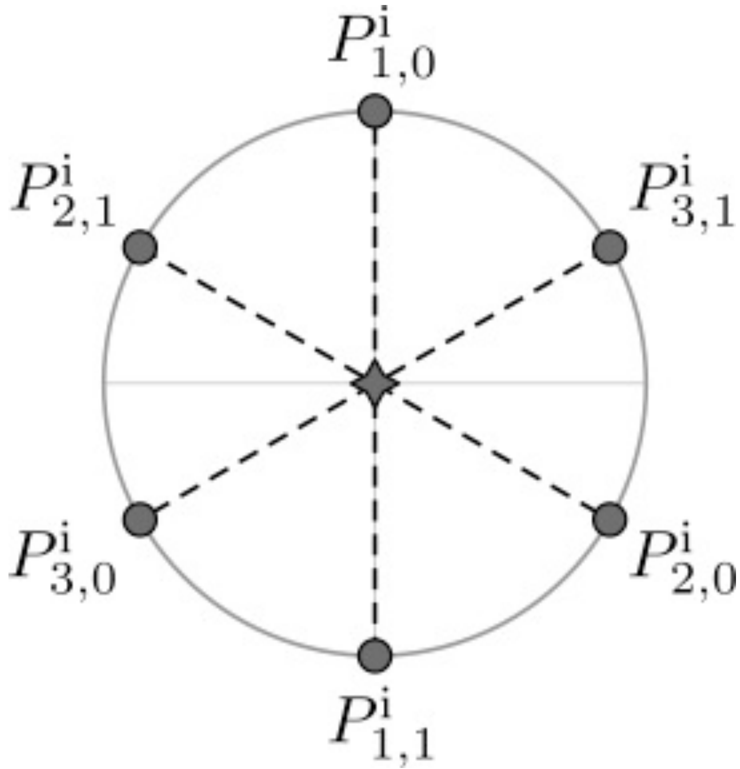


$$\mu(\lambda|P_t) = \frac{1}{2}\mu(\lambda|P_{t,0}) + \frac{1}{2}\mu(\lambda|P_{t,1})$$

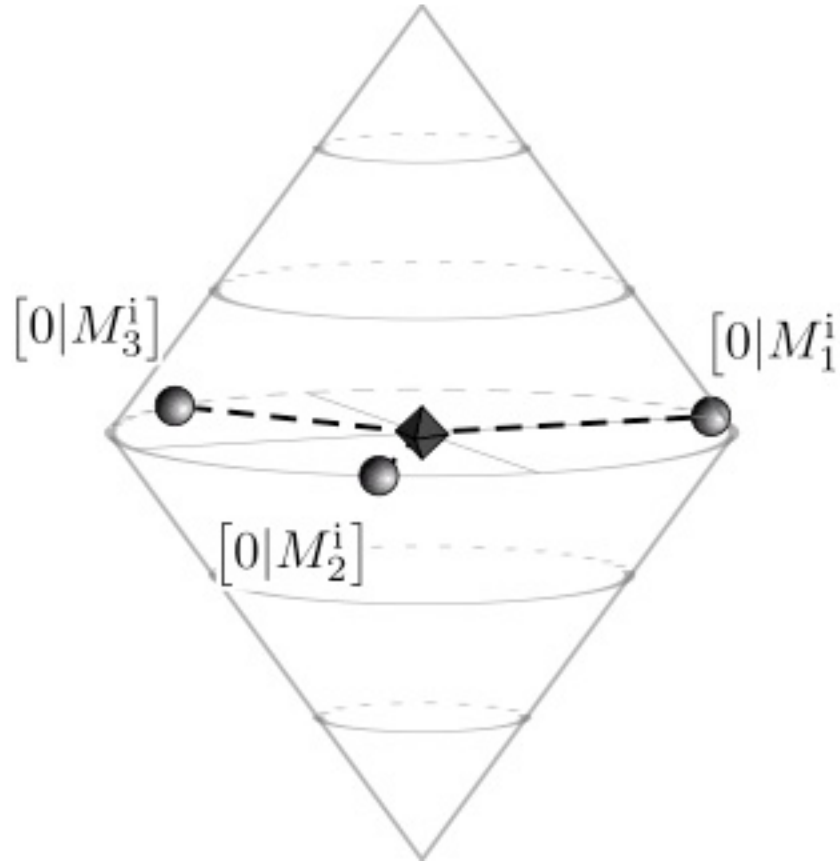


$$\xi(X = b|M_*, \lambda) = \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b|M_t, \lambda)$$

Quantum example



$P^{\mathsf{i}}_1 \simeq P^{\mathsf{i}}_2 \simeq P^{\mathsf{i}}_3$



$M^{\mathsf{i}}_* \simeq$ coin flip

Def'n of average degree of correlation

$$A \equiv \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} p(X = b|M_t, P_{t,b})$$

Theorem: For any $P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1}$

M_1, M_2, M_3

If $P_1 \simeq P_2 \simeq P_3$

$M_* \simeq \text{coin flip}$

Then universal noncontextuality implies

$$A \leq \frac{5}{6}$$

A noncontextuality Inequality

Def'n of average degree of correlation

$$A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b})$$

Lemma: For any $P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1}$
M_1, M_2, M_3

If $P_1 \simeq P_2 \simeq P_3$

$M_* \simeq$ coin flip

Then universal noncontextuality implies

$$A < 1$$

Suppose $A = 1$

Recall $A \equiv \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} p(X = b|M_t, P_{t,b})$

Recall $p(X = b|M_t, P_{t,b}) = \sum_\lambda \xi(X = b|M_t, \lambda)\mu(\lambda|P_{t,b})$

$$A = \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} \sum_\lambda \xi(X = b|M_t, \lambda)\mu(\lambda|P_{t,b})$$

$$A = 1 \longrightarrow \quad \sum_\lambda \xi(X = b|M_t, \lambda)\mu(\lambda|P_{t,b}) = 1$$

$$\longrightarrow \quad \forall\lambda \in \mathsf{supp}(\mu(\cdot|P_{t,b})),\ \ \xi(X = b|M_t, \lambda) = 1$$

$$\xi(X = b \oplus 1|M_t, \lambda) = 0$$

$$\forall\lambda \in \mathrm{supp}(\mu(\cdot|P_{t,b})),\ \xi(X=b|M_t,\lambda)=1$$
$$\xi(X=b\oplus 1|M_t,\lambda)=0$$

By definition of P_t

$$\mu(\lambda|P_t)=\frac{1}{2}\mu(\lambda|P_{t,0})+\frac{1}{2}\mu(\lambda|P_{t,1})$$

$$\longrightarrow \quad \forall\lambda \in \mathrm{supp}(\mu(\cdot|P_t)),\ \xi(X=b|M_t,\lambda)\in\{0,1\}$$

$$\boxed{\mu(\lambda|P_1)=\mu(\lambda|P_2)=\mu(\lambda|P_3)\ \ \forall\lambda\in\Lambda}$$

$$\longrightarrow \forall\lambda \in \mathrm{supp}(\mu(\cdot|P_1))=\mathrm{supp}(\mu(\cdot|P_2))=\mathrm{supp}(\mu(\cdot|P_3))$$
$$\xi(X=b|M_1,\lambda)\in\{0,1\}$$
$$\xi(X=b|M_2,\lambda)\in\{0,1\}$$
$$\xi(X=b|M_3,\lambda)\in\{0,1\}$$

$$\forall \lambda \in \mathsf{supp}(\mu(\cdot|P_1)) = \mathsf{supp}(\mu(\cdot|P_2)) = \mathsf{supp}(\mu(\cdot|P_3))$$

$$\xi(X = b|M_1, \lambda) \in \{0, 1\}$$
$$\xi(X = b|M_2, \lambda) \in \{0, 1\}$$
$$\xi(X = b|M_3, \lambda) \in \{0, 1\}$$

By definition of M*

$$\xi(X = b|M_*, \lambda) = \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b|M_t, \lambda)$$

$$\boxed{\xi(X = b|M_*, \lambda) = \frac{1}{2}, \ \forall \lambda \in \Lambda}$$

Contradiction!

Def'n of average degree of correlation

$$A \equiv \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} p(X=b|M_t, P_{t,b})$$

Lemma: For any $P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1}$

M_1, M_2, M_3

If $P_1 \simeq P_2 \simeq P_3$

$M_* \simeq$ coin flip

Then universal noncontextuality implies

$$A < 1$$

Def'n of average degree of correlation

$$A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b})$$

Theorem: For any $P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1}$
M_1, M_2, M_3

If $P_1 \simeq P_2 \simeq P_3$

$M_* \simeq$ coin flip

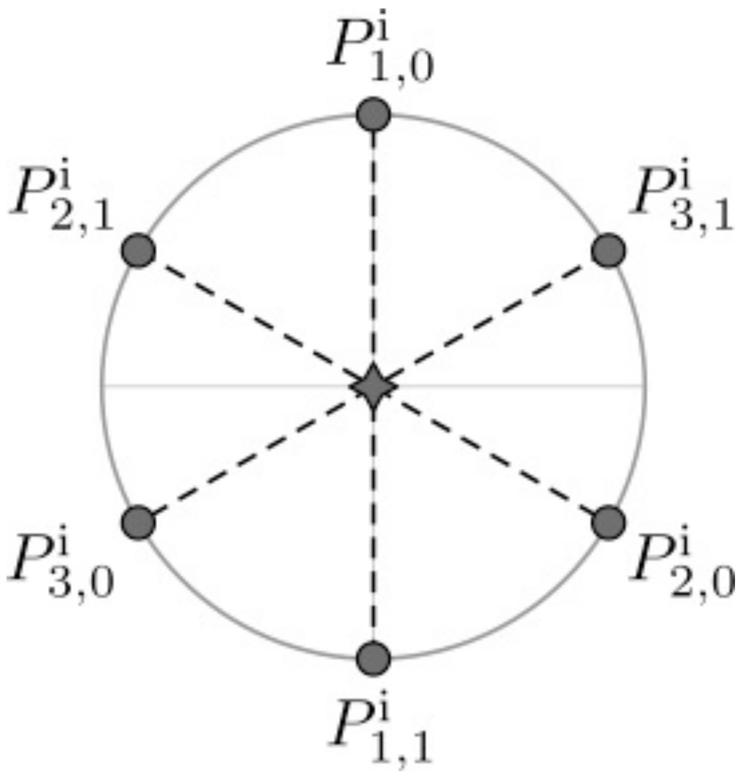
Then universal noncontextuality implies

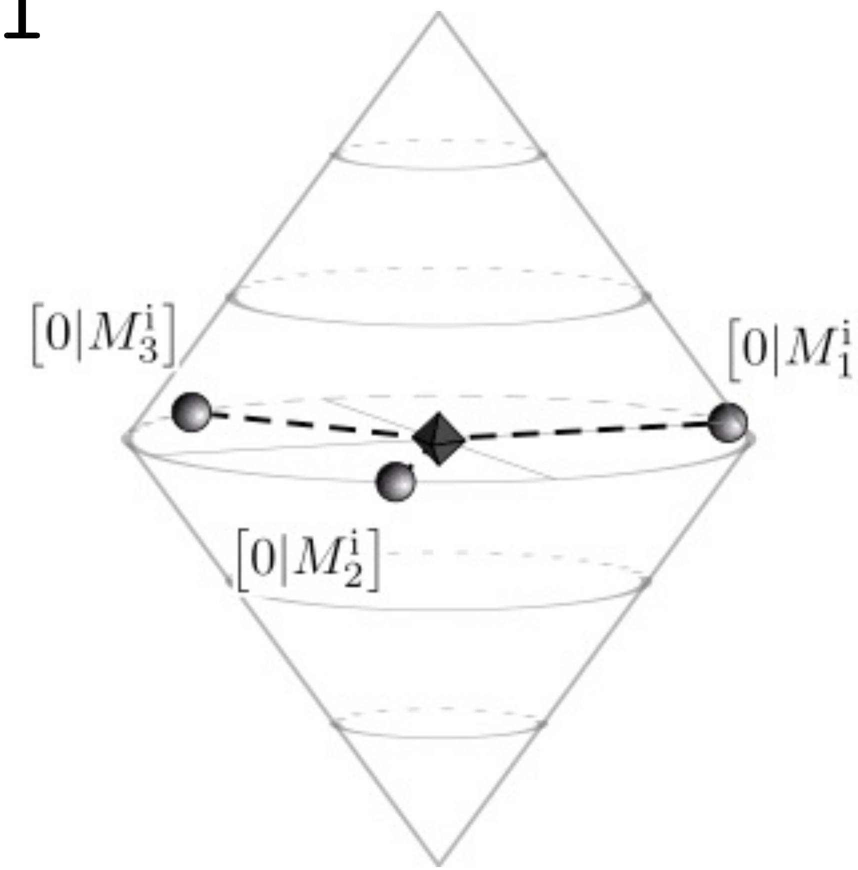
$$A \leq \frac{5}{6}$$

A noncontextuality Inequality

Quantum violation

$$A = 1$$





$P^{\mathsf{i}}_1 \simeq P^{\mathsf{i}}_2 \simeq P^{\mathsf{i}}_3$

$M^{\mathsf{i}}_* \simeq$ coin flip

Robust to noise

Solving the problem of inexactness of operational equivalences

See also:
M. Pusey, arXiv:1506.04178

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality

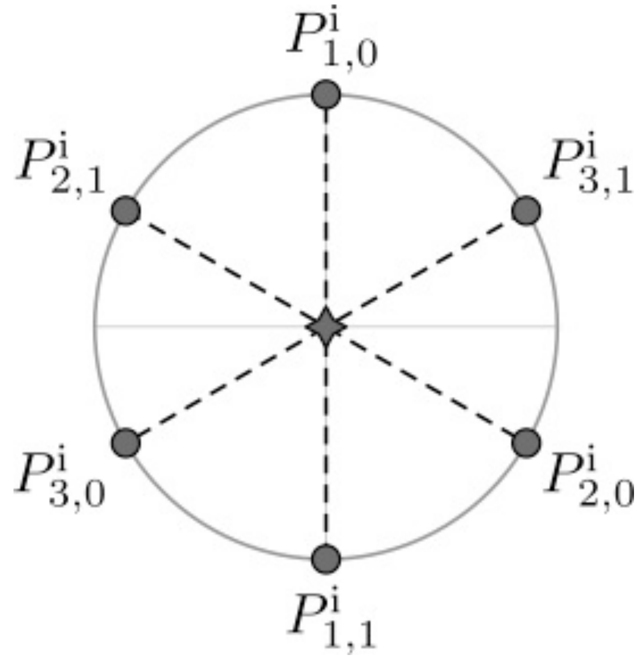
$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

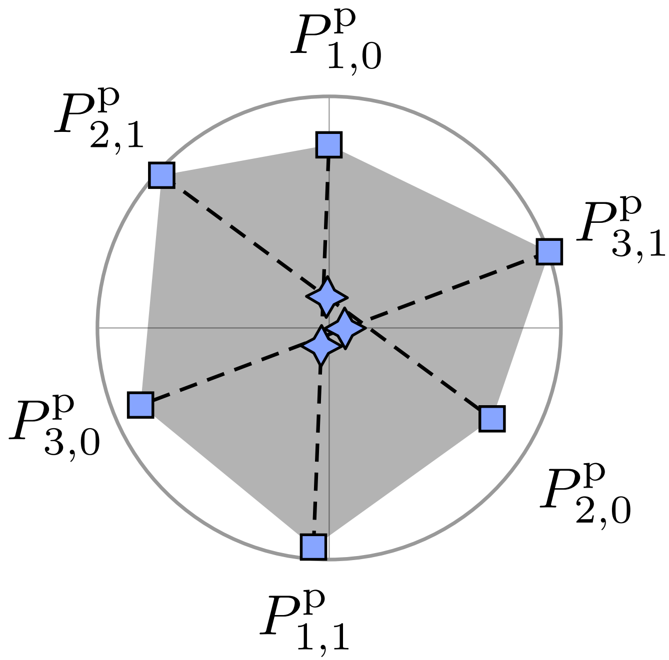
$$\mathsf{M} \simeq \mathsf{M}'$$

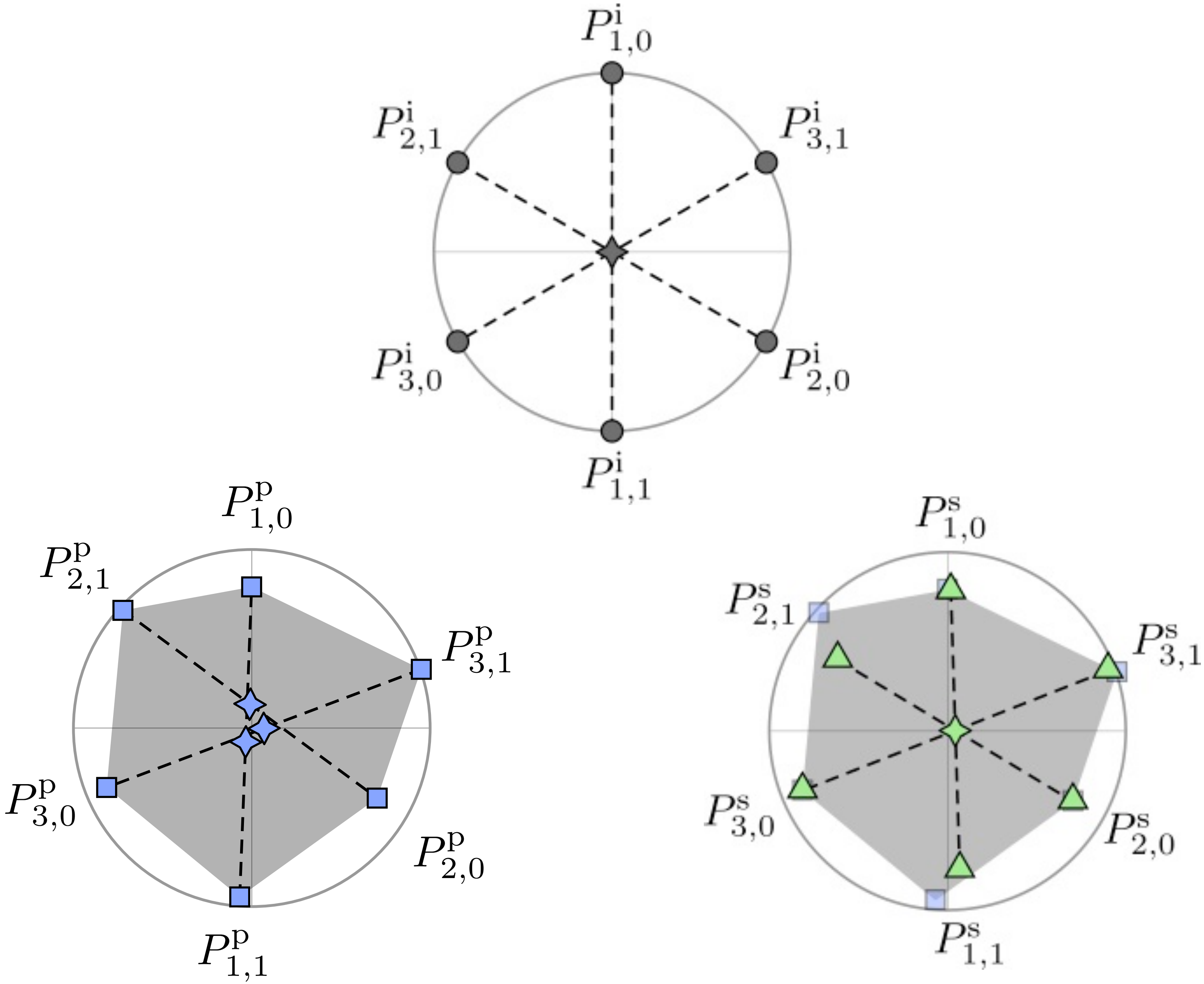
$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$


$P^{\mathrm{i}}_{1,0}$
$P^{\mathrm{i}}_{2,1}$
$P^{\mathrm{i}}_{3,1}$
$P^{\mathrm{i}}_{3,0}$
$P^{\mathrm{i}}_{2,0}$
$P^{\mathrm{i}}_{1,1}$


$P^{\mathrm{p}}_{1,0}$
$P^{\mathrm{p}}_{2,1}$
$P^{\mathrm{p}}_{3,1}$
$P^{\mathrm{p}}_{3,0}$
$P^{\mathrm{p}}_{2,0}$
$P^{\mathrm{p}}_{1,1}$



$P^{\mathrm{i}}_{1,0}$
$P^{\mathrm{i}}_{2,1}$
$P^{\mathrm{i}}_{3,1}$
$P^{\mathrm{i}}_{3,0}$
$P^{\mathrm{i}}_{2,0}$
$P^{\mathrm{i}}_{1,1}$
$P^{\mathrm{p}}_{1,0}$
$P^{\mathrm{p}}_{2,1}$
$P^{\mathrm{p}}_{3,1}$
$P^{\mathrm{p}}_{3,0}$
$P^{\mathrm{p}}_{2,0}$
$P^{\mathrm{p}}_{1,1}$
$P^{\mathrm{s}}_{1,0}$
$P^{\mathrm{s}}_{2,1}$
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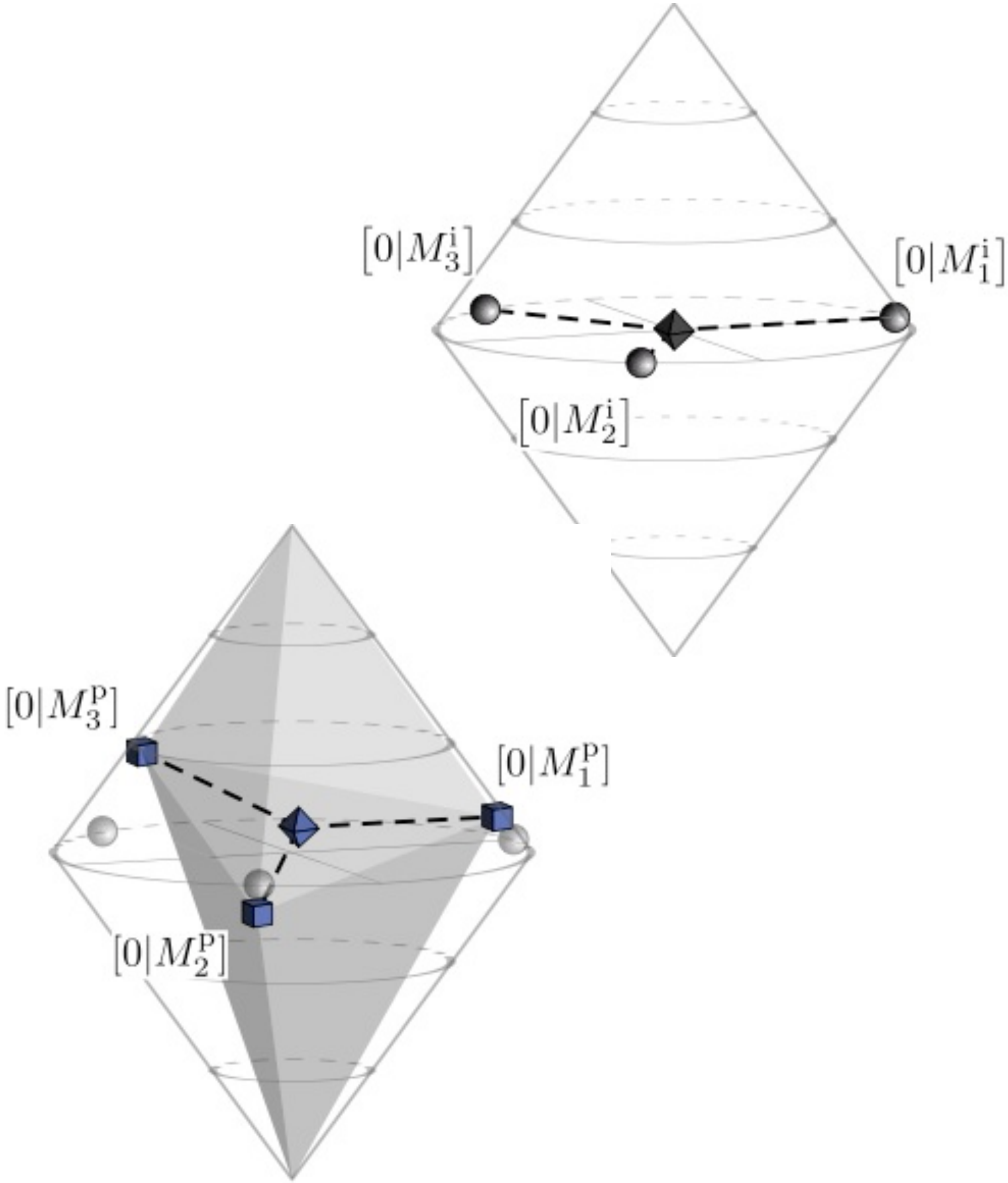
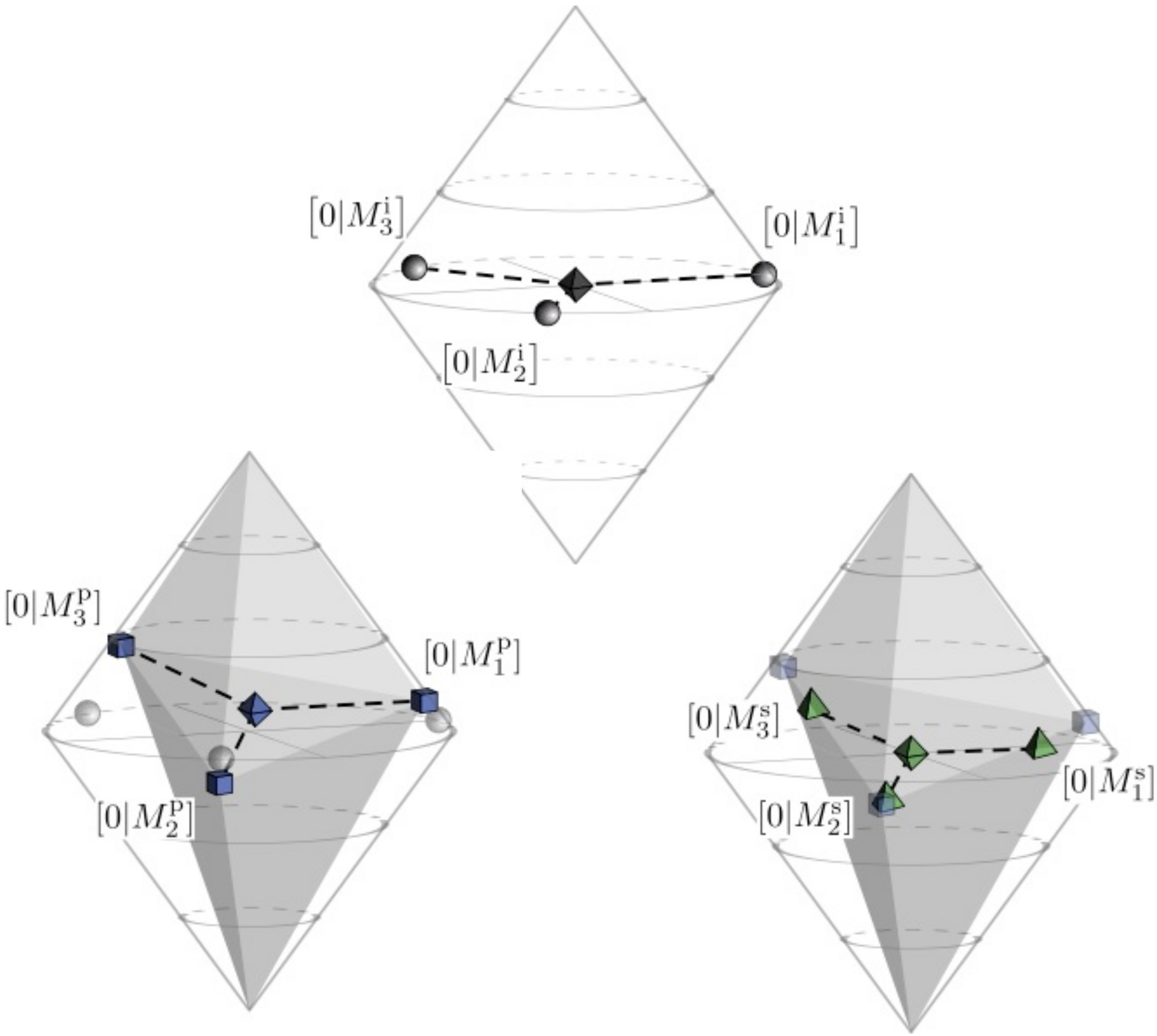
$P^{\mathrm{s}}_{3,0}$
$P^{\mathrm{s}}_{2,0}$
$P^{\mathrm{s}}_{1,1}$


$[0|M_3^{\mathrm{i}}]$
$[0|M_1^{\mathrm{i}}]$
$[0|M_2^{\mathrm{i}}]$
$[0|M_3^{\mathrm{p}}]$
$[0|M_1^{\mathrm{p}}]$
$[0|M_2^{\mathrm{p}}]$


$[0|M_3^{\mathrm{i}}]$
$[0|M_1^{\mathrm{i}}]$
$[0|M_2^{\mathrm{i}}]$
$[0|M_3^{\mathrm{p}}]$
$[0|M_1^{\mathrm{p}}]$
$[0|M_2^{\mathrm{p}}]$
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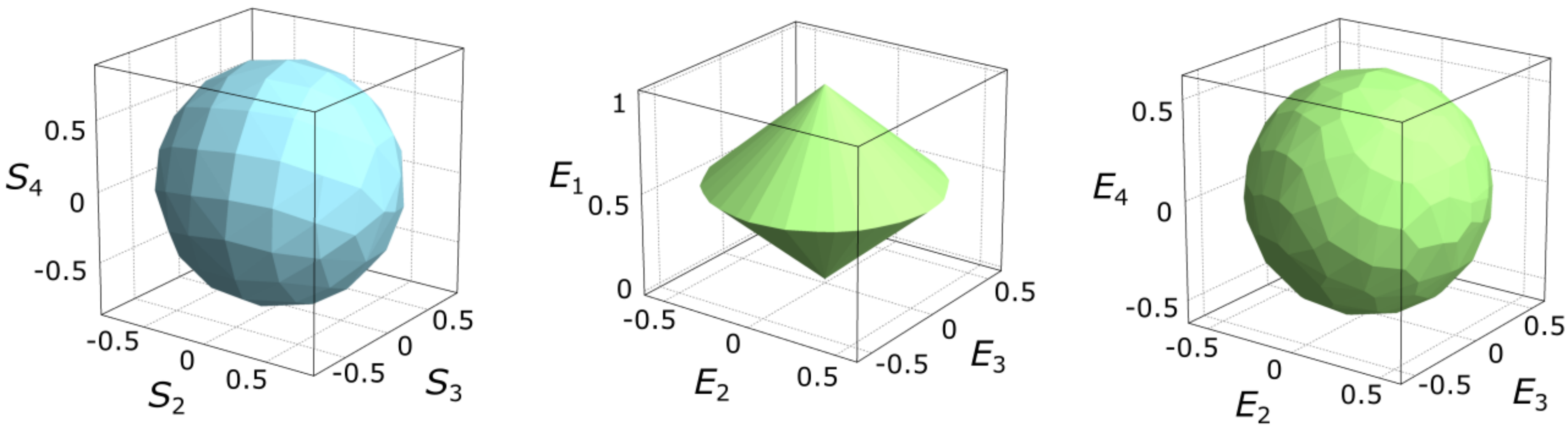
$[0|M_1^{\mathrm{s}}]$
$[0|M_2^{\mathrm{s}}]$



S_4
0.5
0
-0.5
-0.5
0
0.5
S_2
-0.5
0
0.5
S_3
E_1
1
0.5
0
-0.5
0
0.5
E_2
-0.5
0
0.5
E_3
E_4
0.5
0
-0.5
-0.5
0
0.5
E_2
-0.5
0
0.5
E_3

Obstacle #3:
How to verify that a given set of measurements or preparations is tomographically complete?

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\Downarrow$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality

$$\longrightarrow \quad \mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

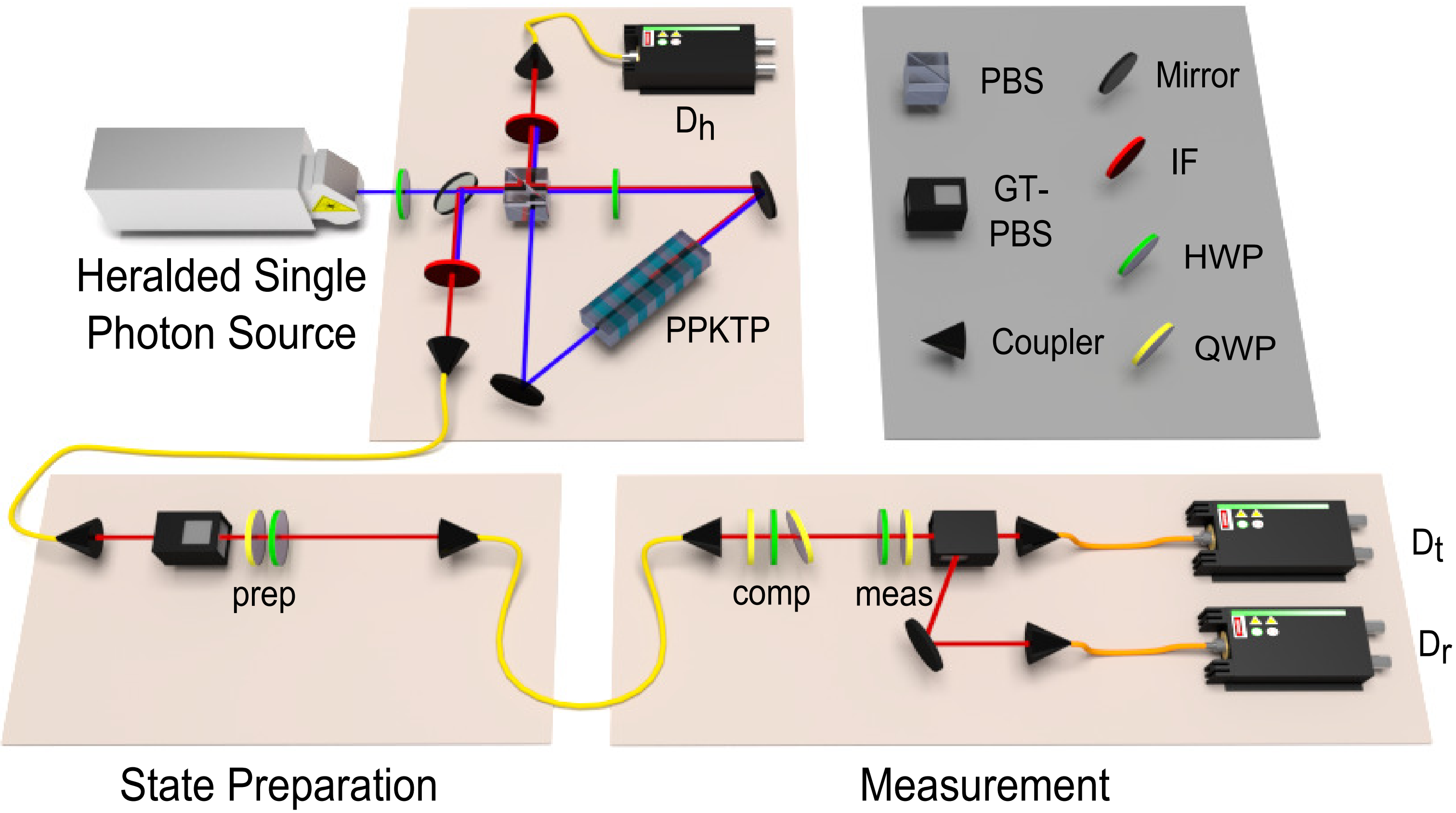
$$\Downarrow$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

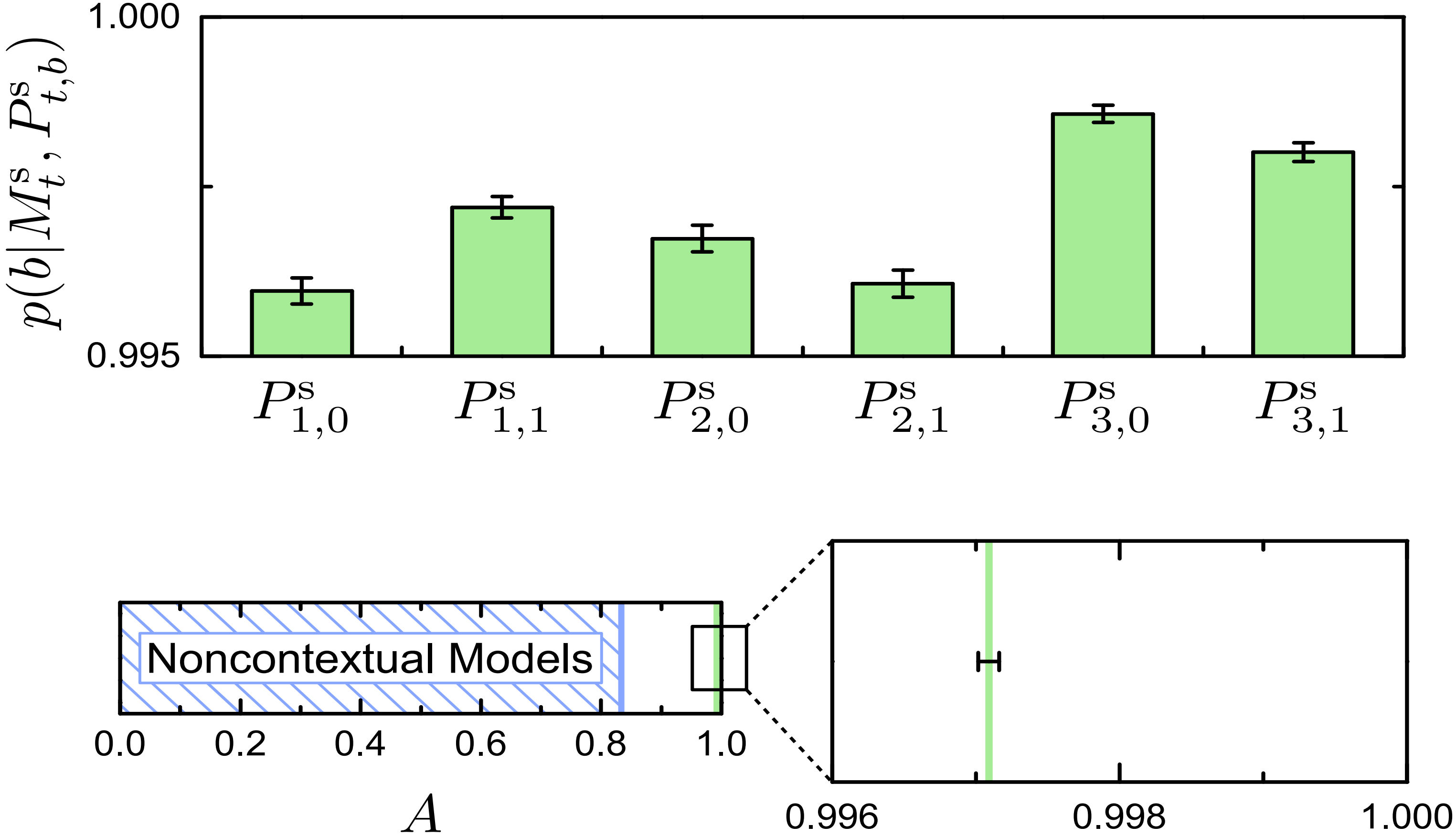
Measurement noncontextuality

$$\longrightarrow \quad \xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

One can accumulate evidence for tomographic completeness
Nonetheless, this is the frontier for expt'l tests of noncontextuality

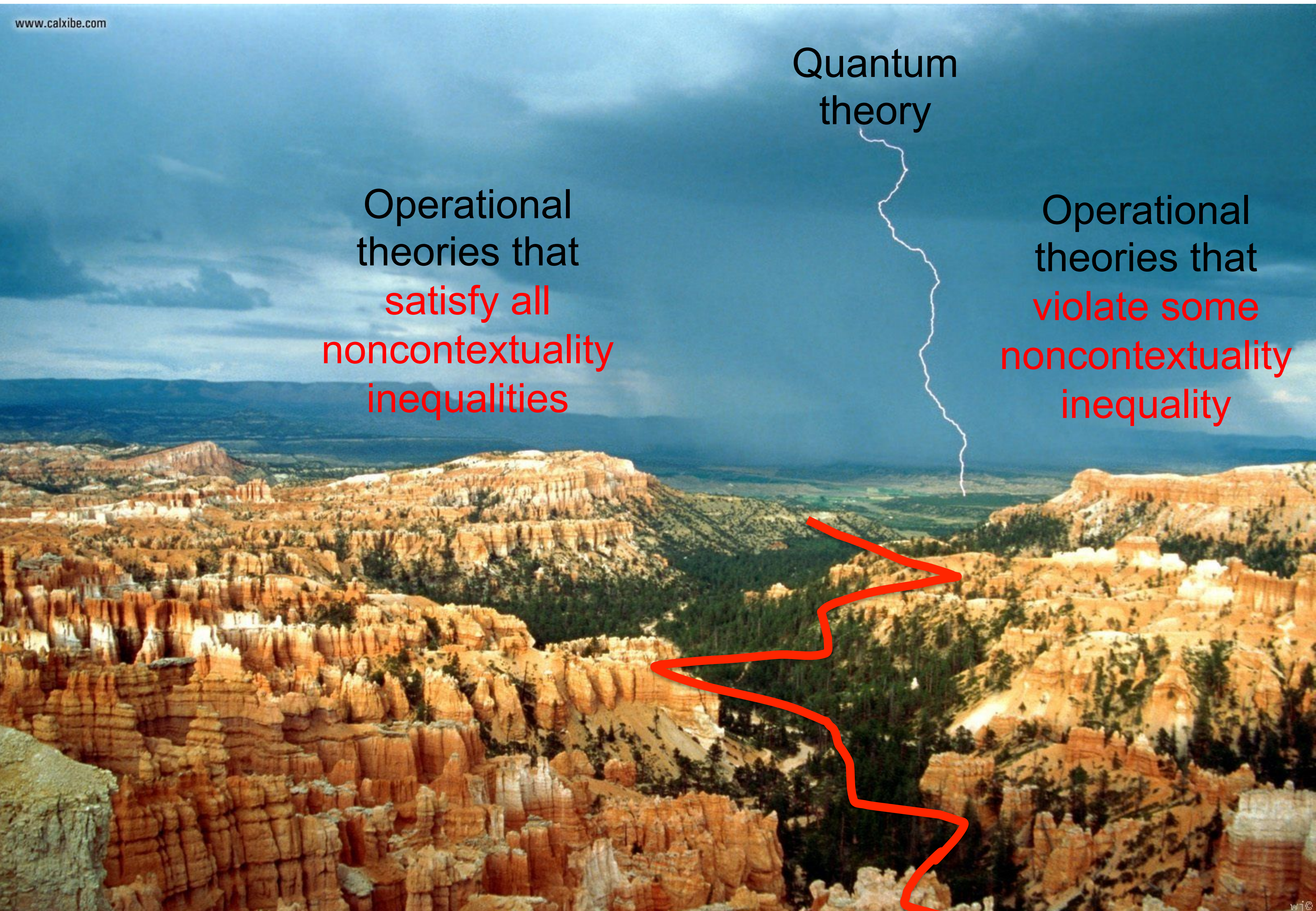


Heralded Single Photon Source
Dh
PPKTP
PBS
Mirror
IF
GT-PBS
HWP
Coupler
QWP
prep
comp
meas
Dt
Dr
State Preparation
Measurement



A = 0.99709 $\pm$ 0.00007

violating the noncontextual bound by 2300 σ


www.calxibe.com
Quantum theory
Operational theories that satisfy all noncontextuality inequalities
Operational theories that violate some noncontextuality inequality

Morals of the story

One can implement tests of universal noncontextuality without unphysical idealizations such as noiseless measurements and exact operational equivalences among the implemented procedures

The secondary procedures trick allows one to overcome the problem of inexact operational equivalences

It also affords a simplification because it allows one to enforce symmetries in the data table

One can accumulate evidence for the tomographic completeness of a set of measurements, which is necessary for evaluating operational equivalences

What are the reasons for doing experiments in physics?

"You're not doing good physics unless you're proposing and performing experiments"

No!

Falsificationism

But I shall certainly admit a system as empirical or scientific only if it is capable of being tested by experience. […] it must be possible for an empirical scientific system to be refuted by experience.
— Karl Popper, 1959

A naïve version of falsificationism

The only criteria by which we need to judge scientific theories is the experimental evidence that exists for them

The problem with naïve falsificationism

--- Auxiliary hypotheses can save any theory from falsification

--- Experiment does not provide a theory-independent court of appeal: all observations are theory-laden

Eyes and ears are bad witnesses for men with barbarian souls

—Heraclitus

The best reasons for doing experiments

- Because we genuinely don’t know what we’ll find
- To help abjudicate between competing theories
- Identify phenomena that resist explanation in current theoretical paradigm

Other reasons for doing experiments

- The discipline required to do so will improve one’s understanding of the theory
- Push the envelope of our technological capabilities

My opinion: Experiments in quantum foundations today don’t teach us much