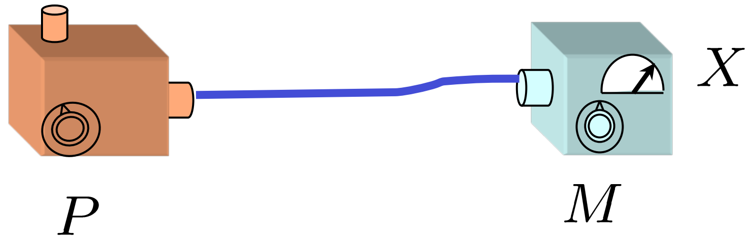
Translating proofs of the Kochen-Specker theorem into noise-robust noncontextuality inequalities

Robert Spekkens
Perimeter Institute for Theoretical Physics

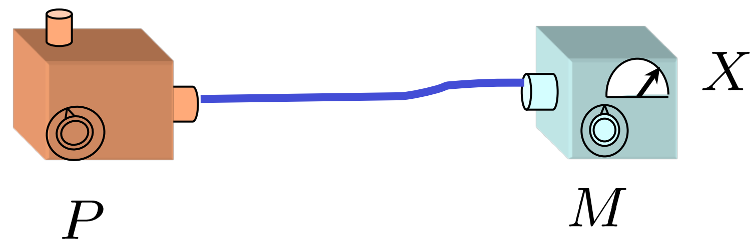
June 22, 2017
Zurich

Operational theory



$$p(X|M,P)$$

Operational theory



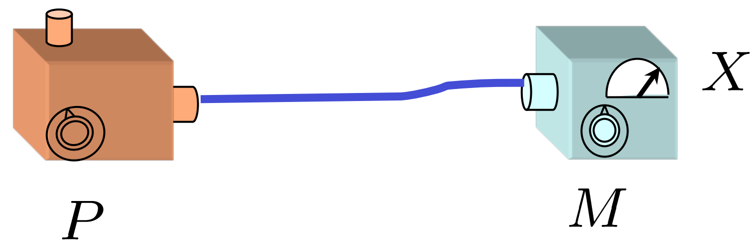
$$p(X|M,P)$$

Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates
between P and M

Operational theory



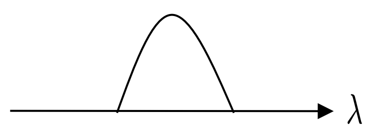
$$p(X|M,P)$$

Ontological model of an operational theory

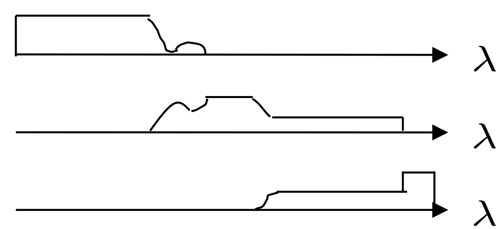
$\lambda \in \Lambda$ Ontic state space

causally mediates
between P and M

$\mathsf{P} \leftrightarrow \mu(\lambda|\mathsf{P})$



$\mathsf{M} \leftrightarrow \xi(X|\mathsf{M},\lambda)$



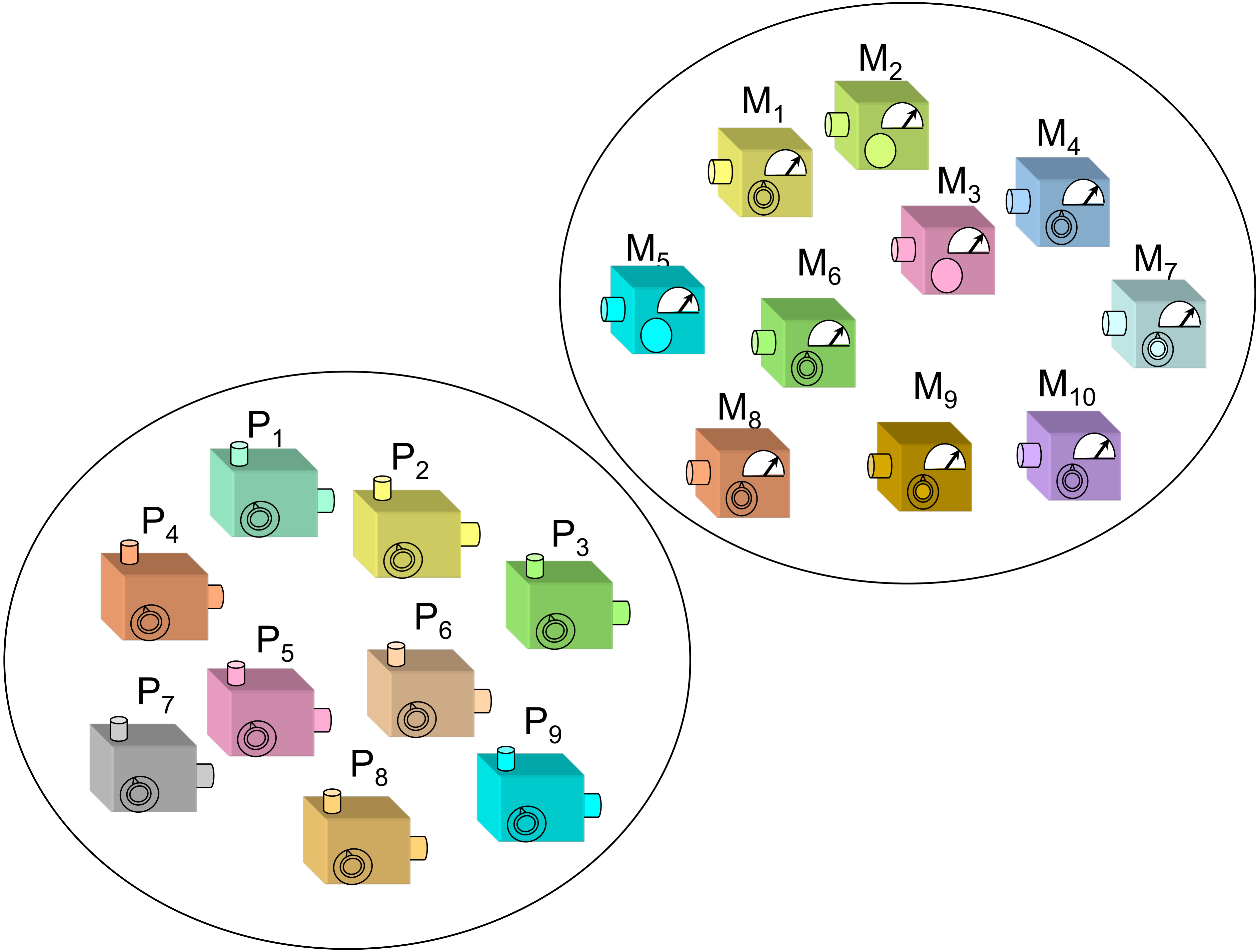
$$p(X|M,P) = \sum_\lambda \xi(X|M,\lambda)\,\mu(\lambda|P)$$

An ontological model of an operational theory is noncontextual if

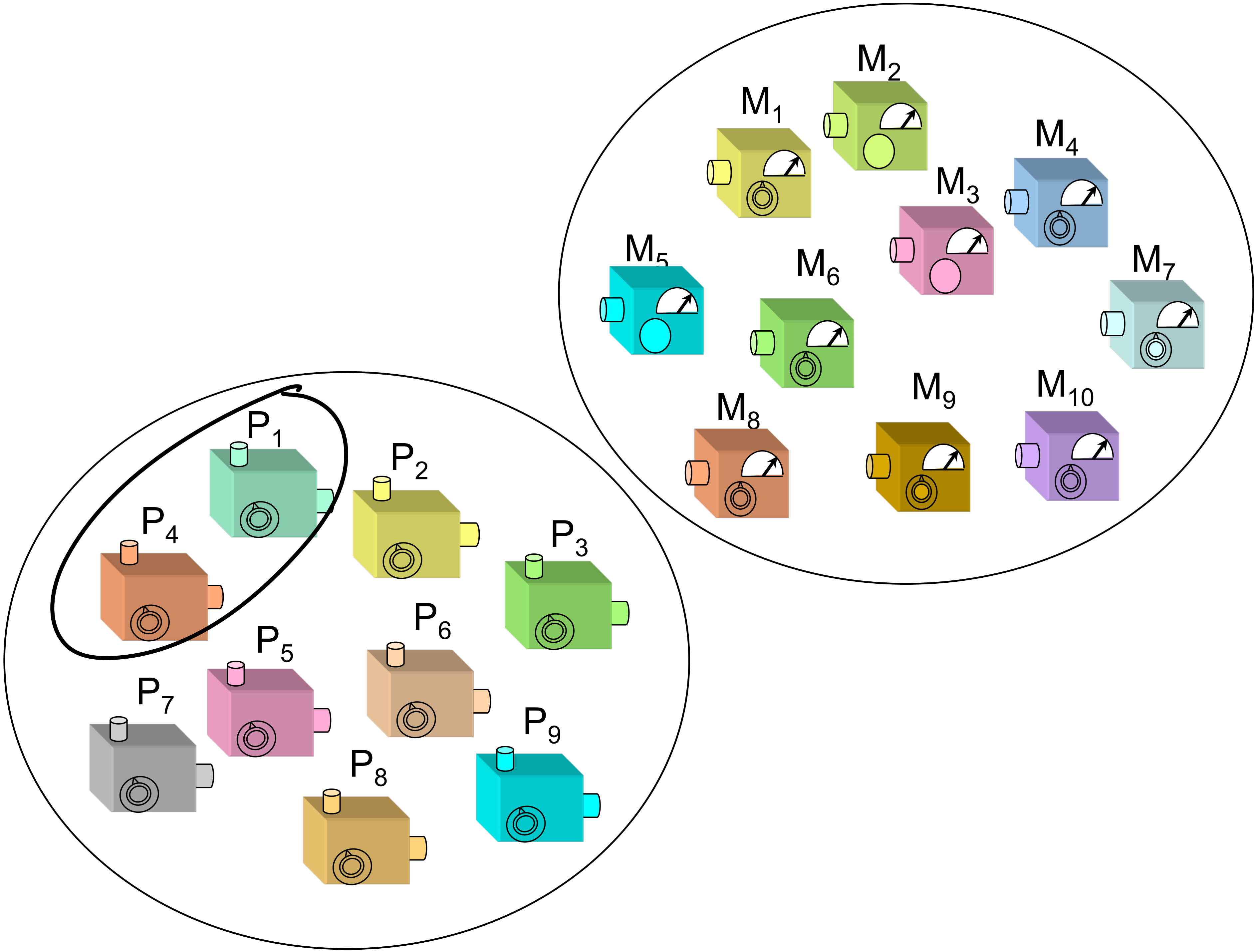
Operational equivalence of two experimental procedures $\longrightarrow$ Equivalent representations in the ontological model

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation Noncontextuality



M1
M2
M4
M3
M5
M6
M7
M8
M9
M10
P1
P2
P4
P3
P6
P5
P7
P9
P8

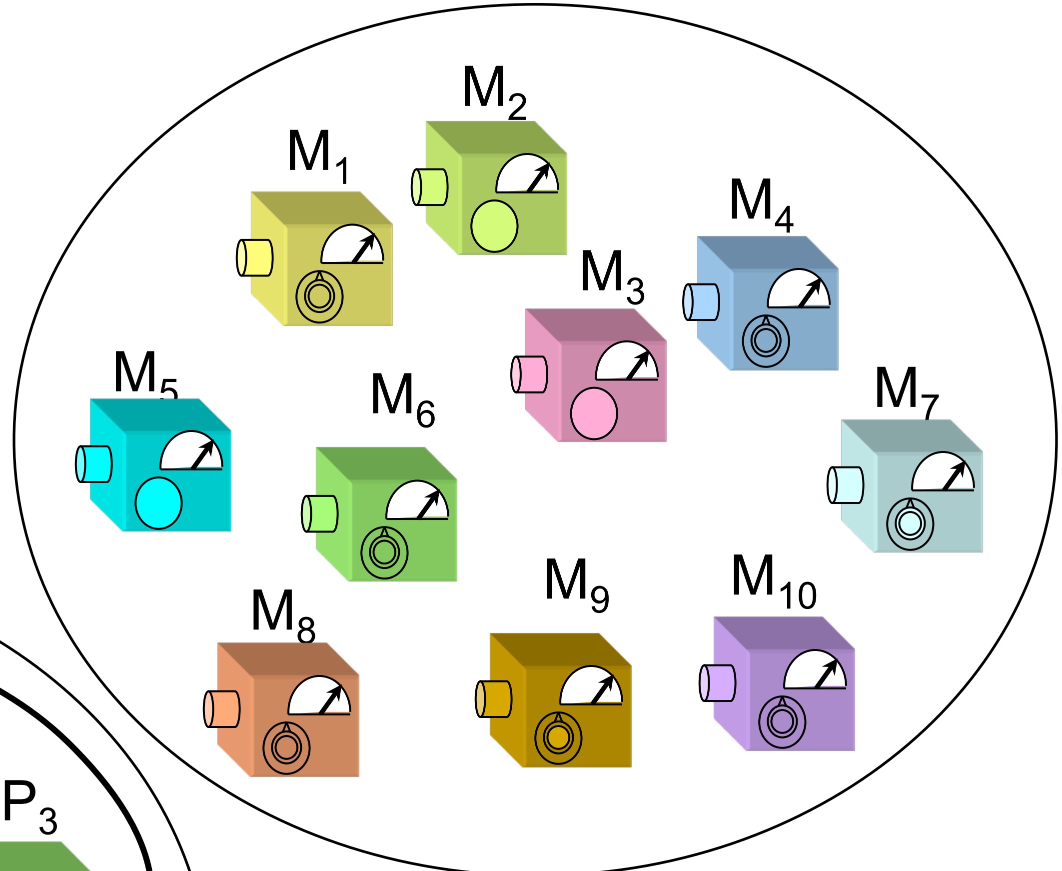


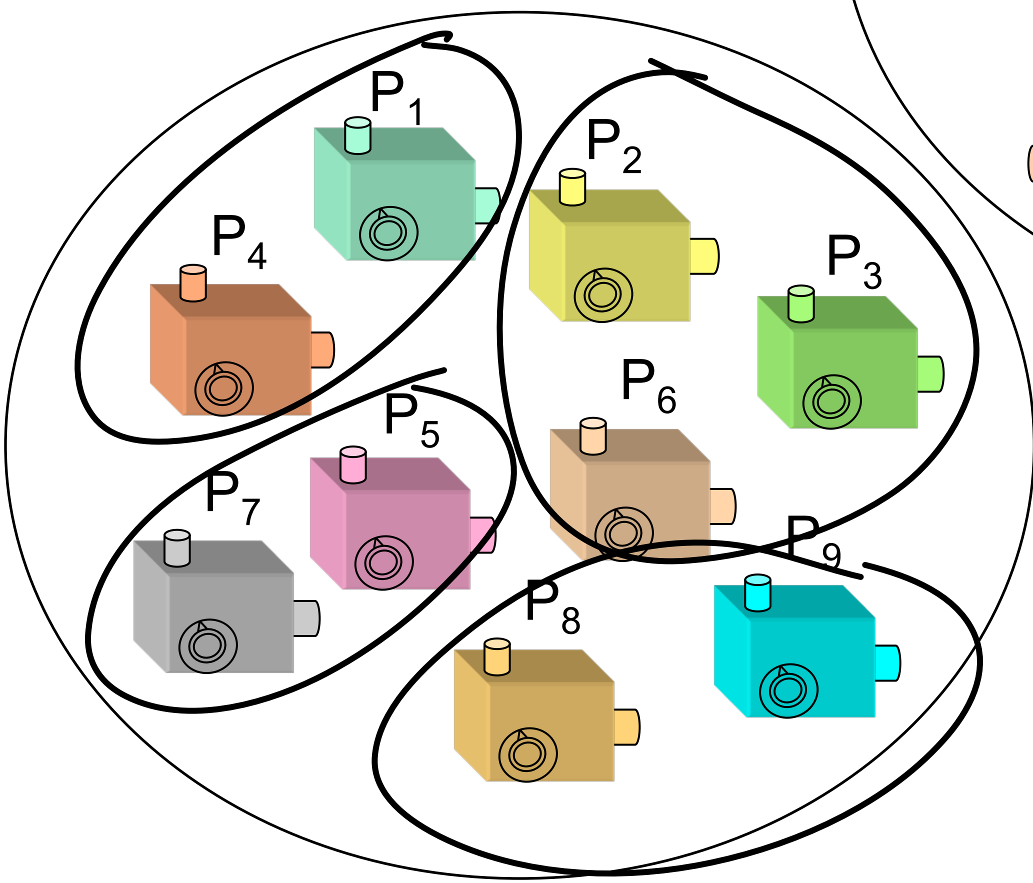
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

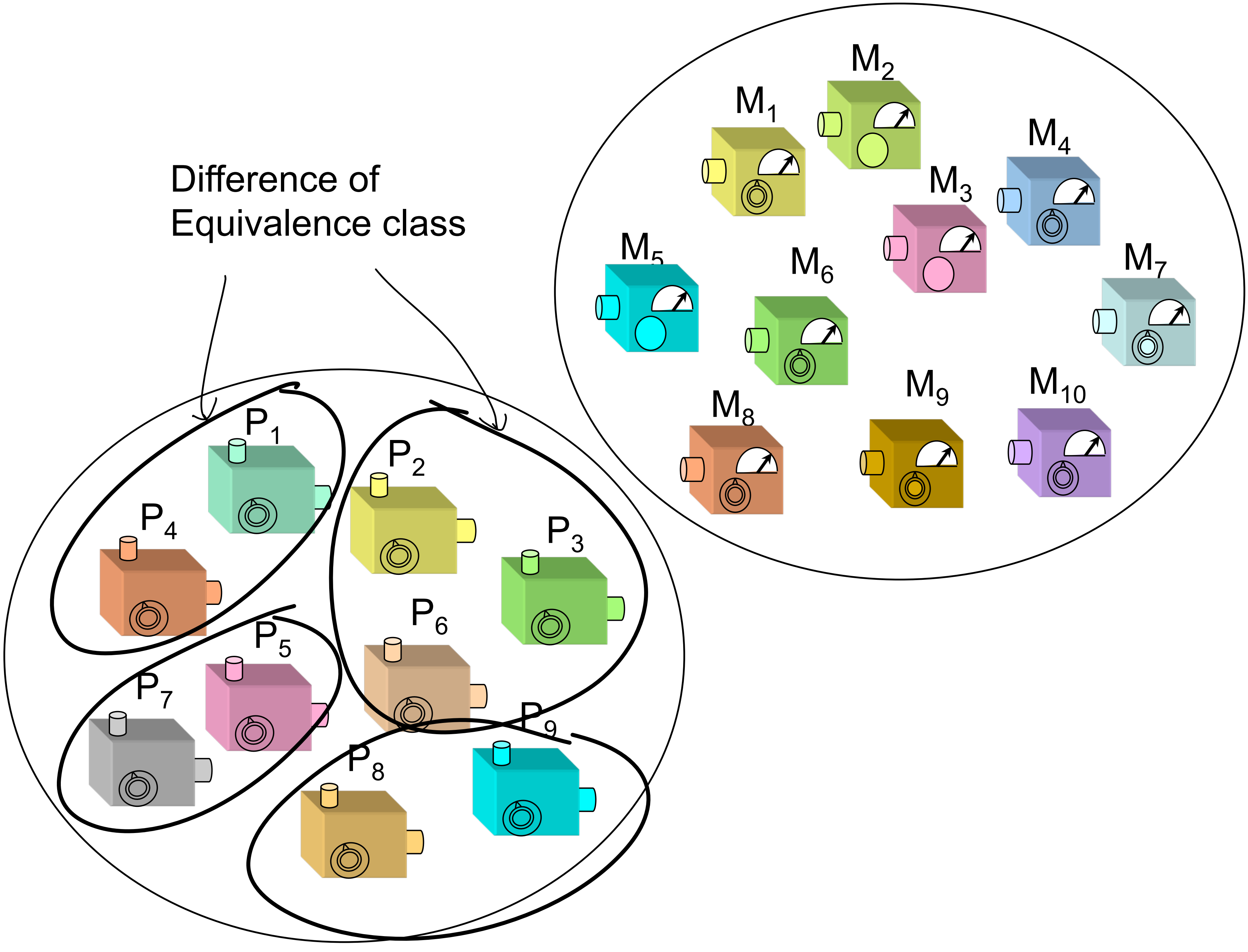
Operational equivalence
classes of preparations

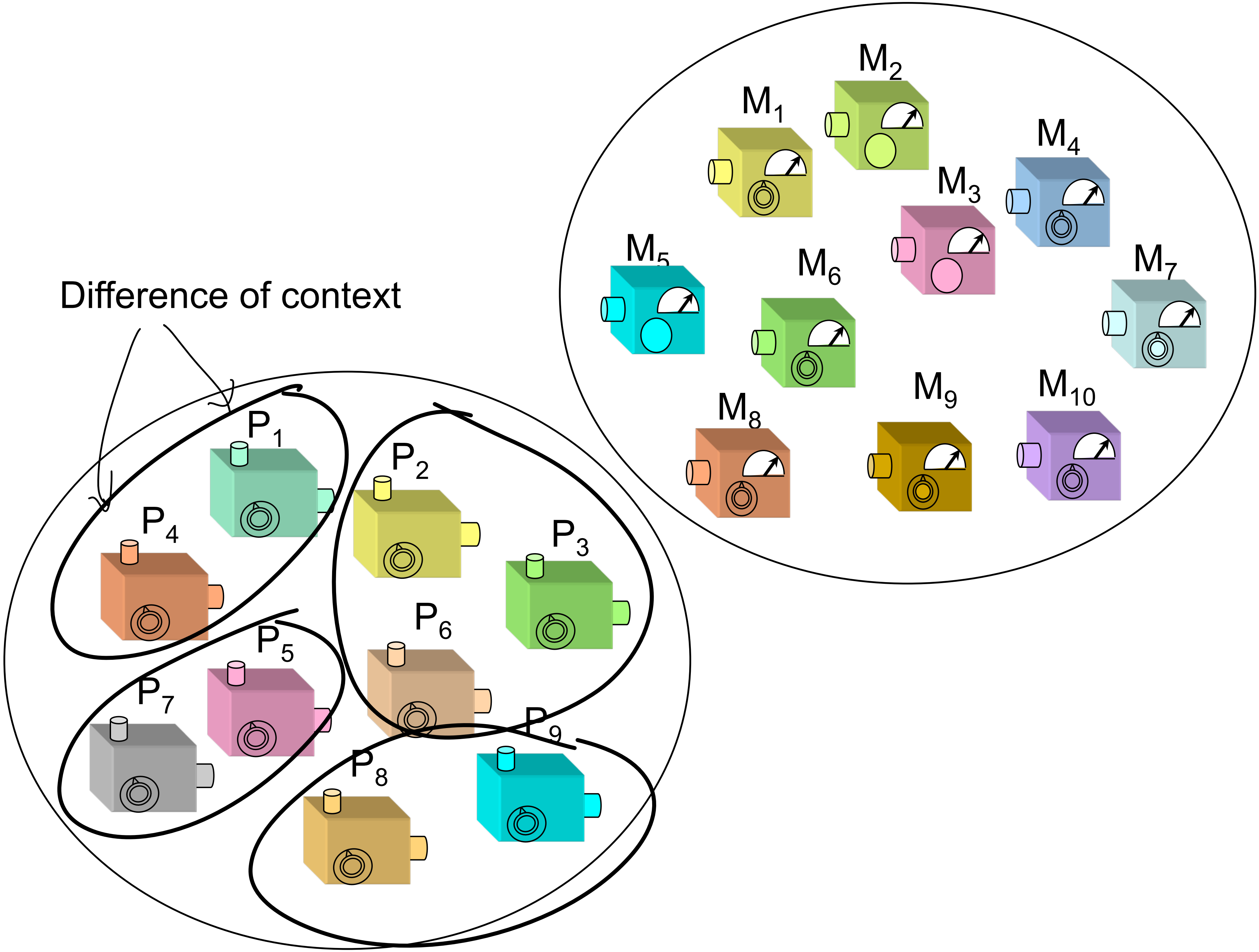
$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$






Difference of
Equivalence class
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}

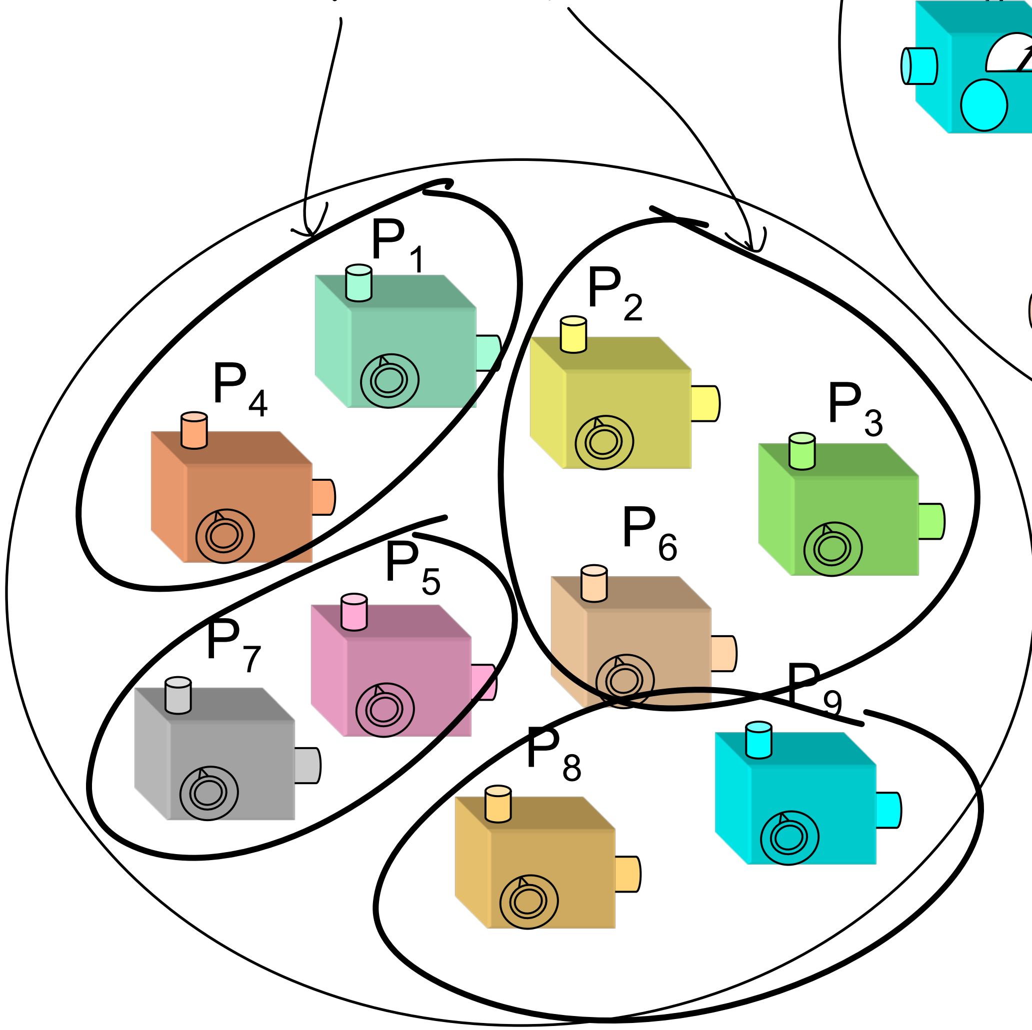


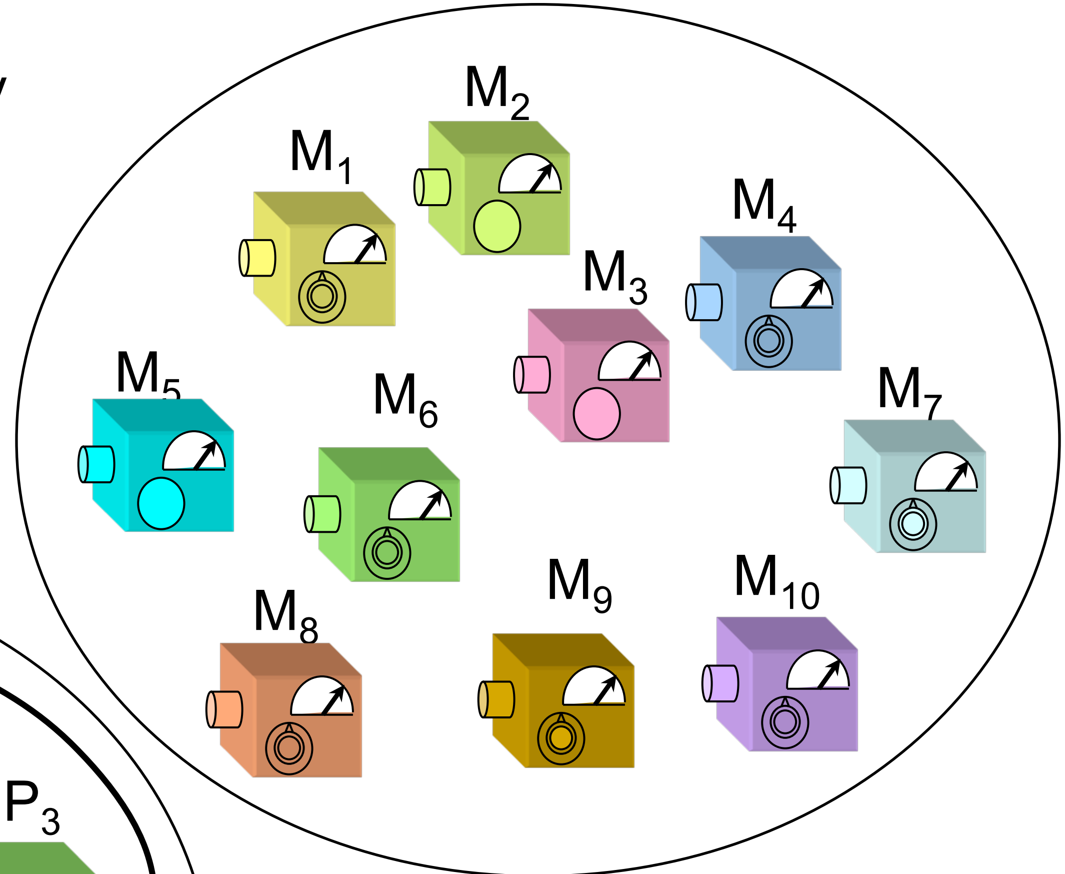
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
Difference of context
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9

Example from quantum theory

Different density op's

ρ ρ'

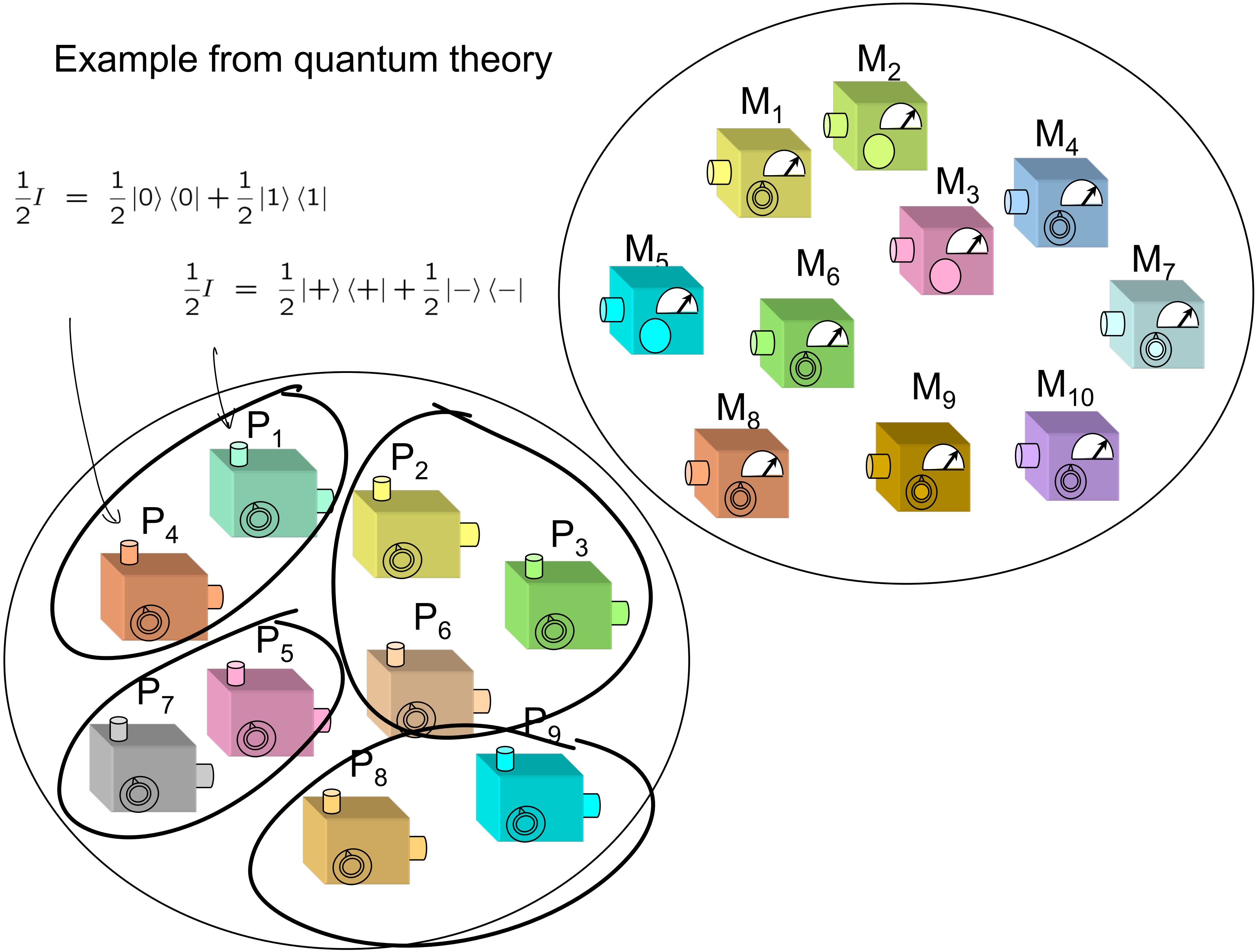




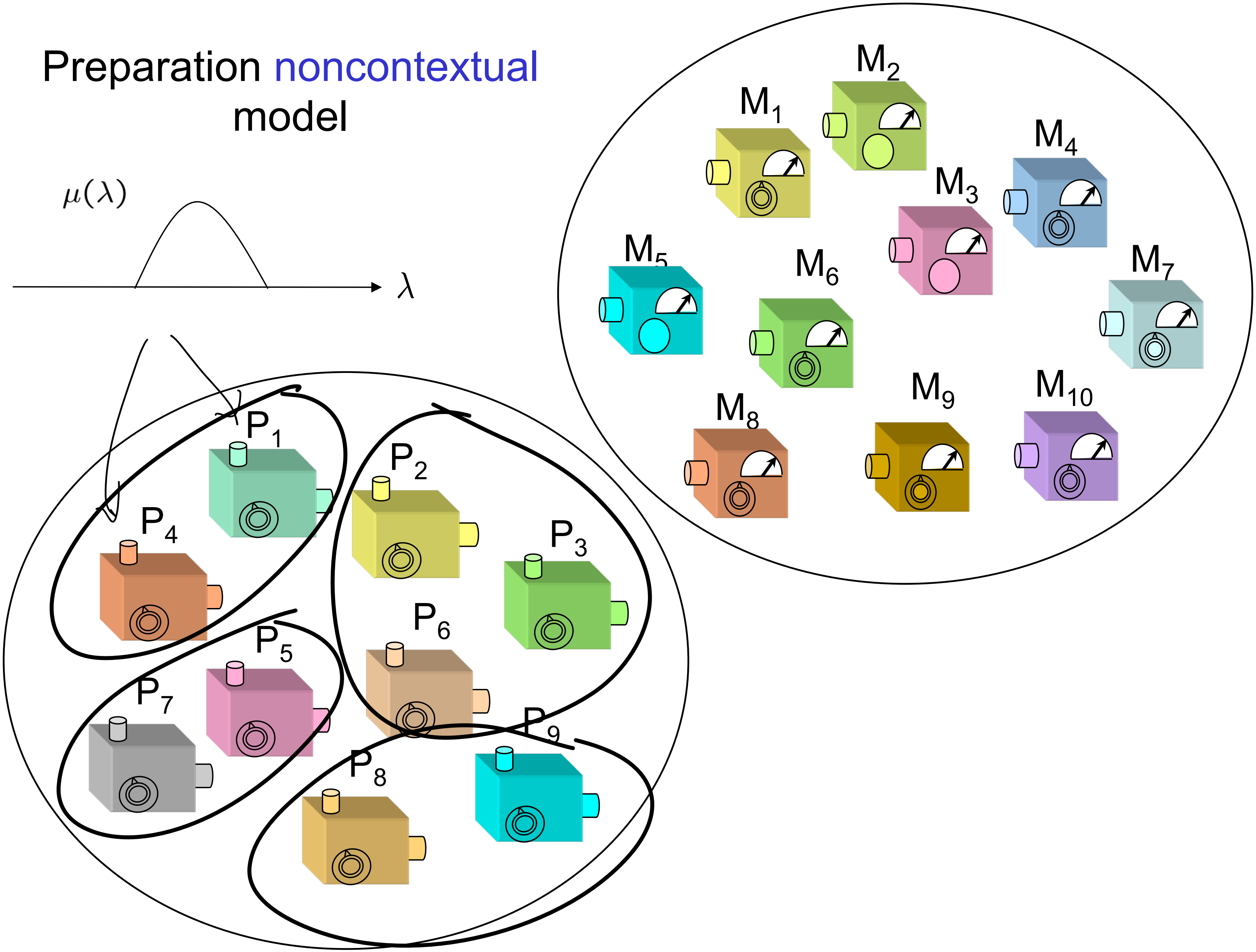
Example from quantum theory

$$\frac{1}{2}I \;=\; \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

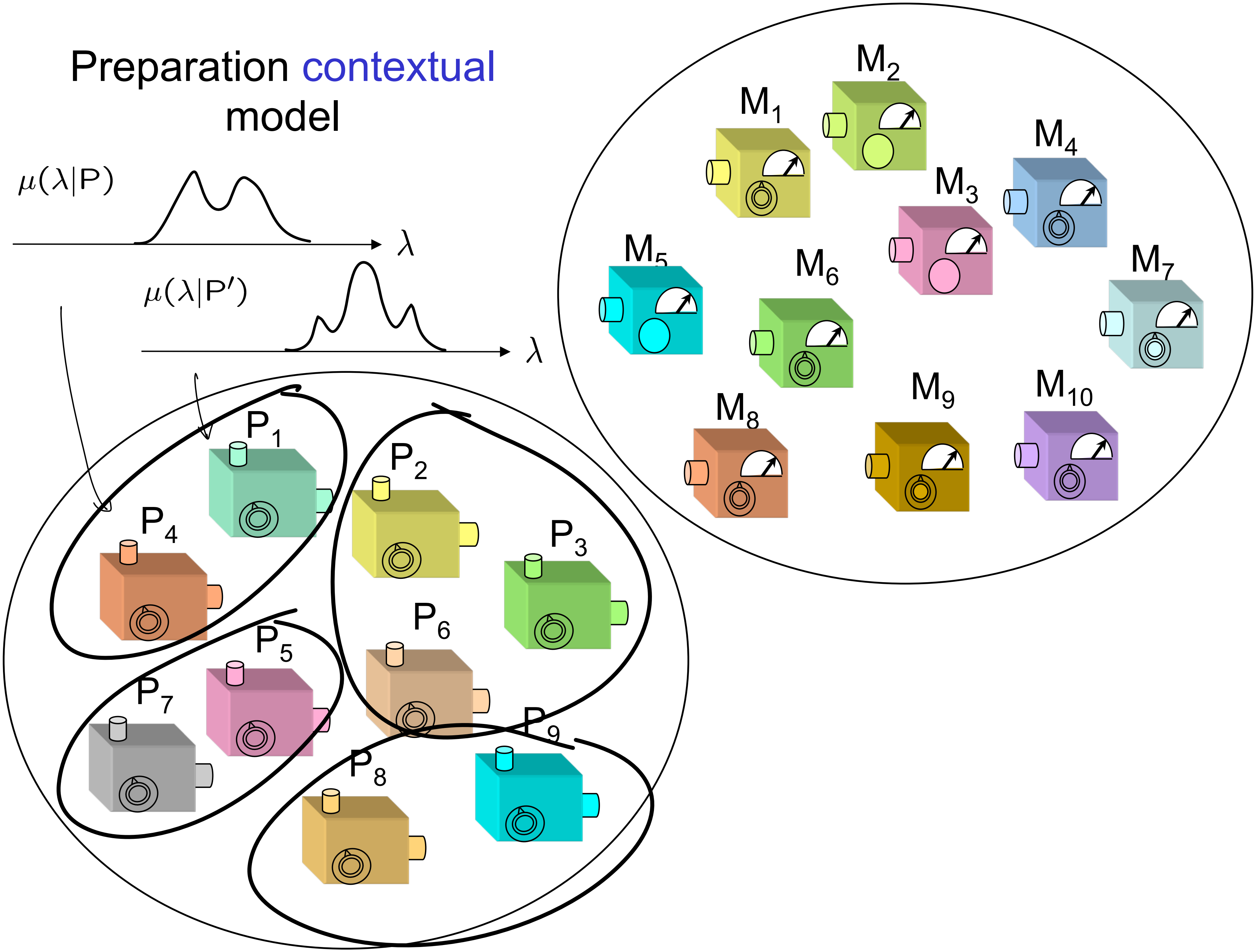
$$\frac{1}{2}I \;=\; \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$



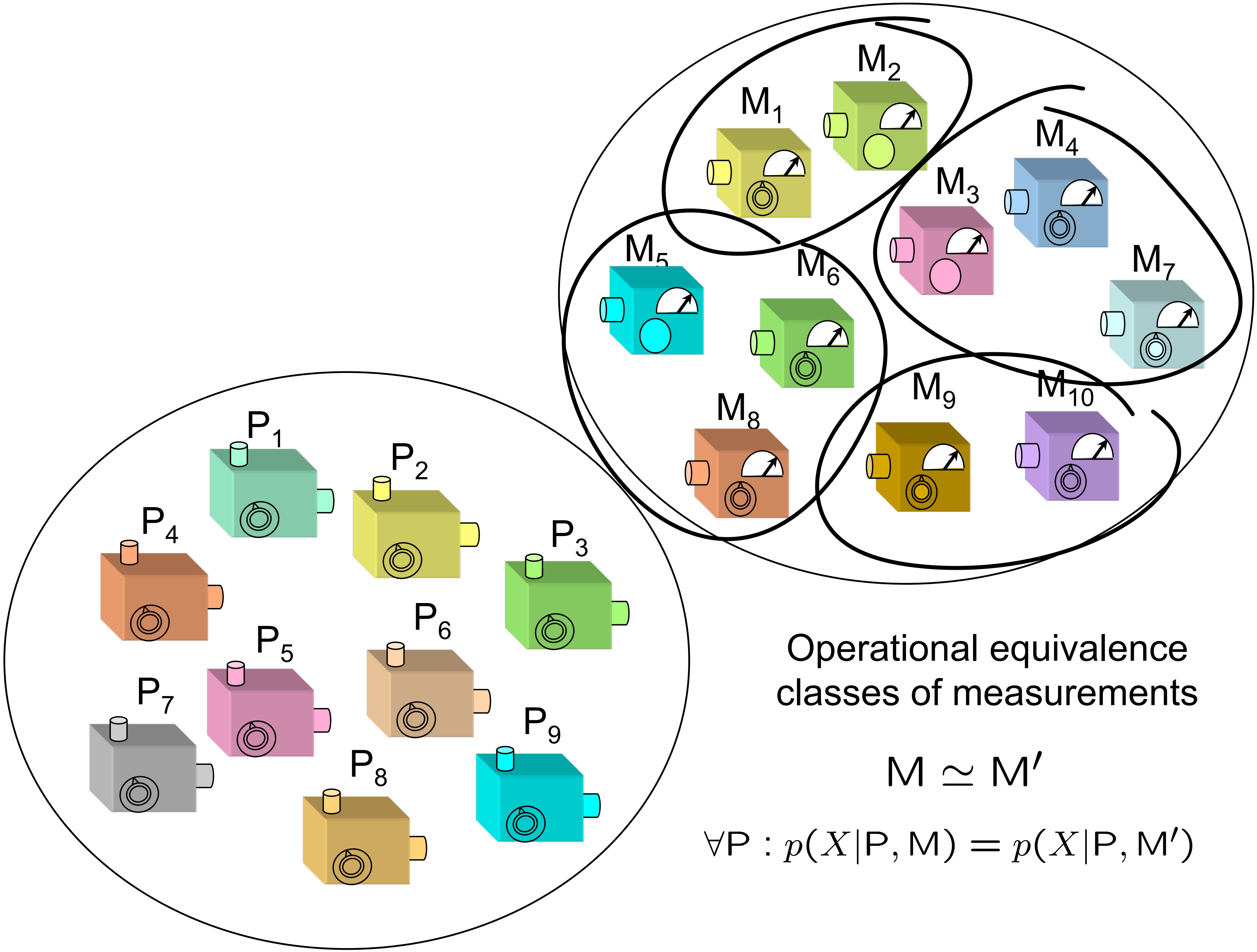
Preparation noncontextual model

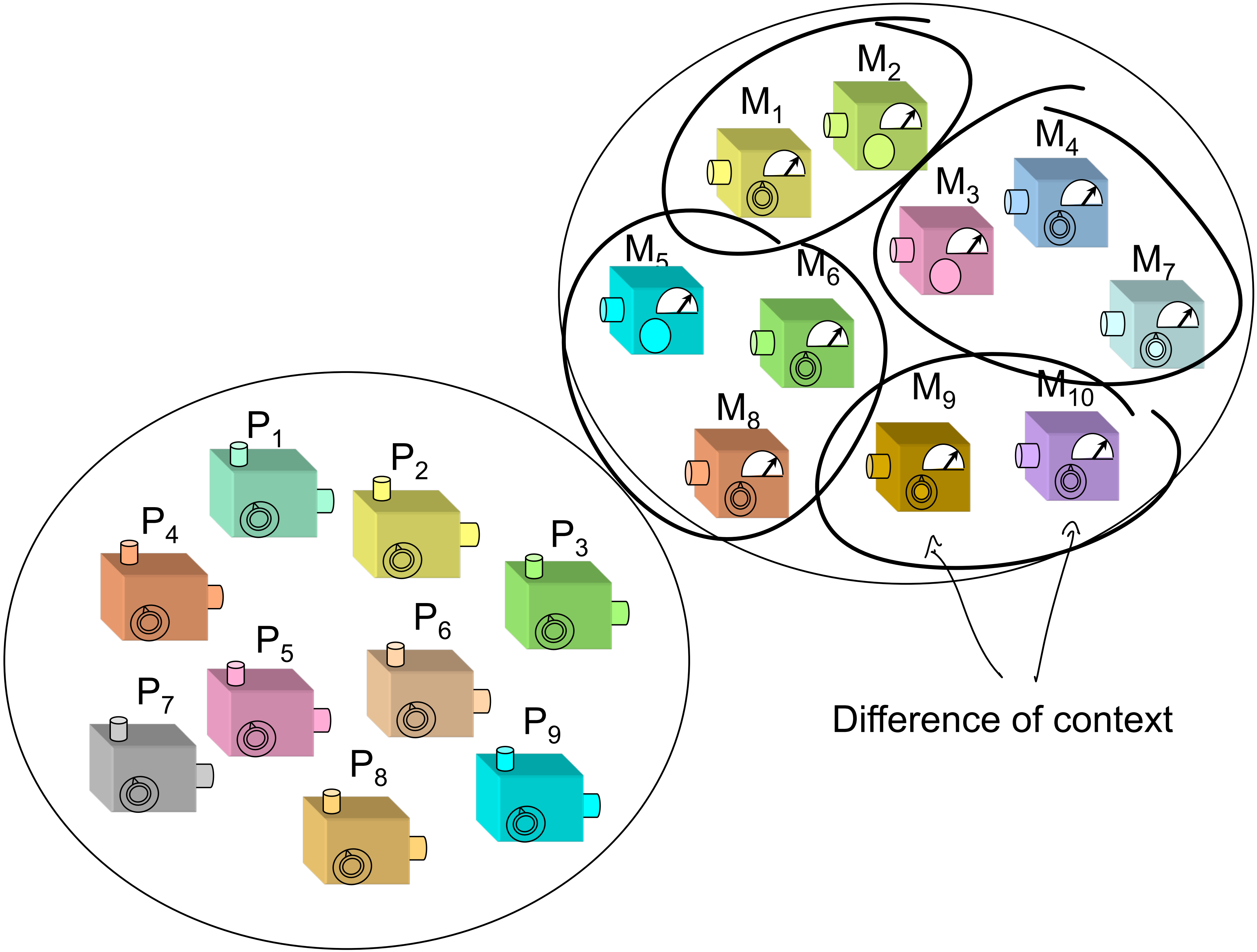


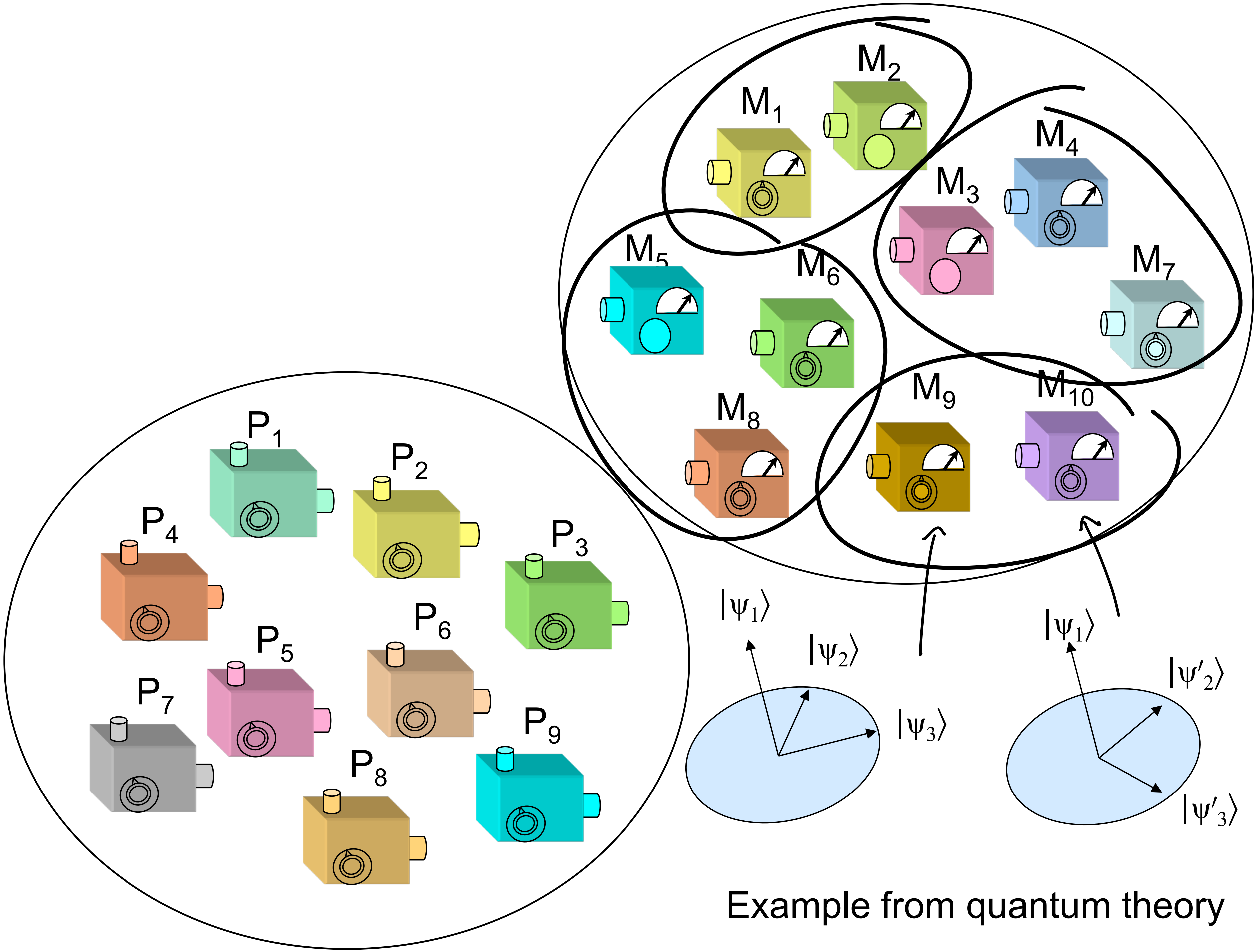
Preparation contextual model



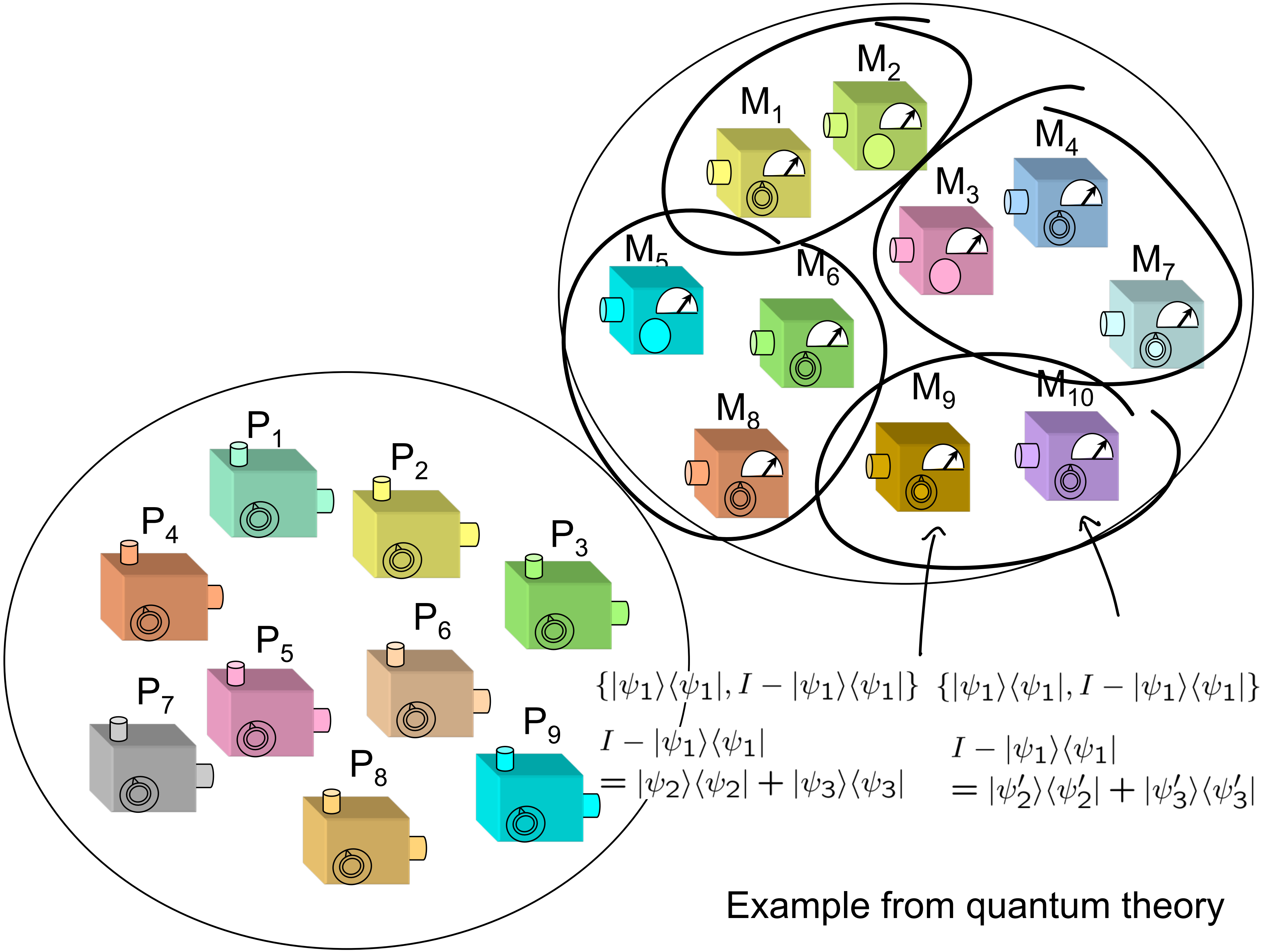
Measurement Noncontextuality


M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
Operational equivalence
classes of measurements
$\mathsf{M} \simeq \mathsf{M}'$
$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$

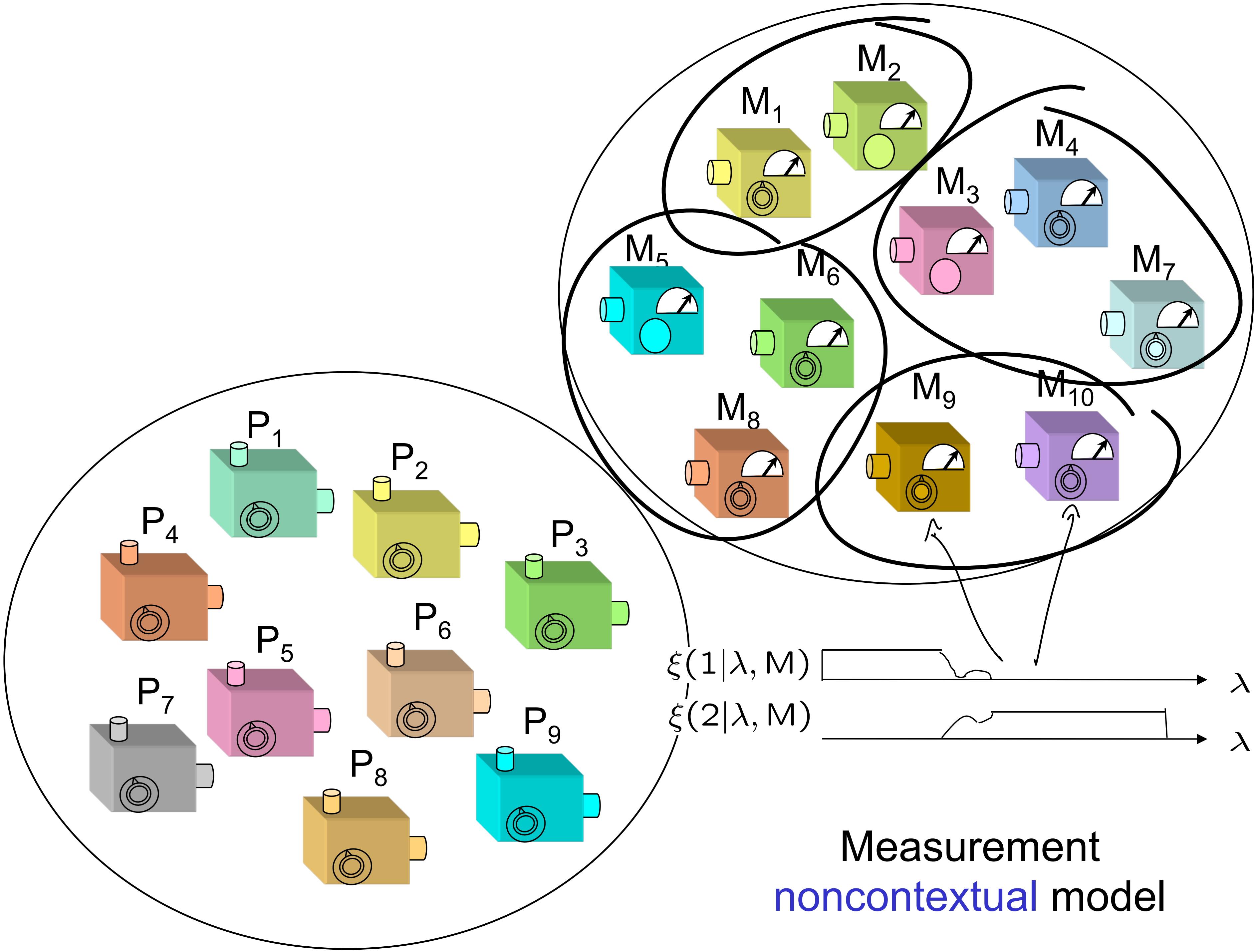

P1
P2
P3
P4
P5
P6
P7
P8
P9
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
Difference of context



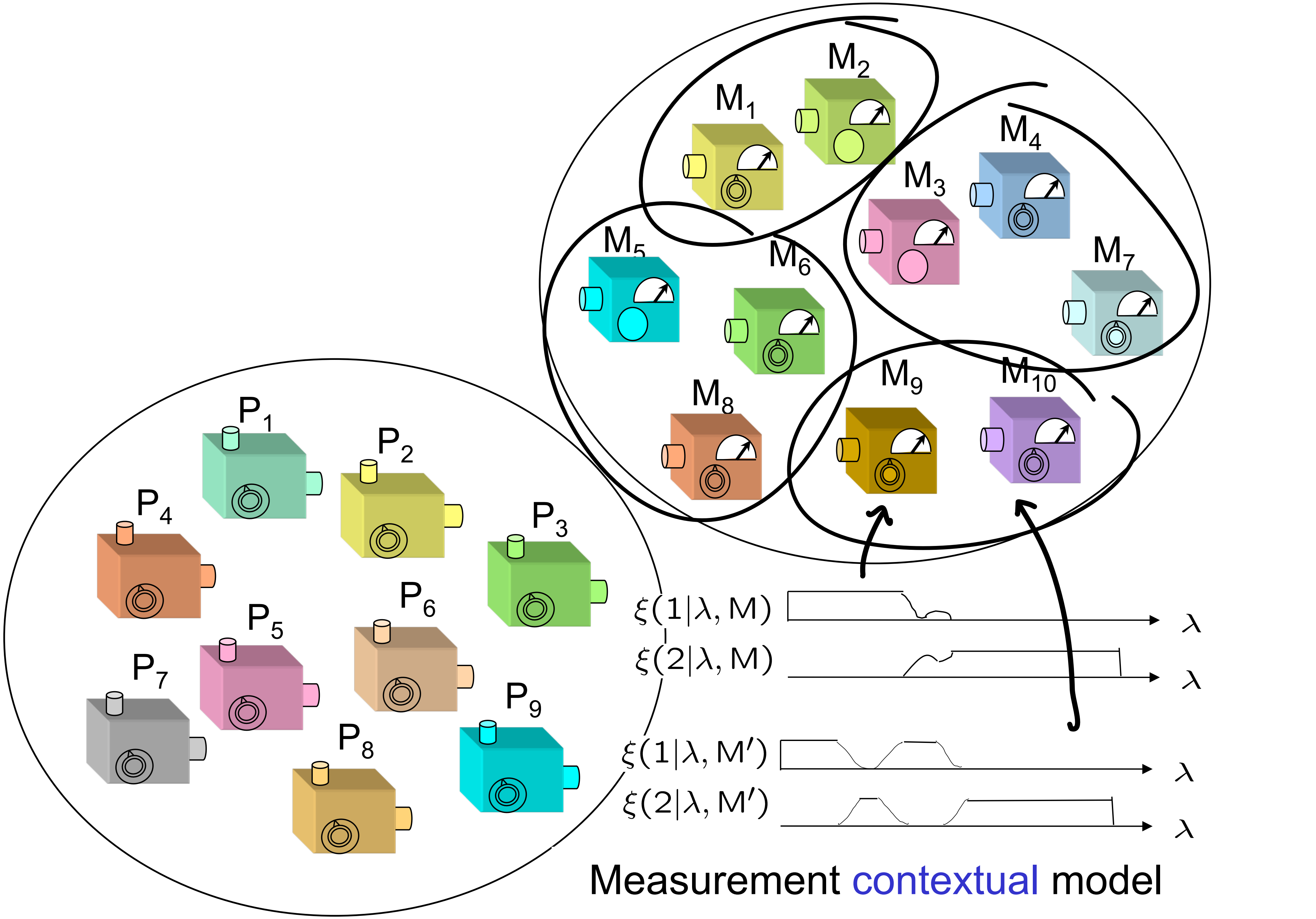
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
Example from quantum theory



Example from quantum theory



M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9
ξ(1|λ, M)
ξ(2|λ, M)
λ
λ
Measurement
noncontextual model



M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9
$\xi(1|\lambda, \mathrm{M})$
λ
$\xi(2|\lambda, \mathrm{M})$
λ
$\xi(1|\lambda, \mathrm{M}')$
λ
$\xi(2|\lambda, \mathrm{M}')$
λ
Measurement contextual model

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality

$$\longrightarrow$$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality

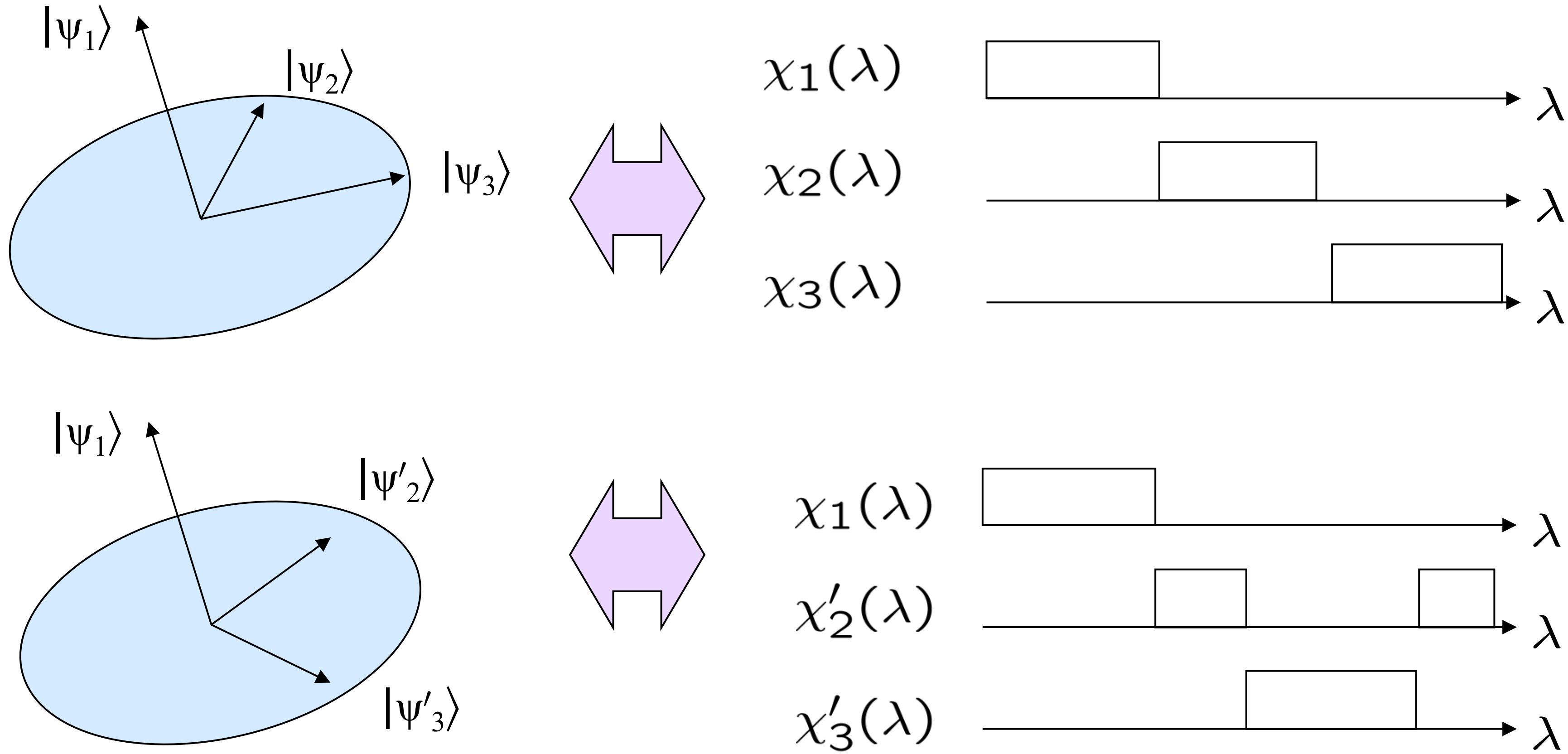
$$\longrightarrow$$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

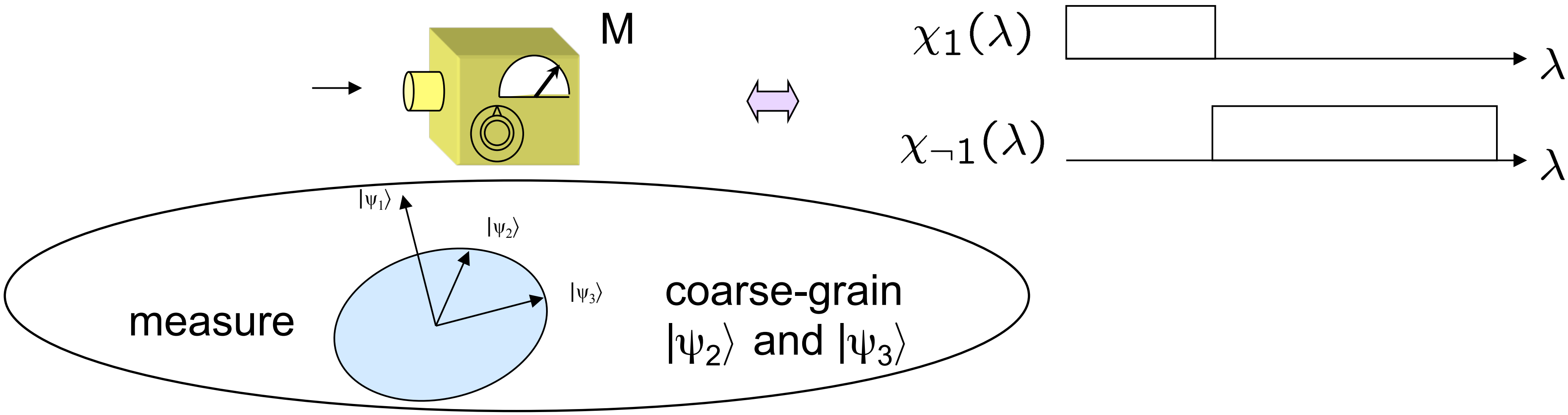
The best explanation of context-independence at the operational level is context-independence at the ontological level

KS noncontextuality
versus
Measurement noncontextuality

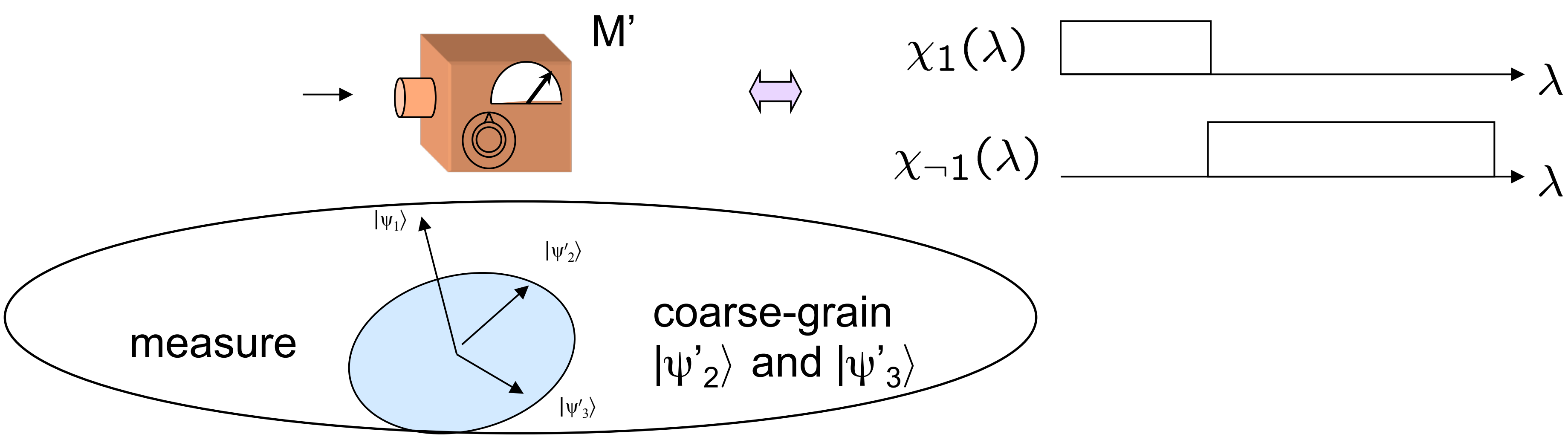
KS noncontextuality:



This is equivalent to assuming:



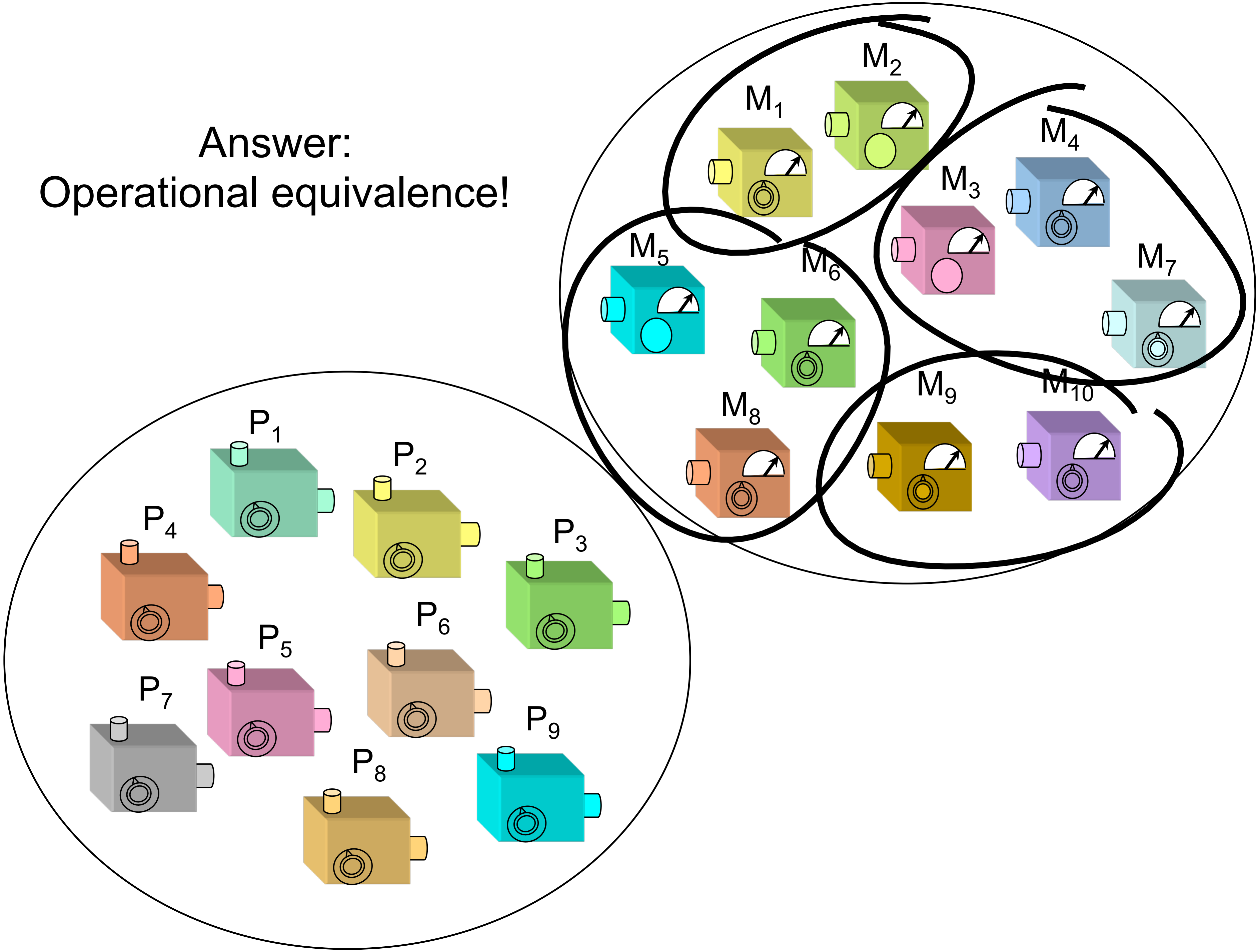
$$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$$

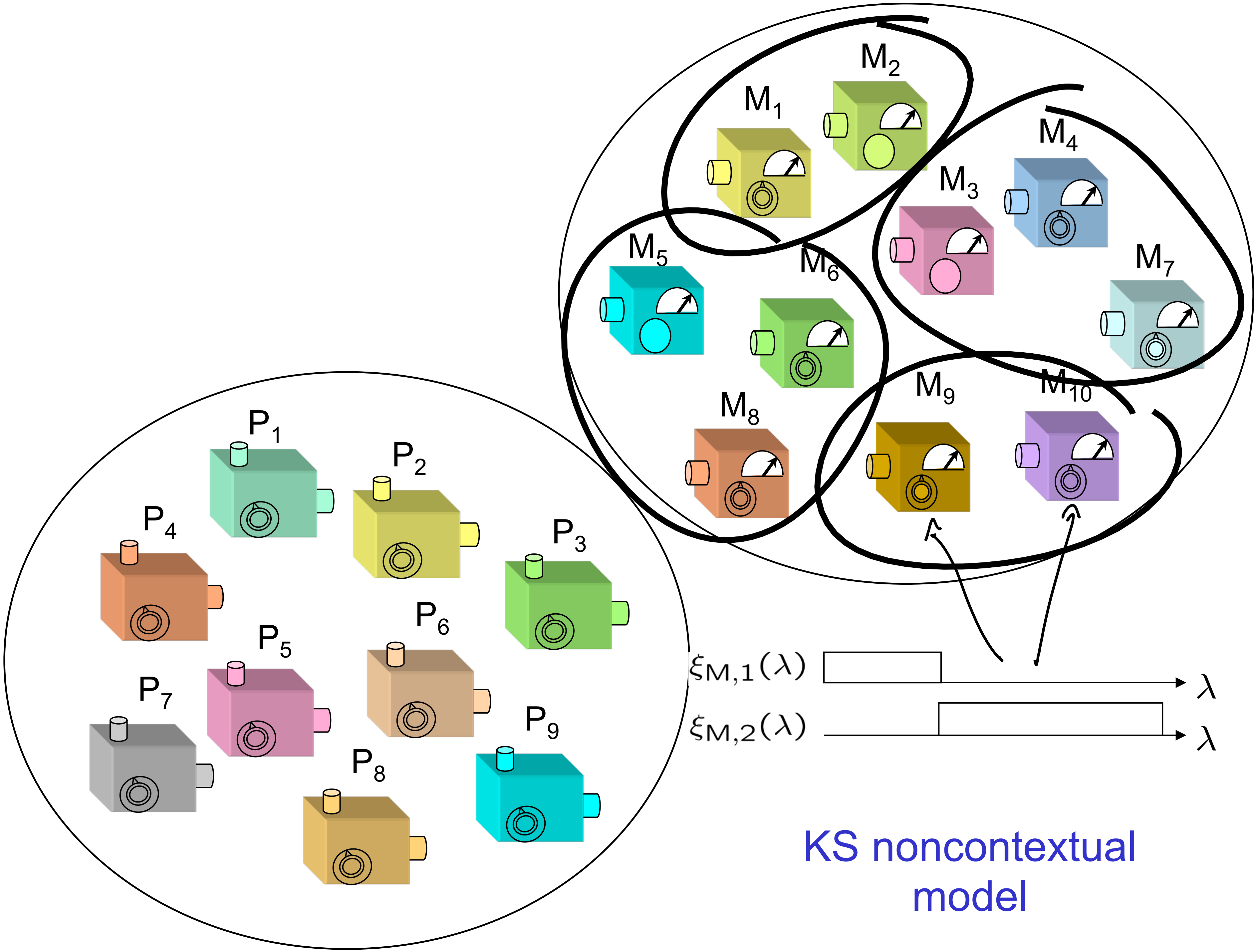


$$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$$

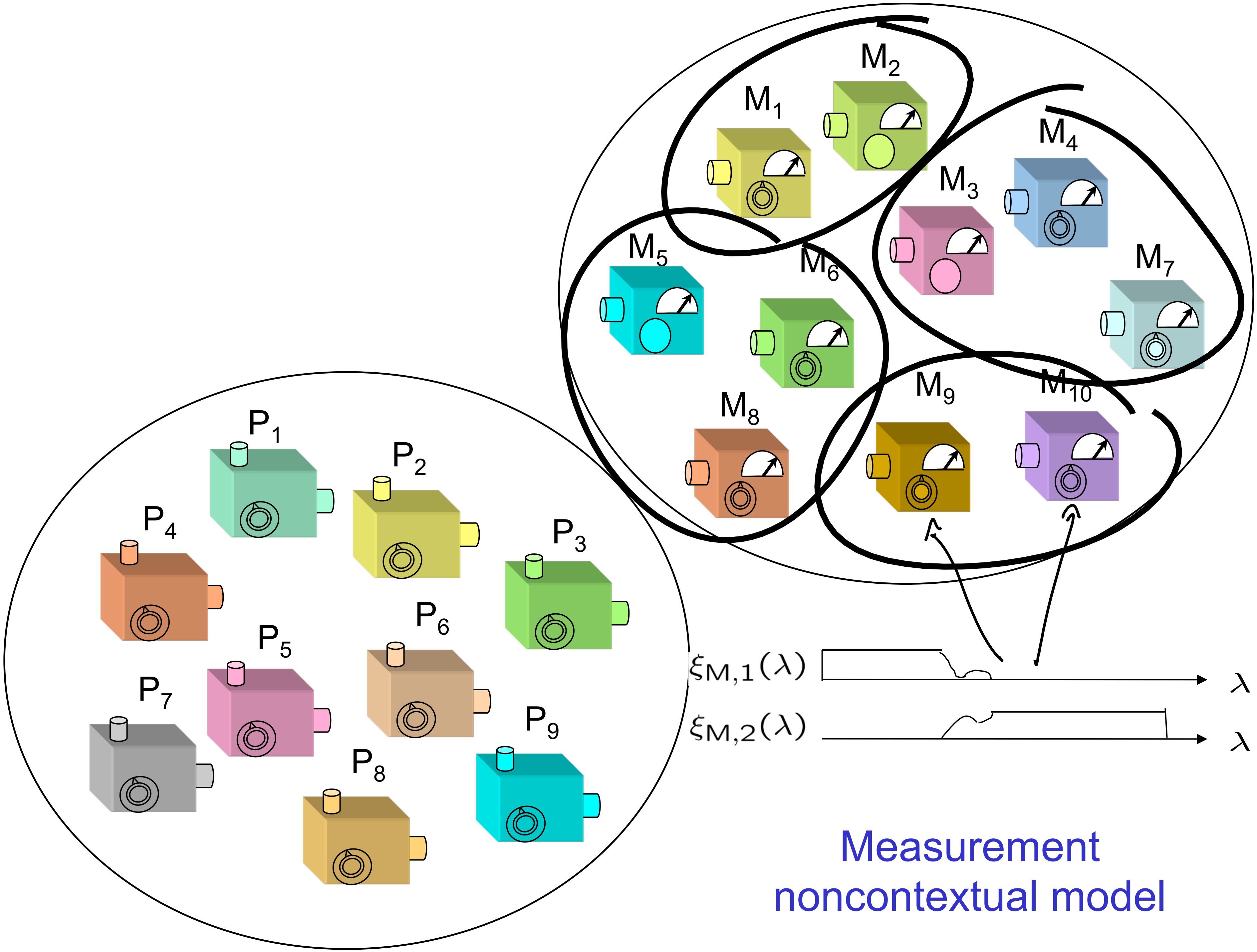
By what grounds do we justify representing two measurement devices by the same set of projectors?

Answer:
Operational equivalence!





M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
$\xi_{M,1}(\lambda)$
λ
$\xi_{M,2}(\lambda)$
λ
KS noncontextual model



Measurement
noncontextual model

measurement noncontextuality

+

outcome determinism

=

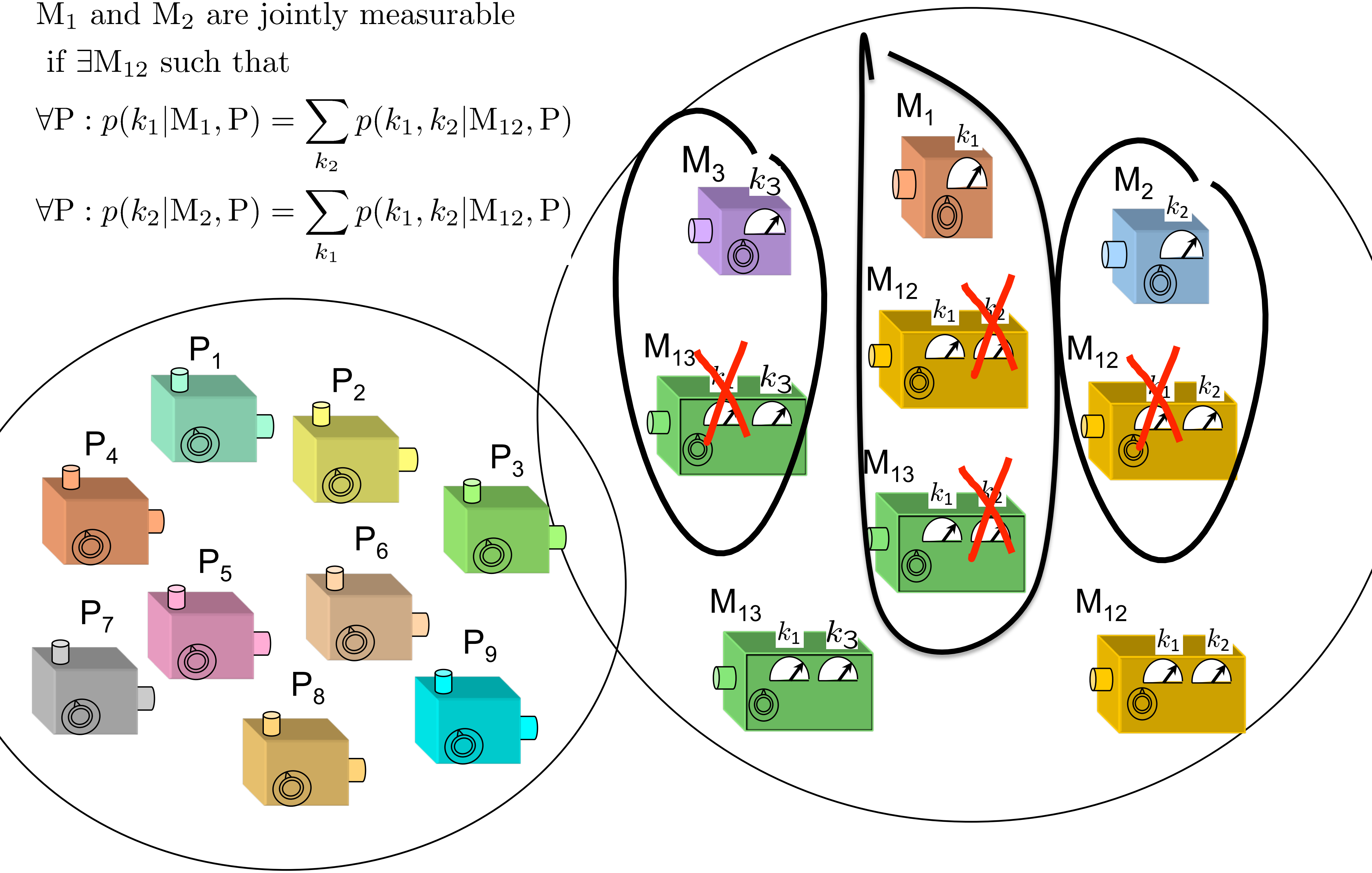
KS-noncontextuality

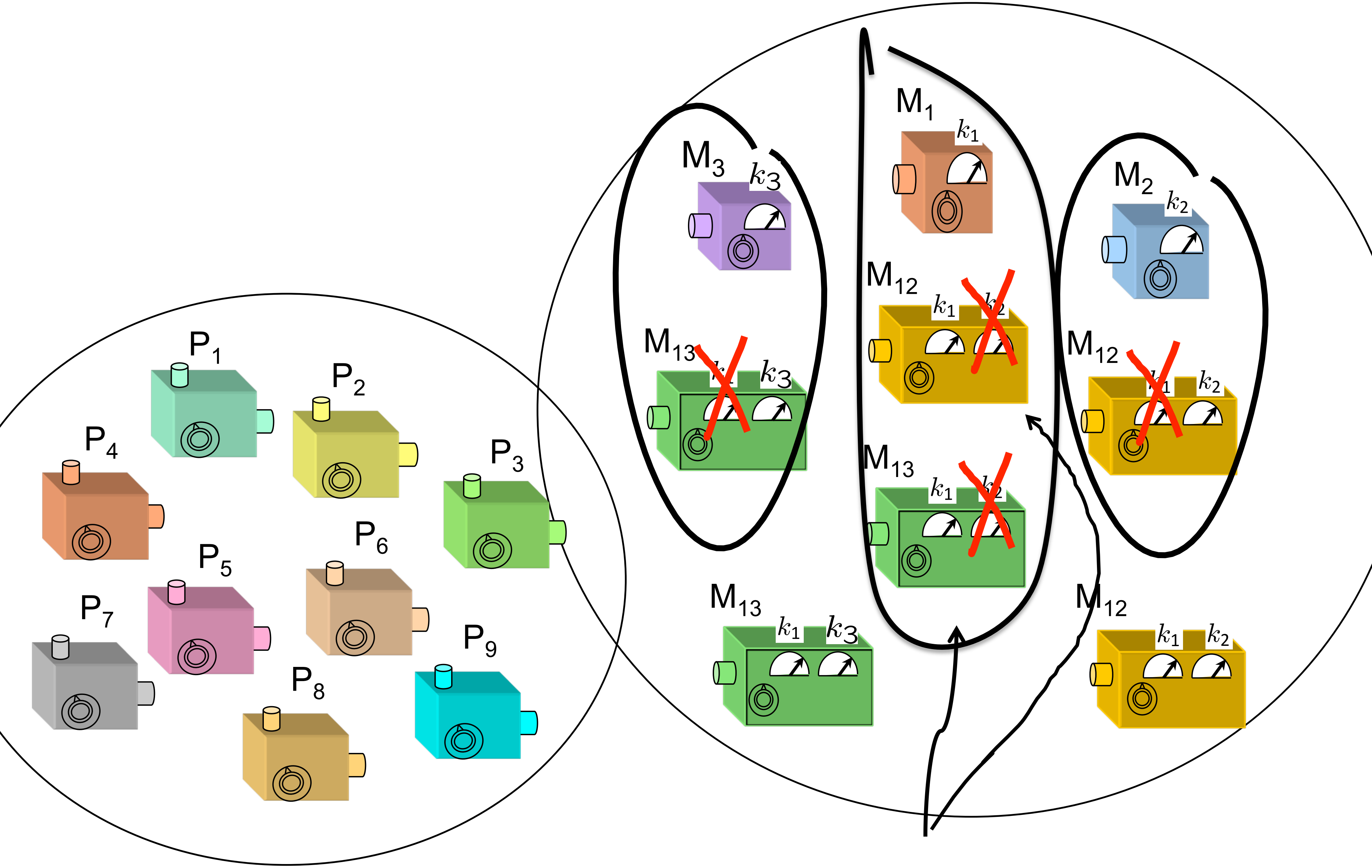
Compatibility of measurements in terms of joint simulatability

M_1 and M_2 are jointly measurable
if $\exists \mathrm{M}_{12}$ such that

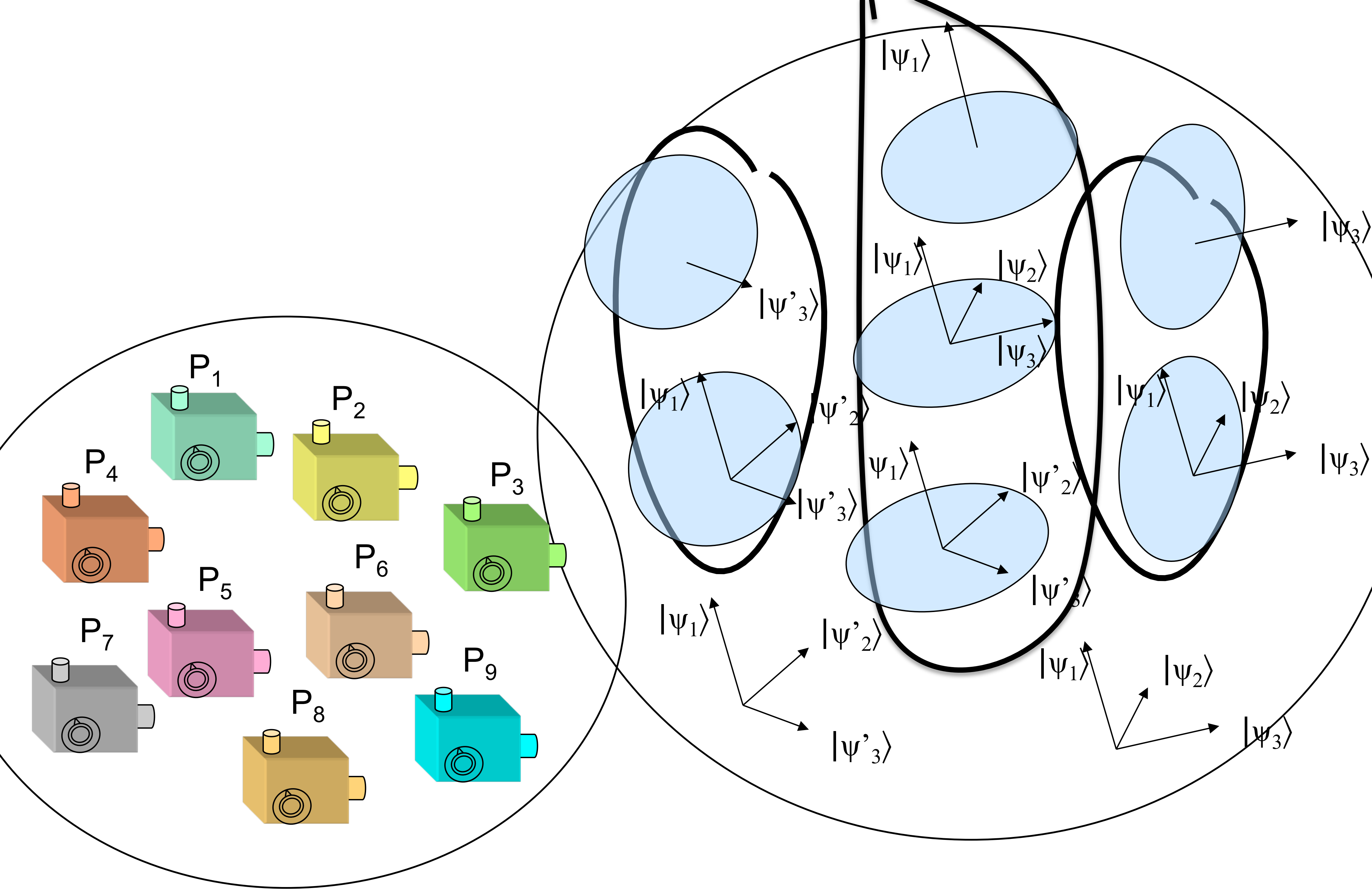
$$\forall \mathrm{P} : p(k_1|\mathrm{M}_1, \mathrm{P}) = \sum_{k_2} p(k_1, k_2|\mathrm{M}_{12}, \mathrm{P})$$

$$\forall \mathrm{P} : p(k_2|\mathrm{M}_2, \mathrm{P}) = \sum_{k_1} p(k_1, k_2|\mathrm{M}_{12}, \mathrm{P})$$





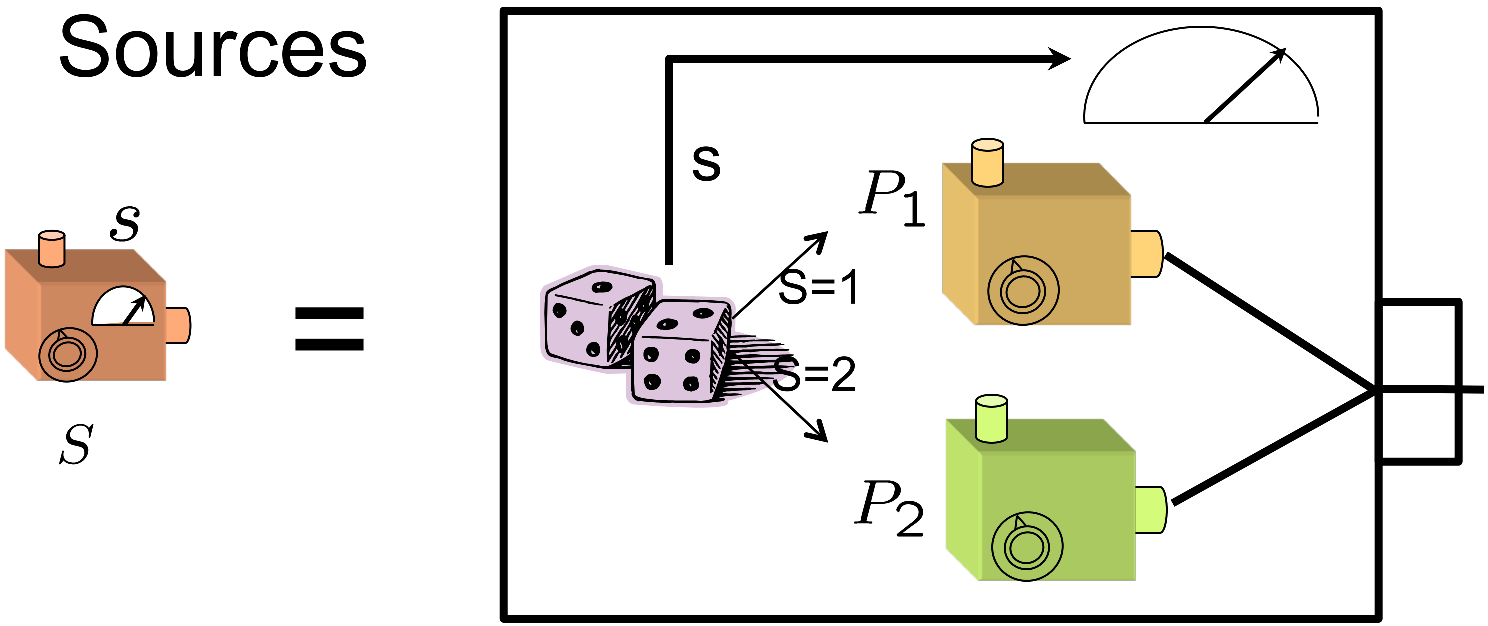
P 1
P 2
P 3
P 4
P 5
P 6
P 7
P 8
P 9
M3
k3
M13
k3
M1
k1
M12
k1
M13
k1
M2
k2
M12
k2
M13
k1
k3
M12
k1
k2
Difference of context



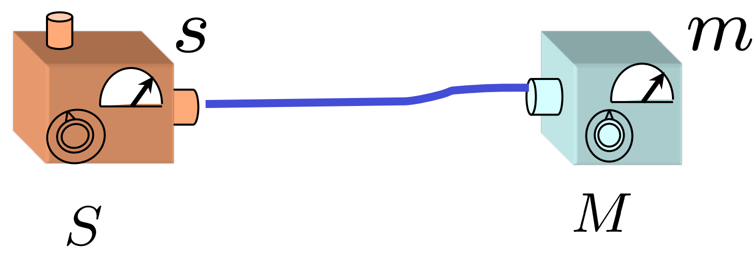
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
$|\psi_1\rangle$
$|\psi'_3\rangle$
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$|\psi_3\rangle$
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$

Sources that are operationally equivalent after marginalization over outcome

Sources



$$
\begin{aligned}
\mu(\lambda, s|S) &= \mu(\lambda|s, S)\mathrm{Pr}(s) \\
&= \mu(\lambda|P_s)\mathrm{Pr}(s)
\end{aligned}
$$



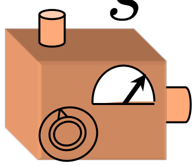
$$\mu(\lambda, s|S) \qquad\qquad \xi(m|M,\lambda)$$

$$\sum_s \sum_{\lambda\in\Lambda} \mu(\lambda, s|S) = 1 \qquad\qquad \sum_m \xi(m|M,\lambda) = 1$$

$$\sum_{\lambda\in\Lambda} \xi(m|M,\lambda)\mu(\lambda, s|S) = \mathrm{pr}(m, s|M, S).$$

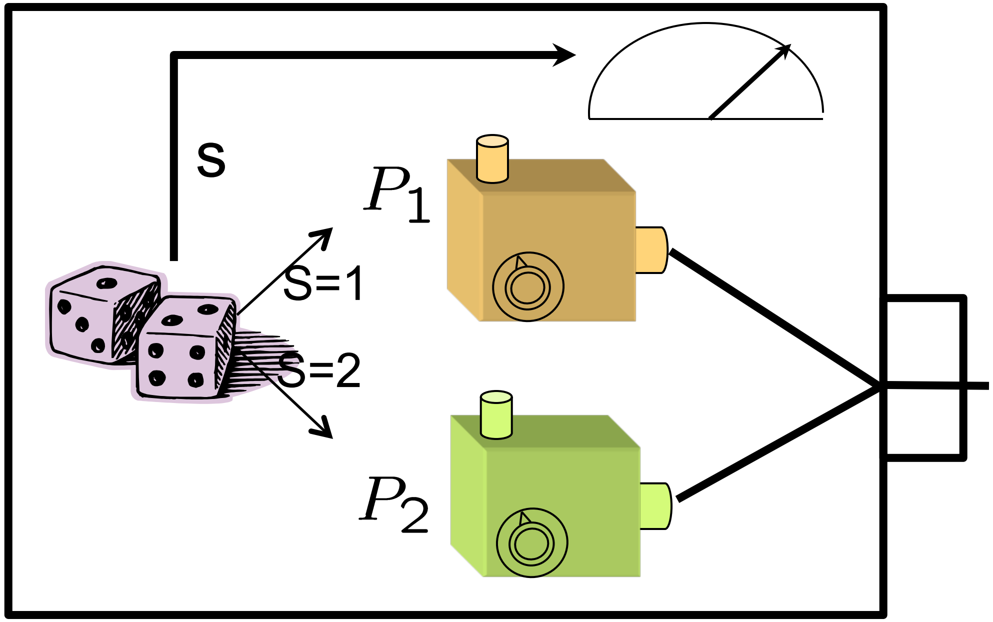
Sources

s



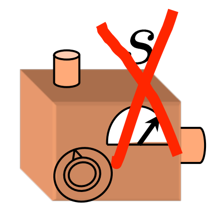
S

$=$



$$\mu(\lambda, s|S) = \mu(\lambda|s, S)\mathrm{Pr}(s)$$
$$= \mu(\lambda|P_s)\mathrm{Pr}(s)$$

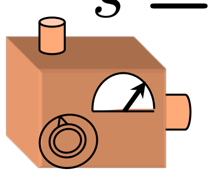
P_S^{ave}



S

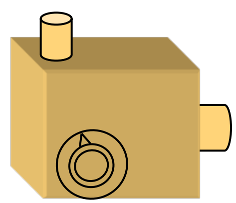
$$\mu(\lambda|P_S^{\text{ave}}) = \sum_s \mu(\lambda, s|S)$$
$$= \sum_s \mu(\lambda|P_s)\mathrm{Pr}(s)$$

$s = 1$

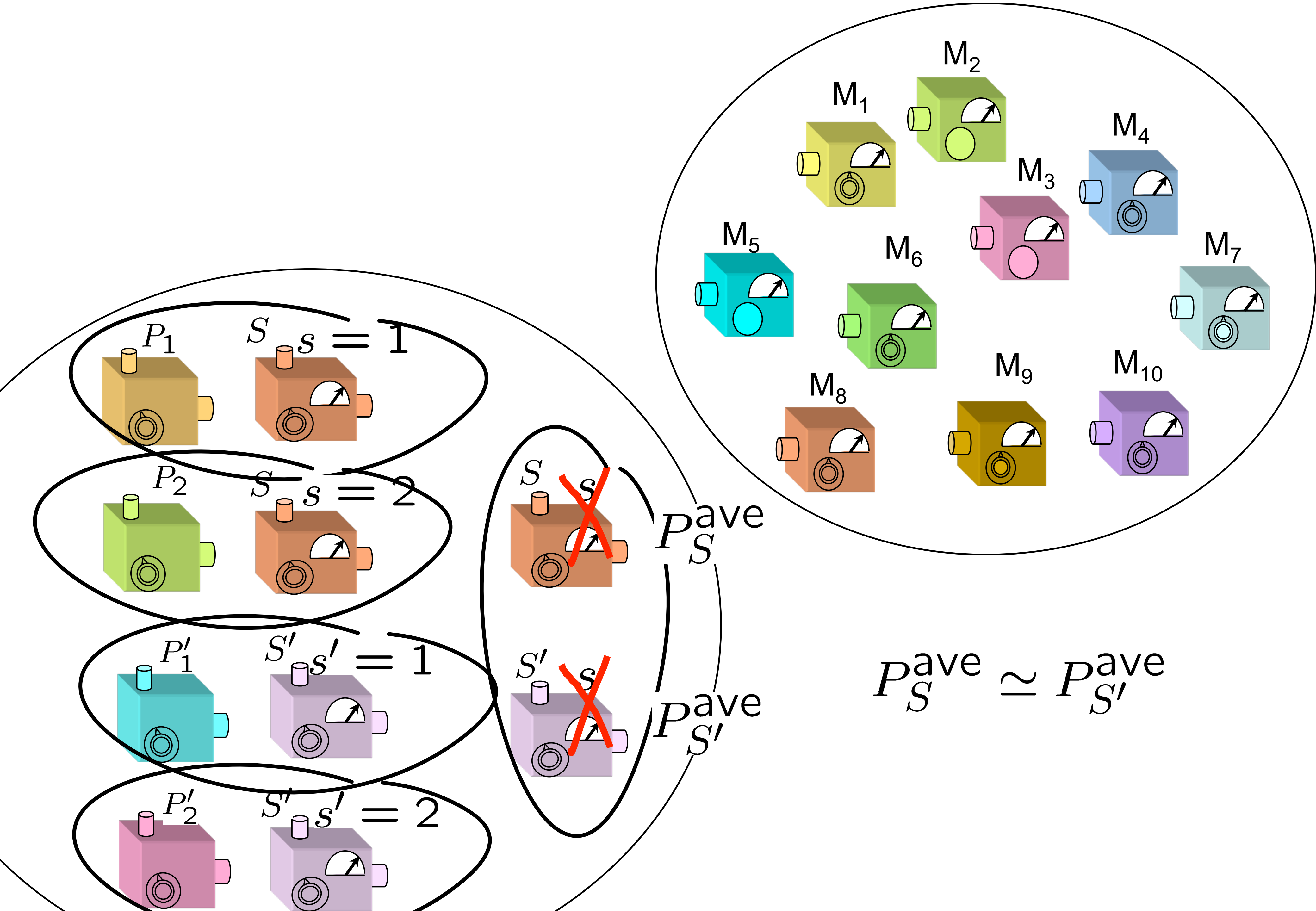


S

$=$ P_1

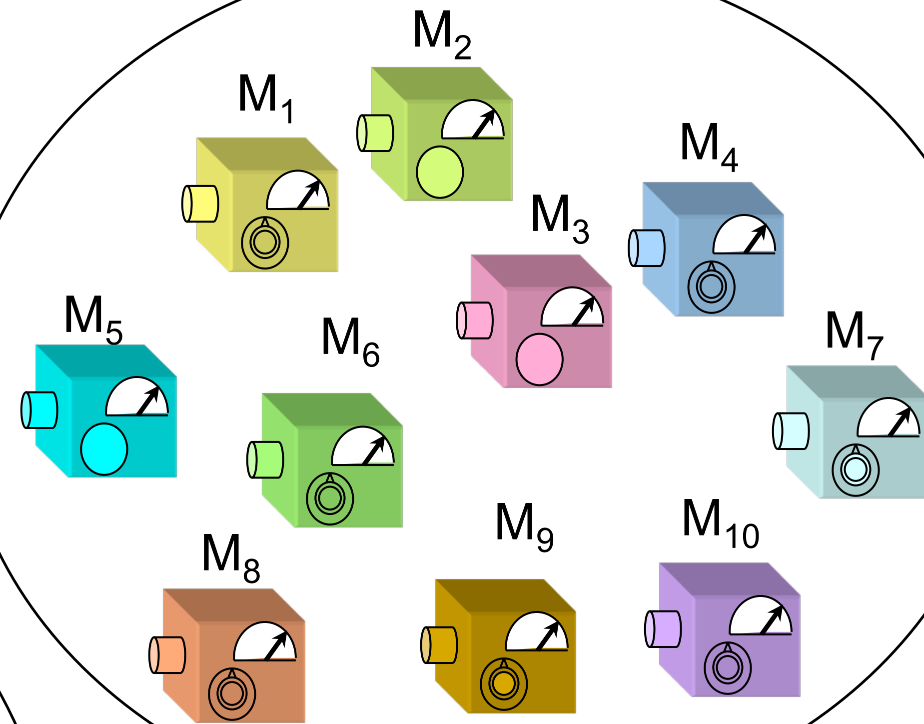


$$\mu(\lambda|s, S) = \mu(\lambda|P_s)$$



M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P_1
S
$s = 1$
P_2
S
$s = 2$
P'_1
S'
$s' = 1$
P'_2
S'
$s' = 2$
S
s
S'
s
P_S^{ave}
$P_{S'}^{\text{ave}}$
$P_S^{\text{ave}} \simeq P_{S'}^{\text{ave}}$

Quantum example:



$|0\rangle\langle 0|$

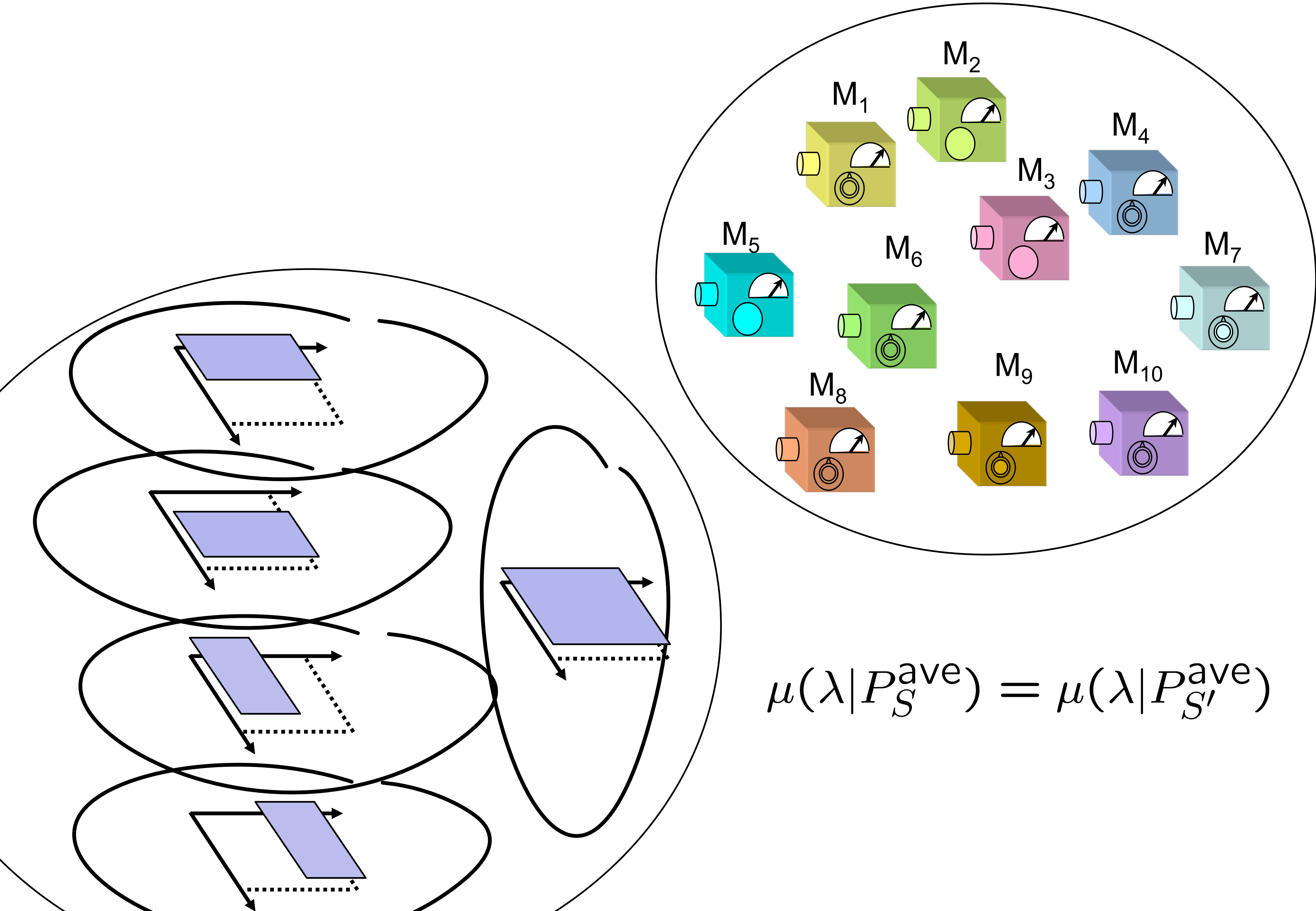
$|1\rangle\langle 1|$

$|+\rangle\langle +|$

$|-\rangle\langle -|$

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$



M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
$\mu(\lambda|P_S^{\mathrm{ave}}) = \mu(\lambda|P_{S'}^{\mathrm{ave}})$

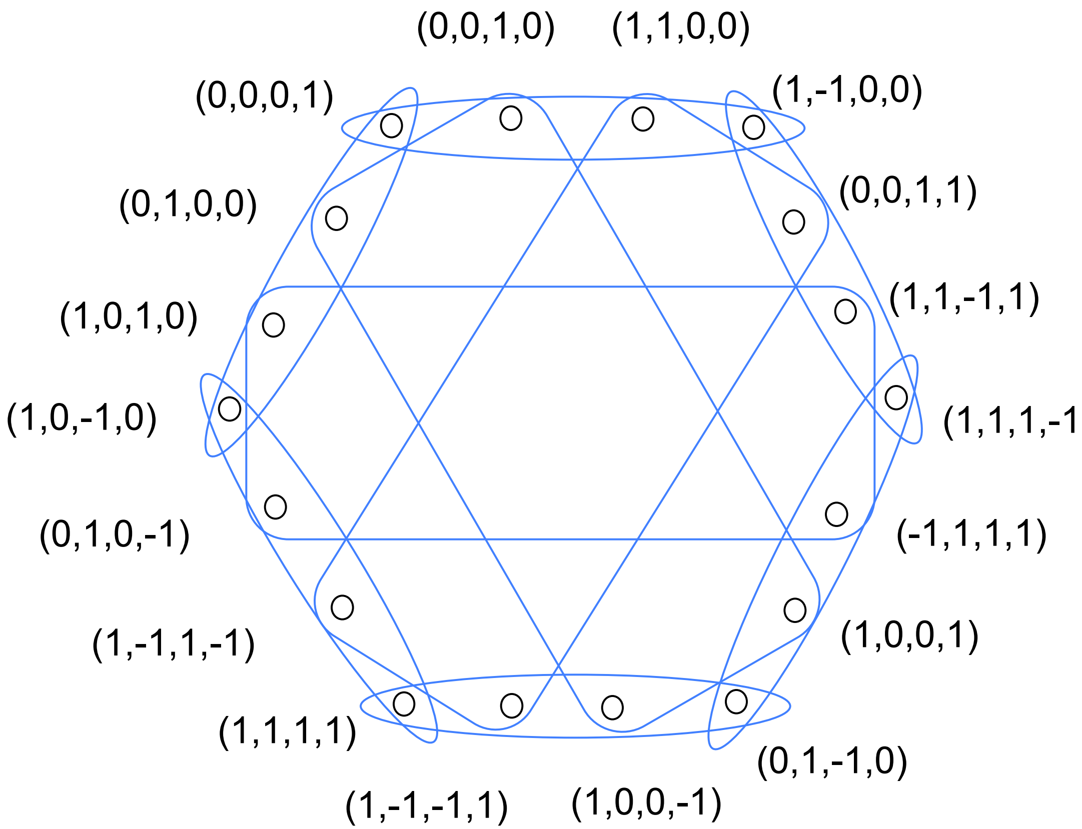
Robust noncontextuality inequality from state-independent proof of KS theorem

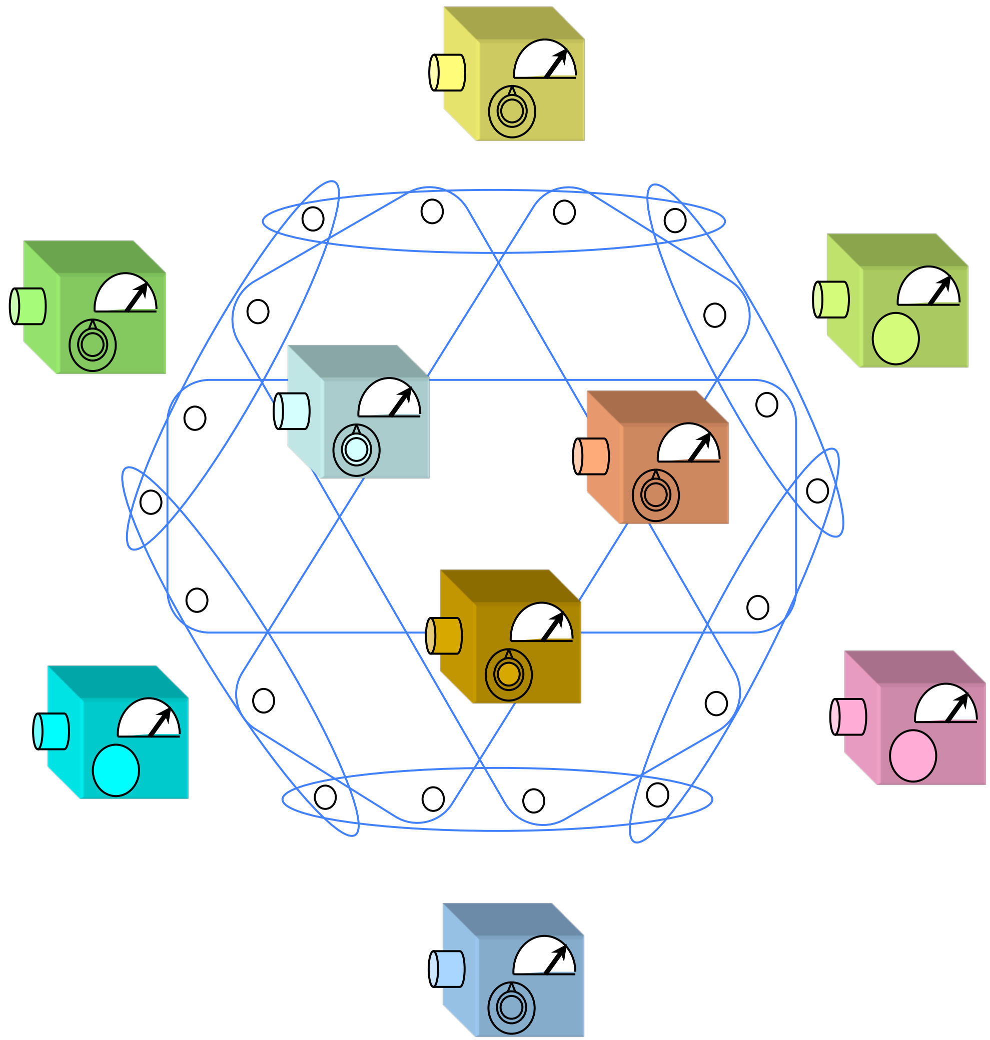
Kunjwal and RWS, PRL 115, 110403 (2015)

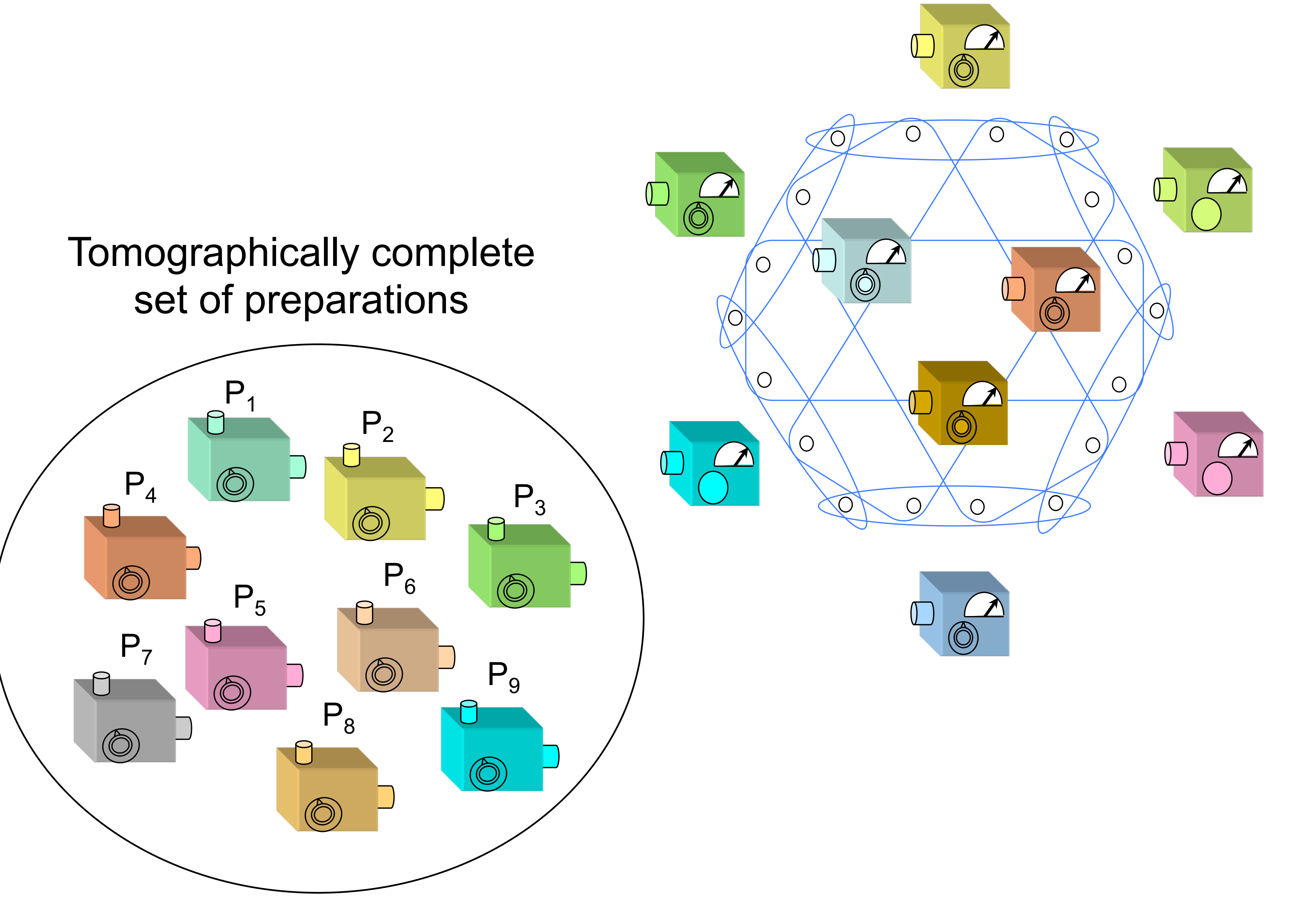
State-independent proof of no-go

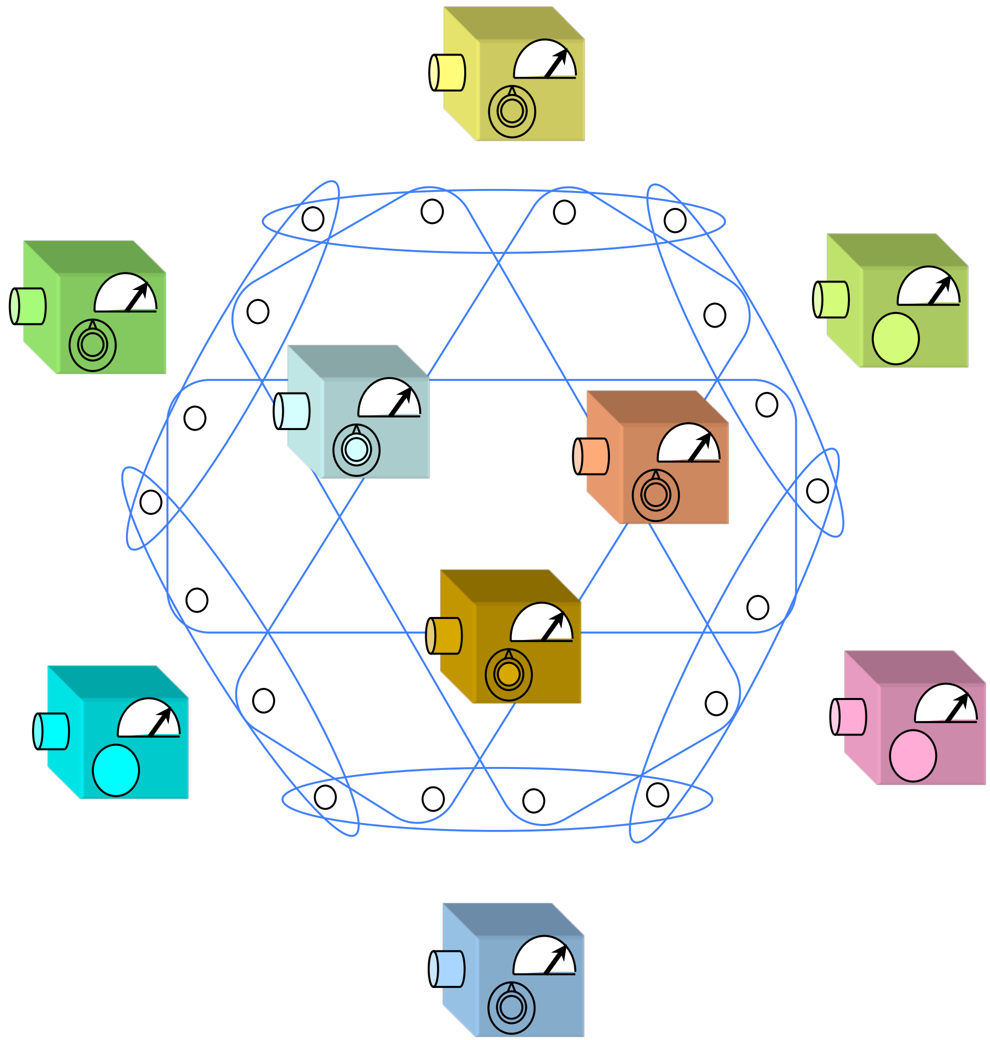
18 ray proof in 4d

Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)

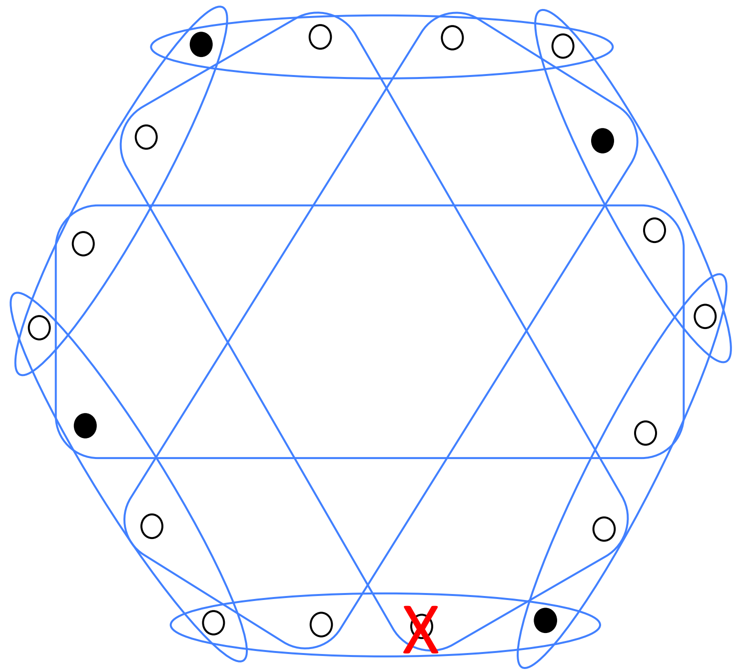





Tomographically complete
set of preparations
P1
P2
P3
P4
P5
P6
P7
P8
P9



No KS noncontextual assignments



○ : value 0
● : value 1

KS-noncontextuality

+

Operational equivalences
in the set of mmts

$\Rightarrow$ Contradiction

KS-noncontextuality

+

Operational equivalences
in the set of mmts

$\Rightarrow$ Contradiction

measurement
noncontextuality
+
outcome
determinism

=

KS-noncontextuality

State independent no-go

measurement noncontextuality

+

outcome determinism

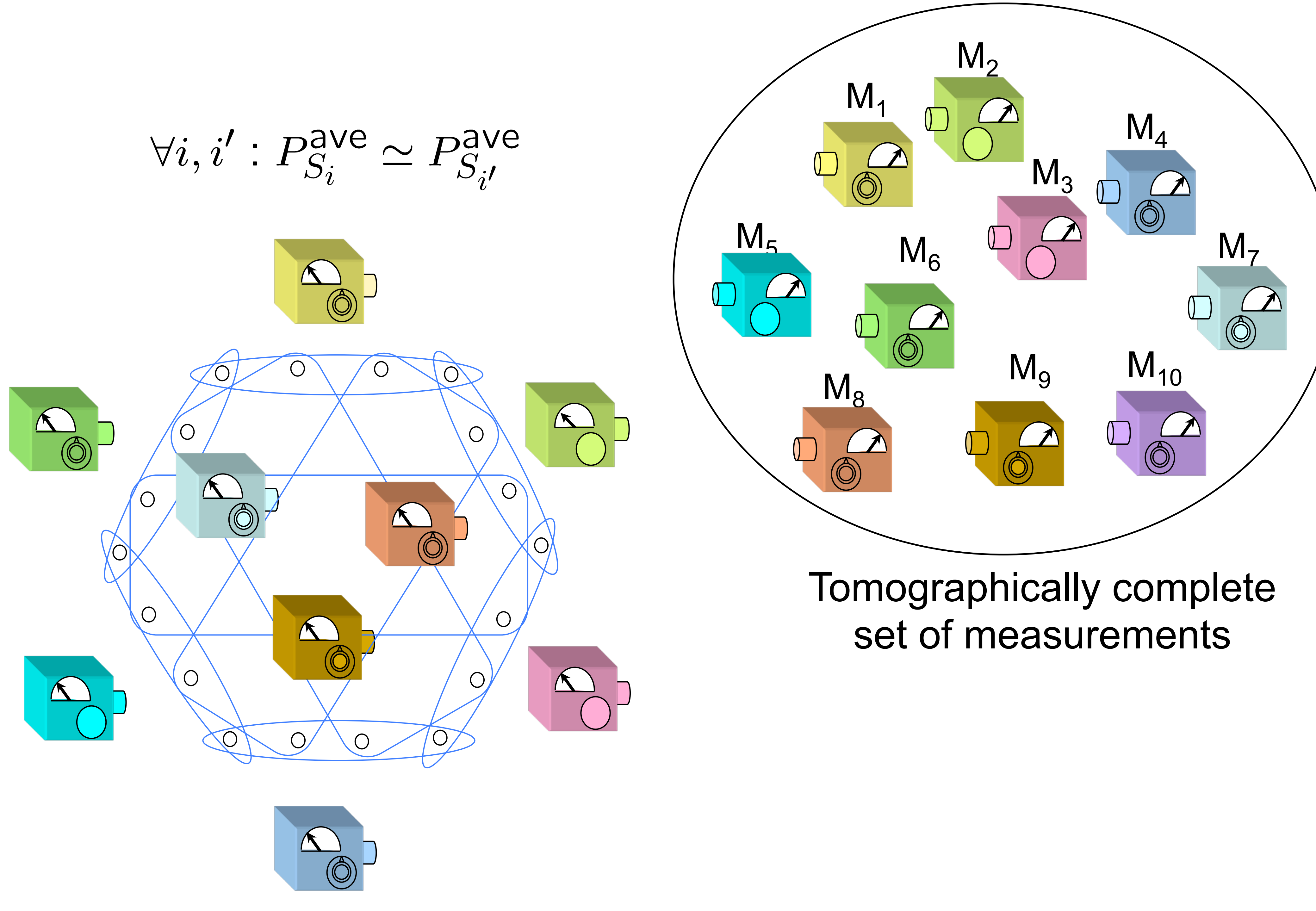
+

Operational equivalences in the sets of mmts

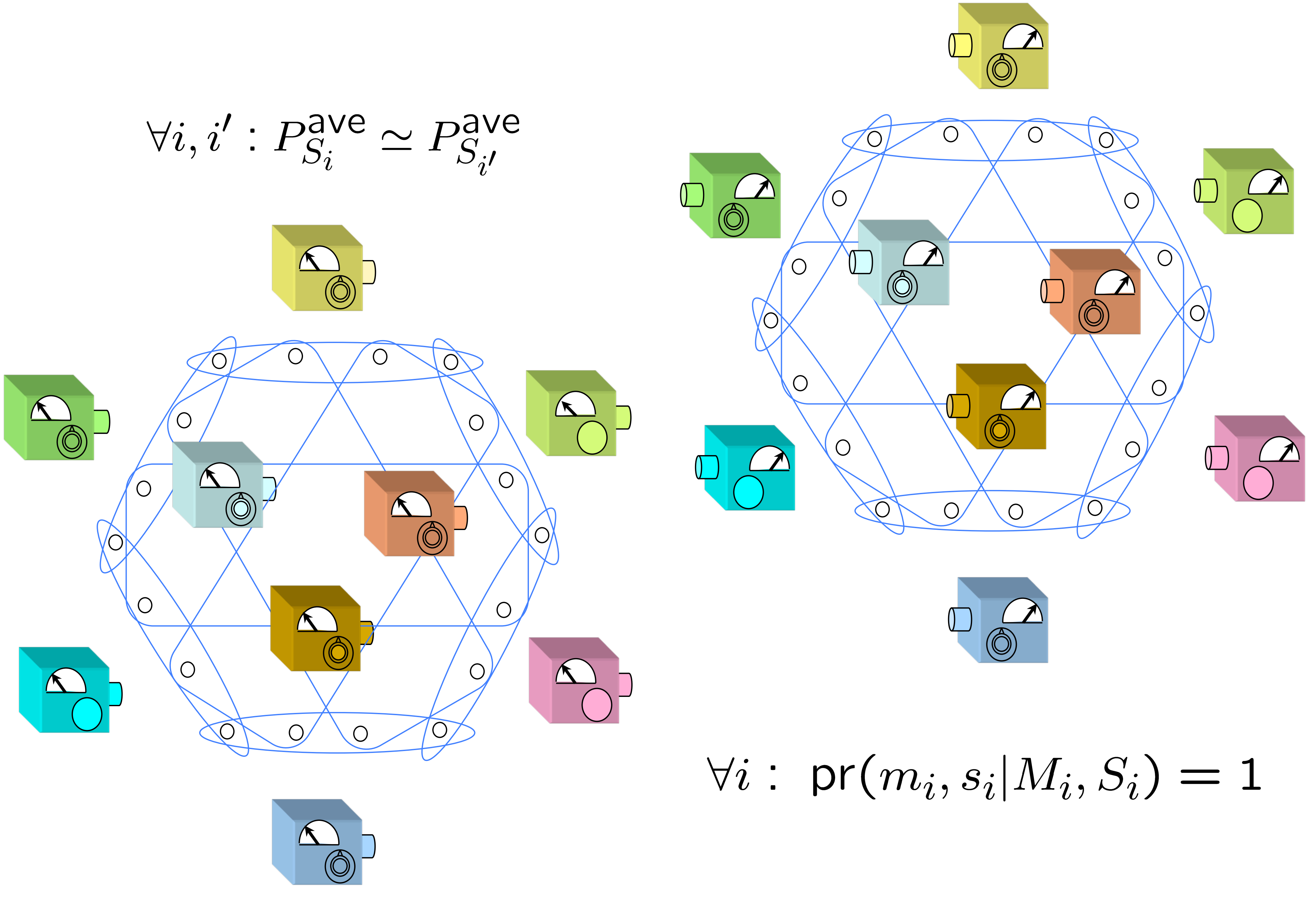
$\Rightarrow$ Contradiction

In face of contradiction, we could give up outcome determinism

Justifying outcome determinism



$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
Tomographically complete
set of measurements


$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$
$\forall i : \ \text{pr}(m_i, s_i | M_i, S_i) = 1$

Operational grounds for assuming outcome determinism

Preparation noncontextuality

+

Operational equivalences in the set of sources

+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Outcome determinism

1. Preparation noncontextuality

$$P_S^{\text{ave}} \simeq P_{S'}^{\text{ave}} \implies \mu(\lambda|S) = \mu(\lambda|S')$$

2. Operational equivalences in the set of sources

$$\forall i, i' : P_{S_i}^{\text{ave}} \simeq P_{S_{i'}}^{\text{ave}}$$

(1) and (2) $\longrightarrow$ $\forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$

$$\begin{aligned}\text{pr}(m_i, s_i|M_i, S_i) &= \sum_{\lambda\in\Lambda} \xi(m_i|M_i, \lambda)\mu(s_i, \lambda|S_i) \\ &= \sum_{\lambda\in\Lambda} \xi(m_i|M_i, \lambda)\mu(s_i|\lambda, S_i)\mu(\lambda|S_i) \\ &= \sum_{\lambda\in\Lambda} \xi(m_i|M_i, \lambda)\mu(s_i|\lambda, S_i)\nu(\lambda)\end{aligned}$$

$$\mathrm{pr}(m_i, s_i | M_i, S_i) = \sum_{\lambda \in \Lambda} \xi(m_i | M_i, \lambda)\mu(s_i | \lambda, S_i)\nu(\lambda)$$

3. Perfect Correlation between outcomes of S_i and M_i for all i

$$\forall i: \ \mathrm{pr}(m_i = s_i | M_i, S_i) = 1$$

$$\longrightarrow \qquad \forall \lambda \in \mathrm{supp}(\nu) : \xi(m_i | M_i, \lambda) \in \{0, 1\}$$

Outcome determinism

Note: precisely parallel to Bell's argument for outcome determinism from perfect correlations and local causality

Justifying outcome determinism

Preparation noncontextuality

+

Operational equivalences in the set of sources

+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Outcome determinism

State independent no-go

measurement noncontextuality

+

outcome determinism

+

Operational equivalences in the sets of mmts

$\Rightarrow$ Contradiction

Justifying outcome determinism

Preparation noncontextuality

+

Operational equivalences in the set of sources

+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Outcome determinism

State independent no-go

Universal noncontextuality

+

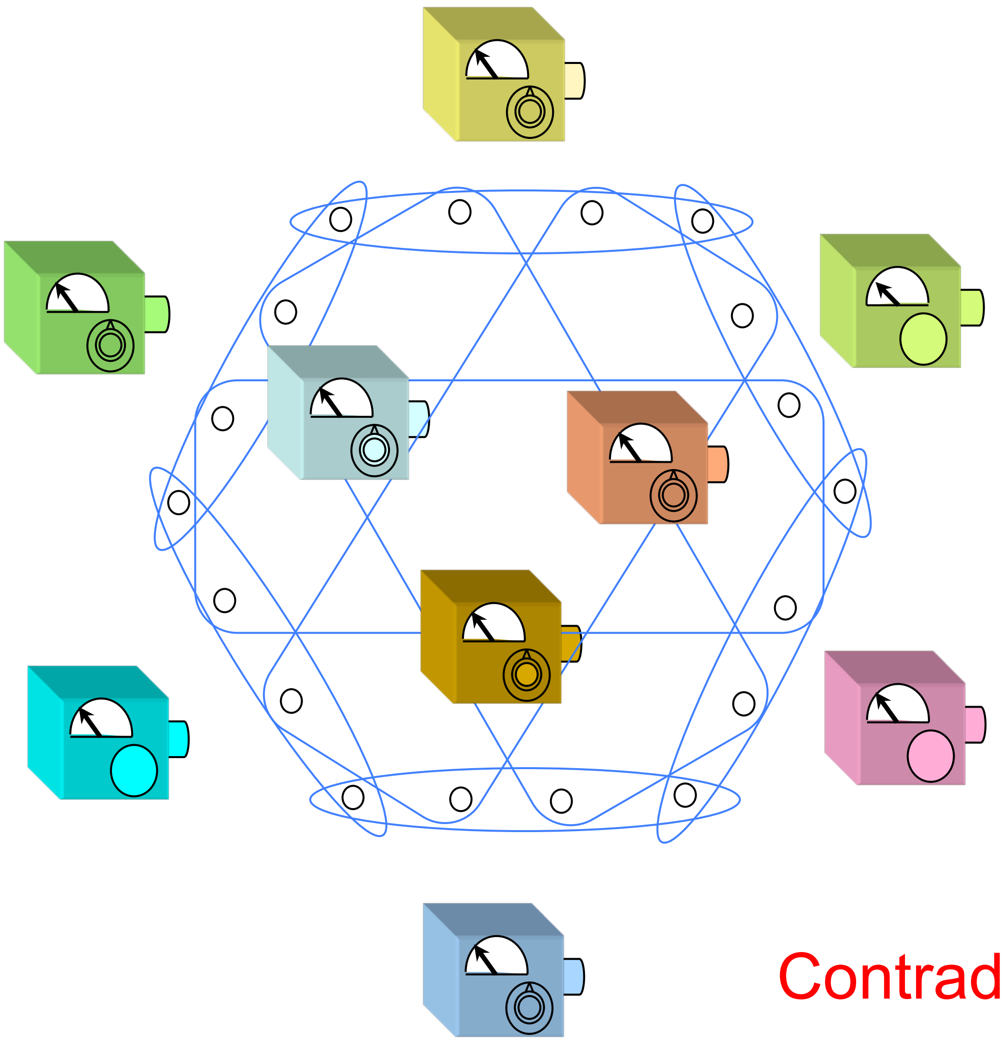
Operational equivalences in the sets of mmts and sources

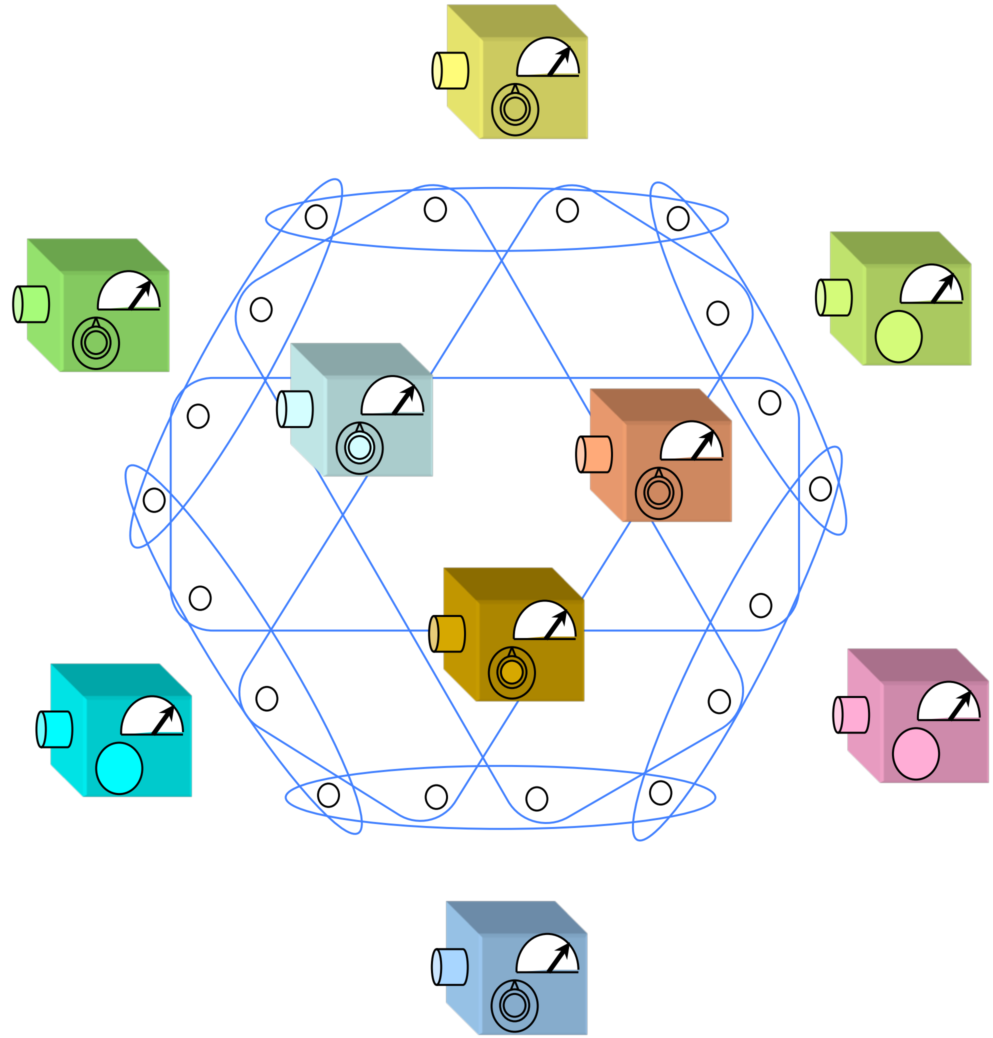
+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Contradiction

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$$





$$\text{Corr} \equiv \frac{1}{9}\sum_{i=1}^{9} \text{pr}(m_i = s_i | M_i, S_i)$$

Contradiction w/ universal noncontextuality

Note: the no-go isn't state independent anymore

Translating this
state-independent no-go result
into
a noncontextuality inequality

State independent no-go

Universal noncontextuality

+

Operational equivalences in the sets of mmts and sources

+

Perfect Correlation between outcomes of S_i and M_i for all i. $\Longleftrightarrow$ $\mathrm{Corr} = 1$

$\Rightarrow$ Contradiction

State independent no-go

Universal noncontextuality

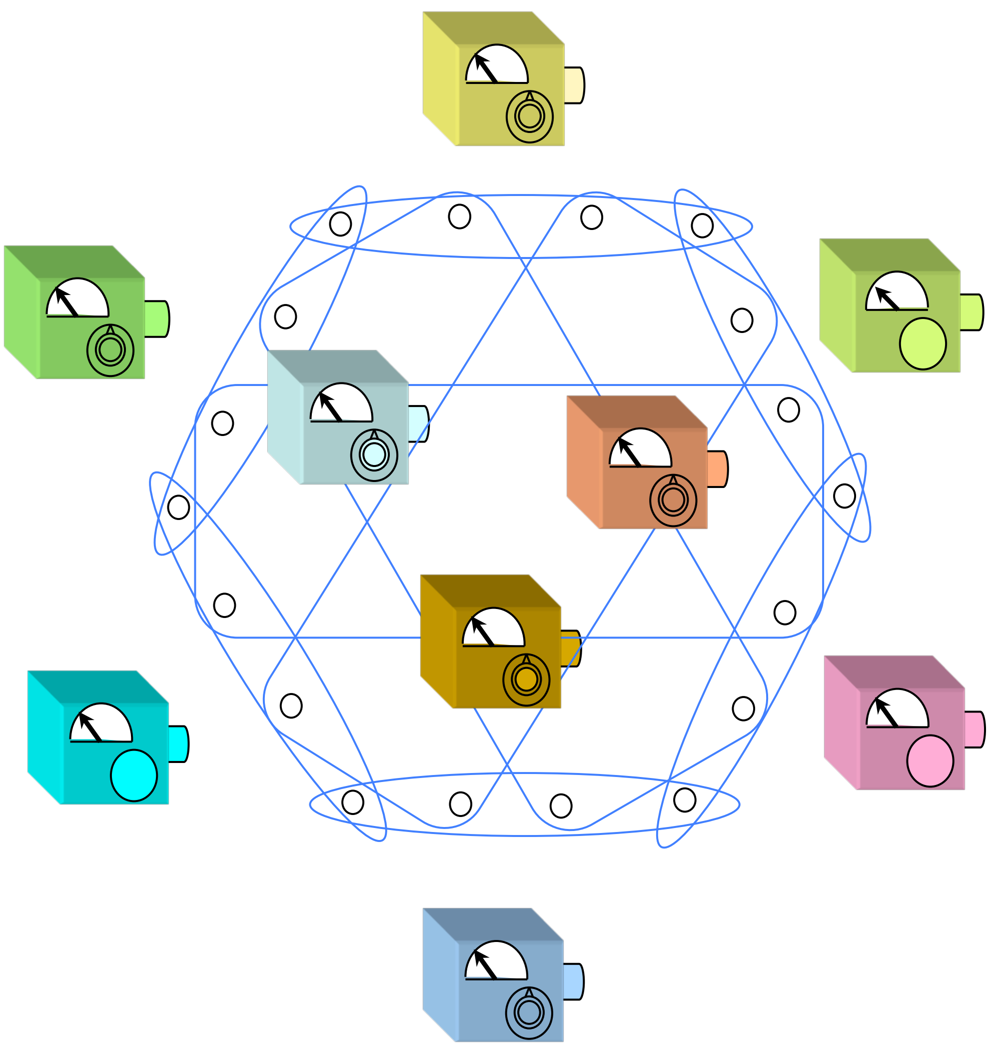
+

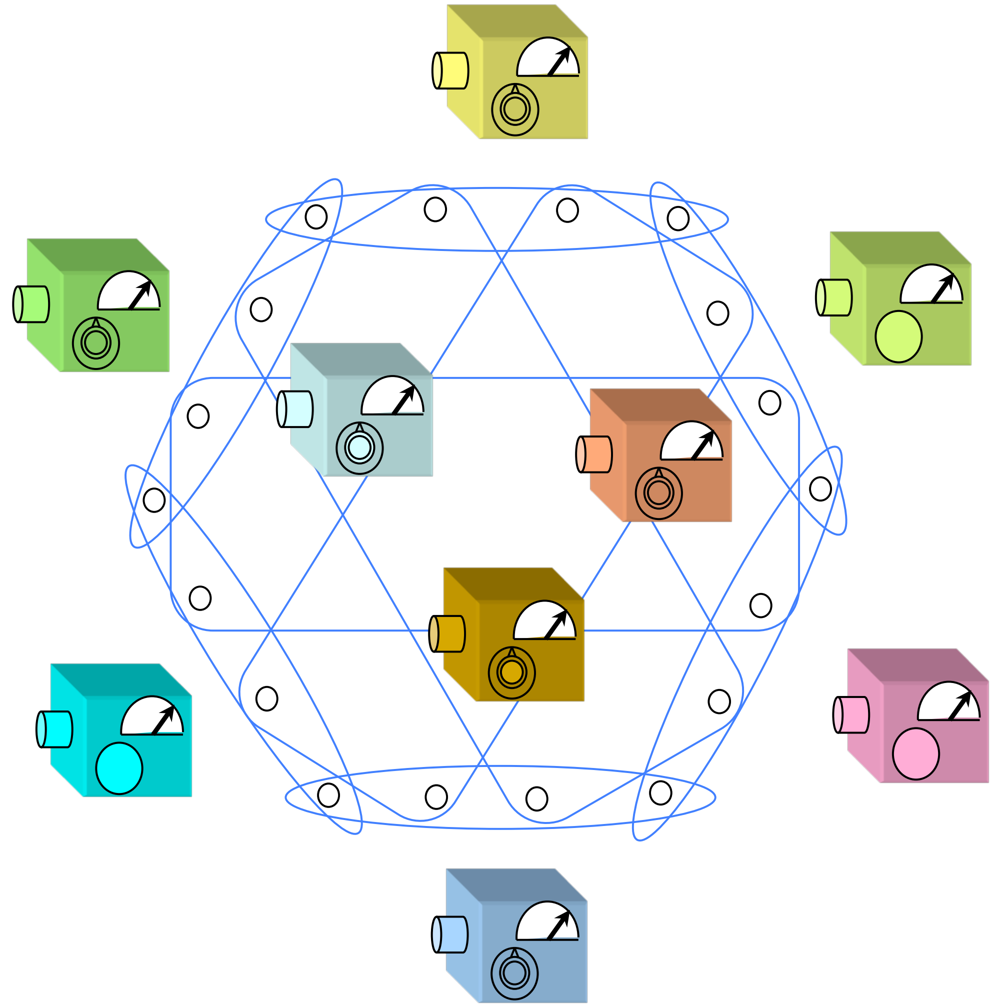
Operational equivalences in the sets of mmts and sources

$$\Downarrow$$

Imperfect Correlation between outcomes of S_i and M_i for all i. $\Leftrightarrow$ $\mathrm{Corr} < 1$

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$$





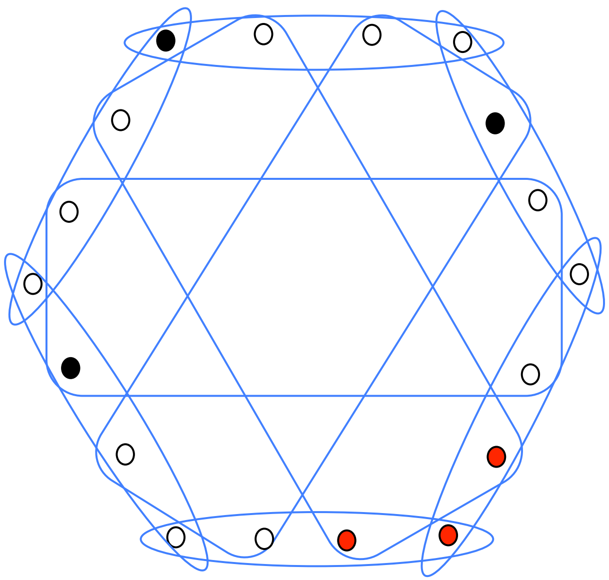
$$\text{Corr} \equiv \frac{1}{9}\sum_{i=1}^{9} \text{pr}(m_i = s_i | M_i, S_i)$$

$$\boxed{\text{Corr} \leq \frac{5}{6}}$$

Noncontextuality inequality

Proof: $$\text{Corr} \equiv \frac{1}{9}\sum_{i=1}^{9} \text{pr}(m_i = s_i | M_i, S_i)$$

$$\text{pr}(m_i = s_i | M_i, S_i) = \sum_{\lambda\in\Lambda}\sum_{m_i,s_i} \delta_{m_i,s_i}\xi(m_i|M_i,\lambda)\mu(s_i|\lambda,S_i)\nu(\lambda)$$



$$\text{Corr} \equiv \sum_{\lambda\in\Lambda} \text{Corr}(\lambda)\nu(\lambda)$$

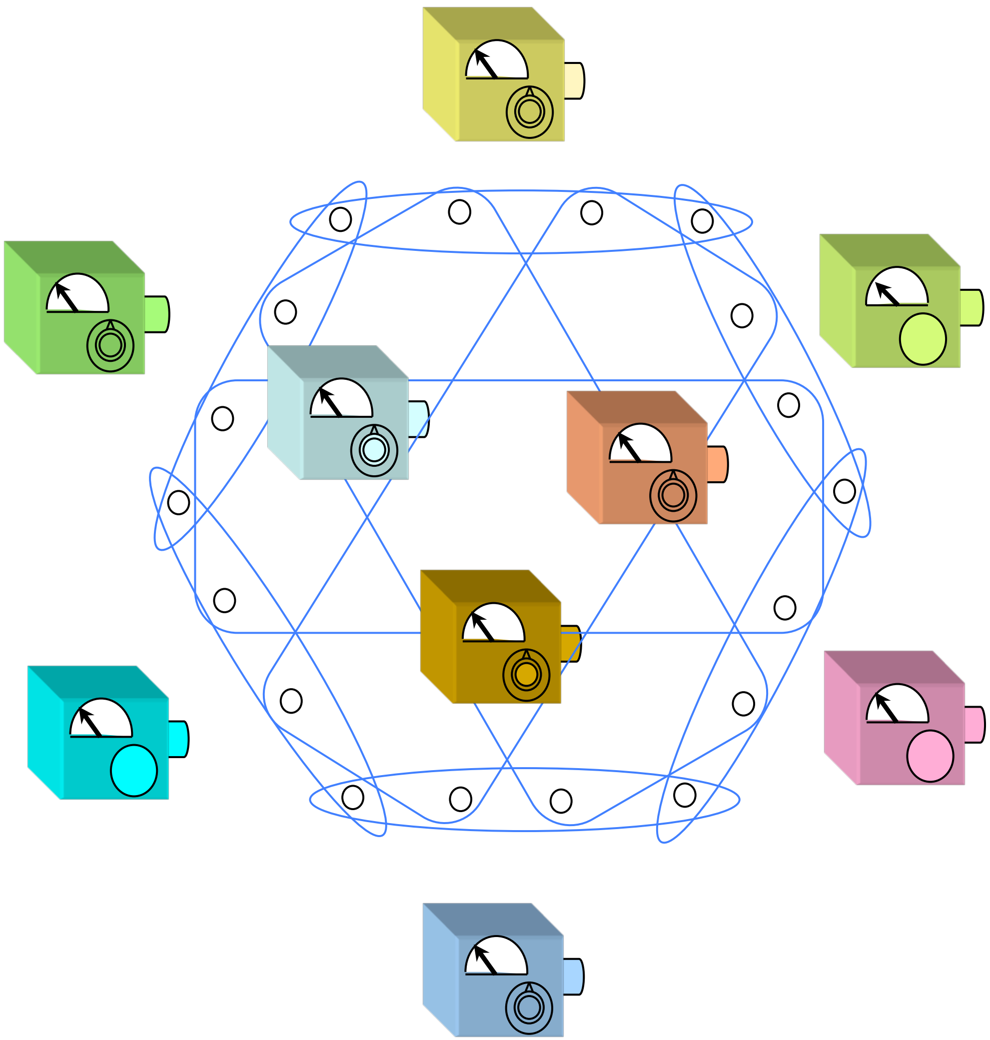
$$\text{Corr}(\lambda) = \sum_{i=1}^{9}\sum_{m_i,s_i} \delta_{m_i,s_i}\xi(m_i|M_i,\lambda)\mu(s_i|\lambda,S_i)$$

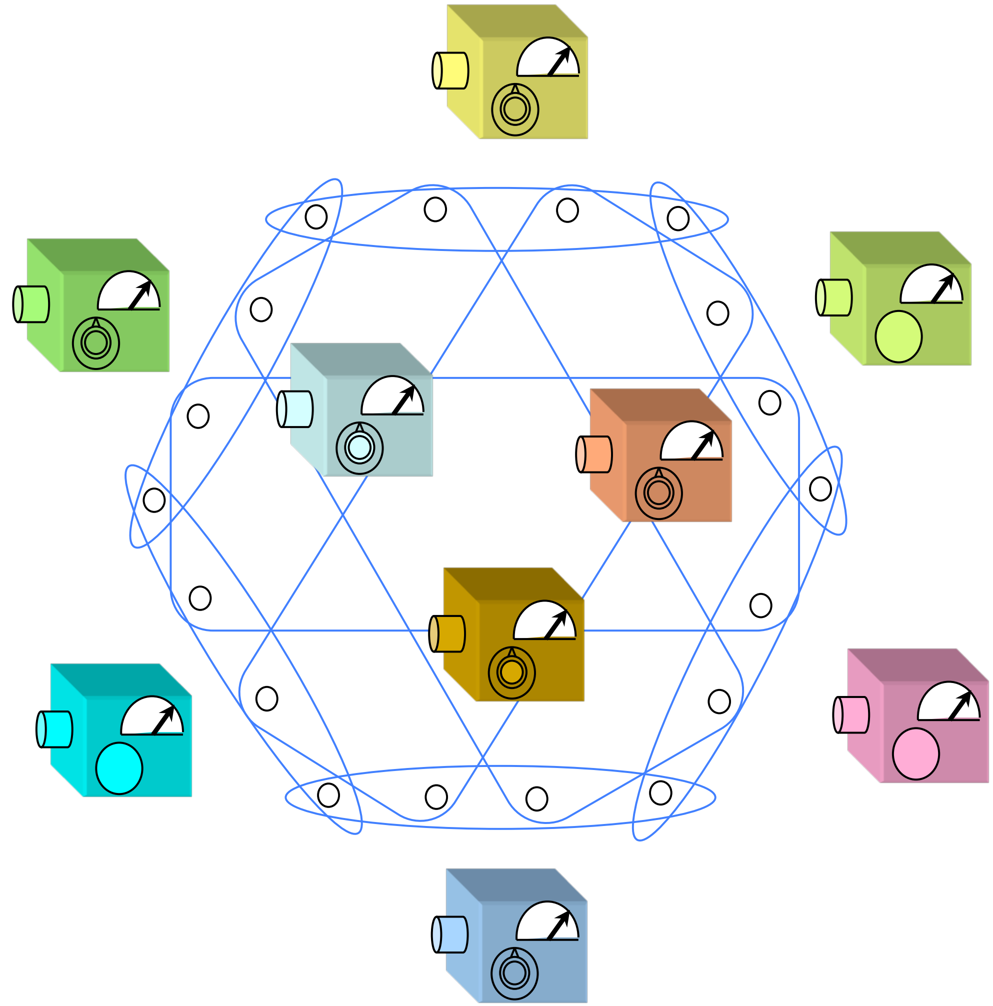
The polytope of indeterministic NC assignments is 9d with 146 vertices

$$\begin{aligned}\text{Corr}(\lambda) &= \frac{1}{9}\left(6\cdot 1 + 3\cdot\frac{1}{2}\right)\\ &= \frac{5}{6}\end{aligned}$$

$$\boxed{\text{Corr} \leq \frac{5}{6}}$$

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$$



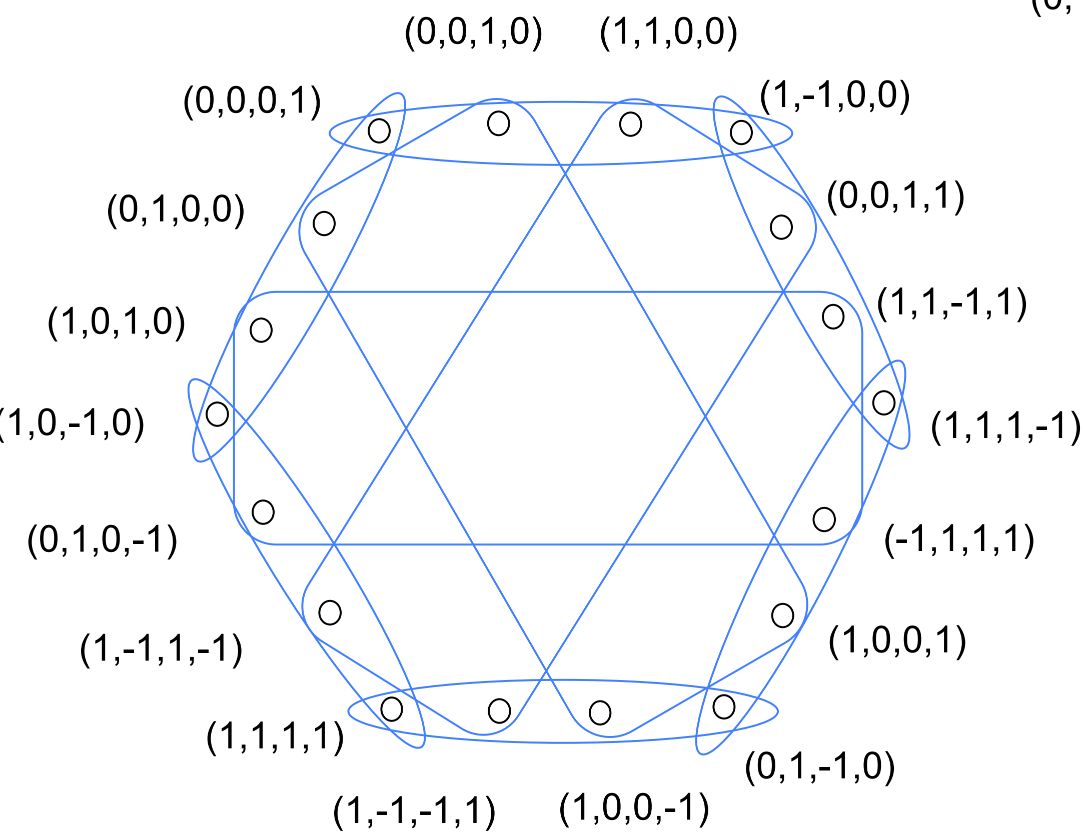


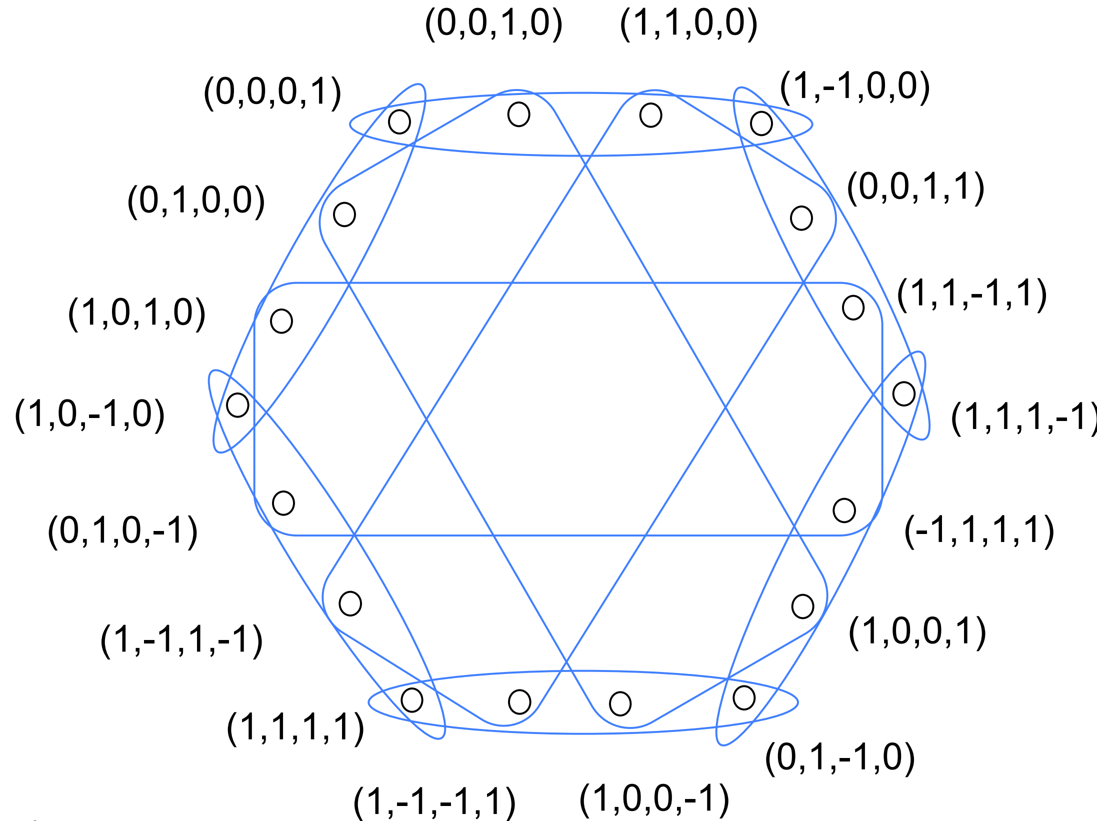
$$\text{Corr} \equiv \frac{1}{9} \sum_{i=1}^{9} \text{pr}(m_i = s_i | M_i, S_i)$$

$$\boxed{\text{Corr} \leq \frac{5}{6}}$$

Noncontextuality inequality

Quantum violation of noncontextuality inequality





$$\mathrm{Corr} = 1 > \frac{5}{6}$$

Noise-robust!

Robust noncontextuality inequality from state-dependent proof of KS theorem

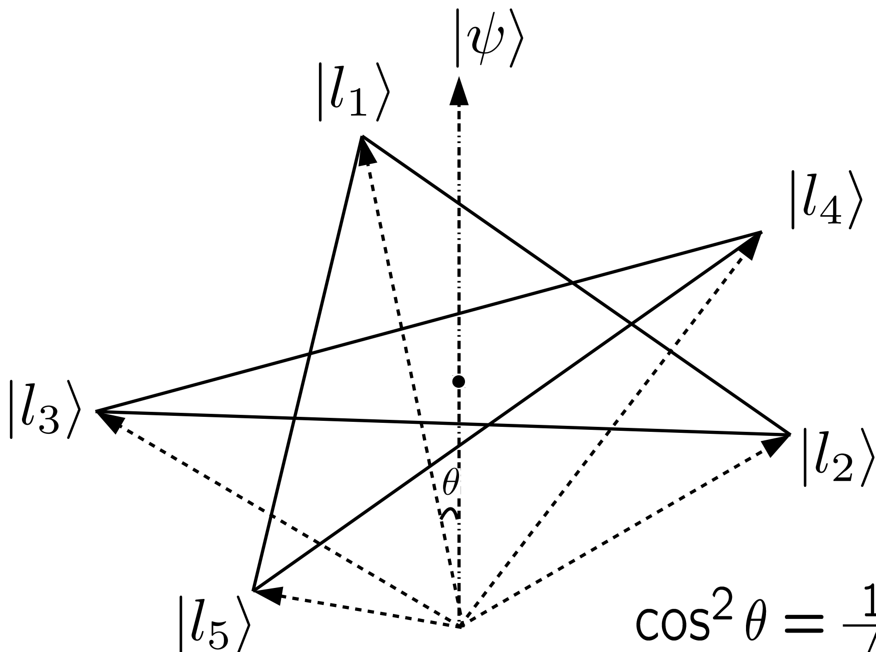
Kunjwal and RWS, forthcoming

State-dependent proof of no-go

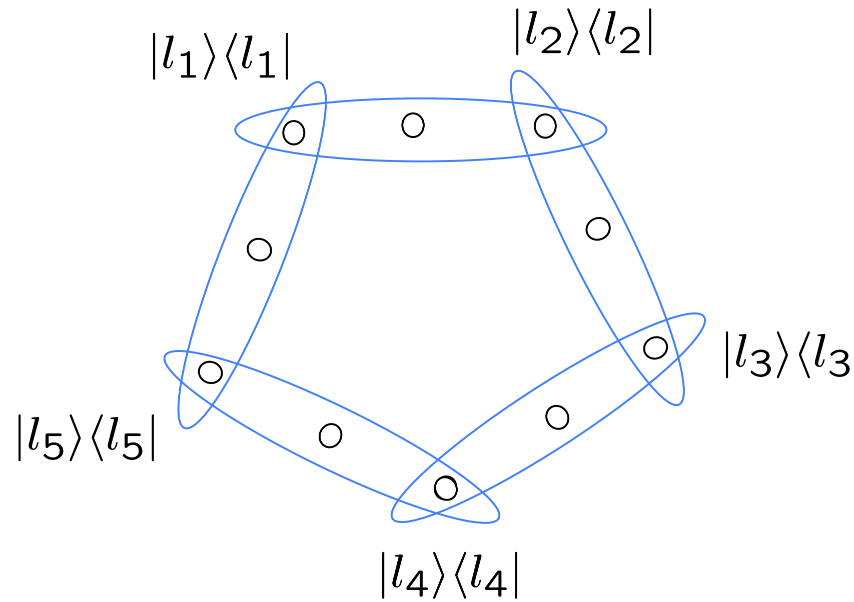
Klyachko, Can, Biniciolu, Shumovsky, PRL 101, 020403 (2008)

Special preparation

$$|\psi\rangle\langle\psi|$$



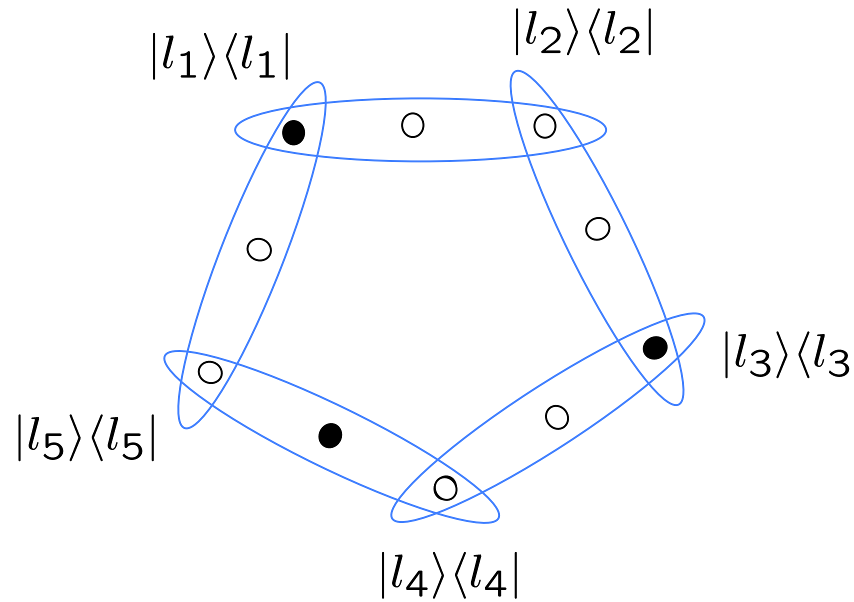
$$\cos^2\theta = \frac{1}{\sqrt{5}}$$



Measurements

Special preparation

$$|\psi\rangle\langle\psi|$$



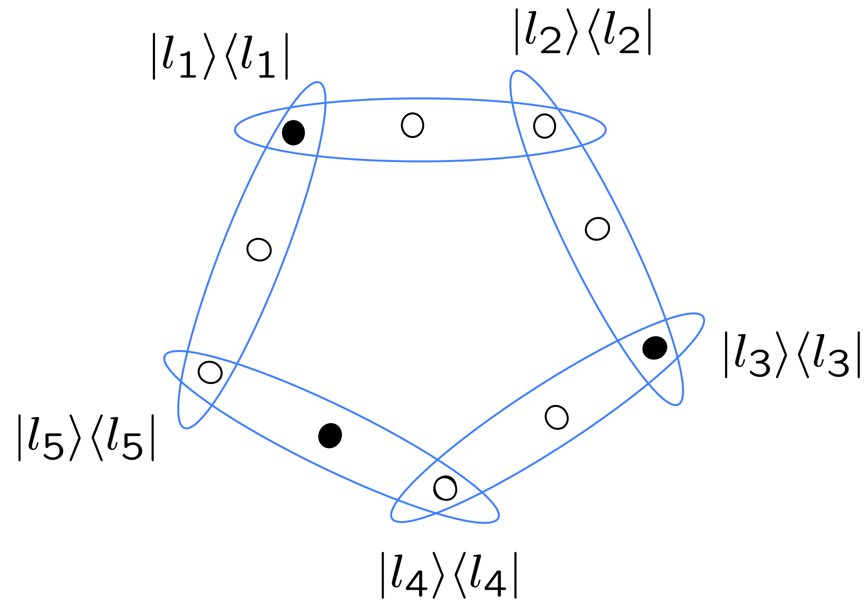
Measurements

R = average over mmts of probability of obtaining outer outcomes

$$R \leq \frac{4}{5}$$ KS inequality

Special preparation

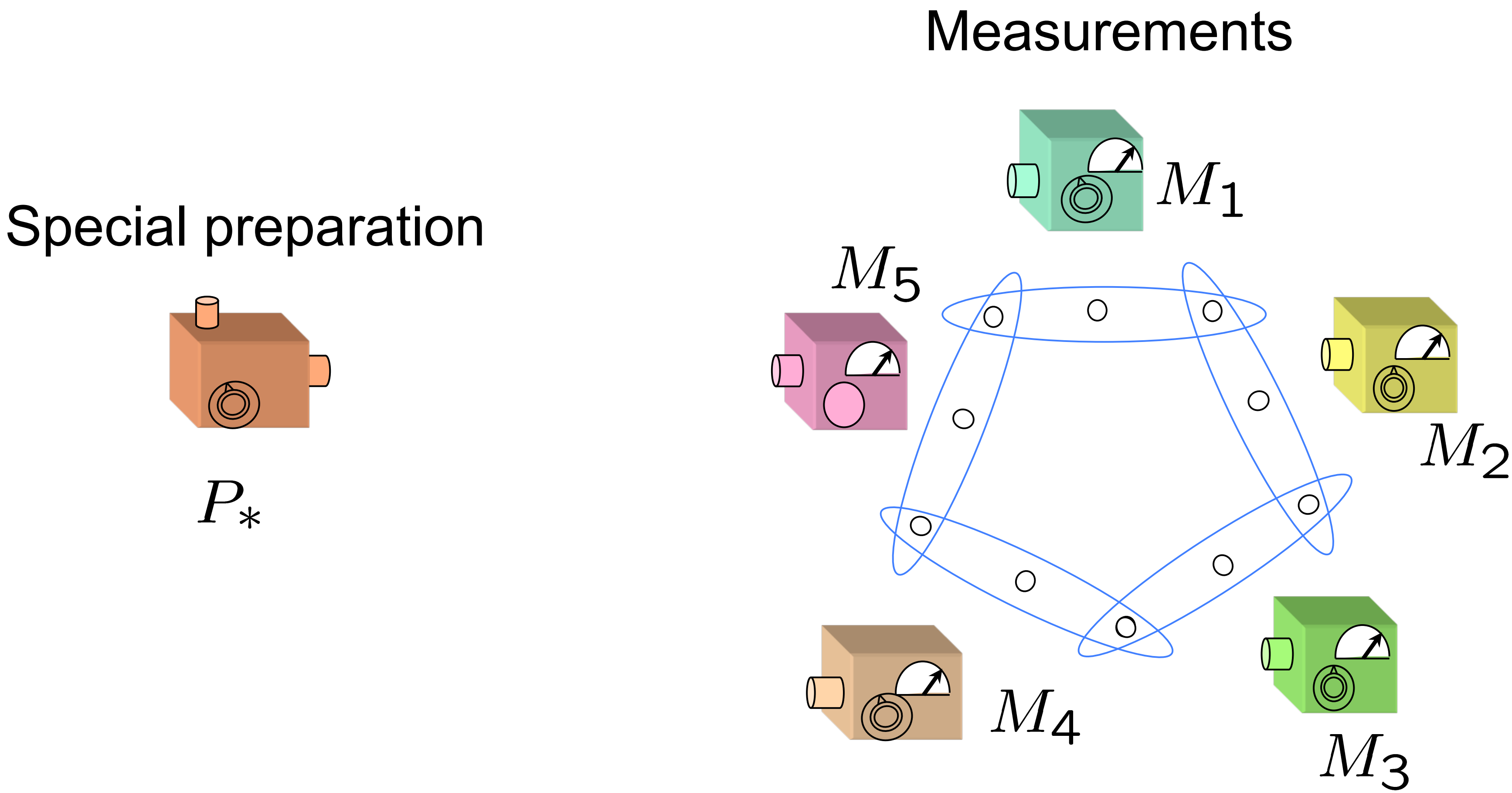
$$|\psi\rangle\langle\psi|$$



Measurements

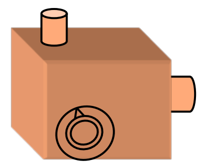
R = average over mmts of probability of obtaining outer outcomes

$$R = \frac{2}{\sqrt{5}} \simeq 0.89$$ Quantum



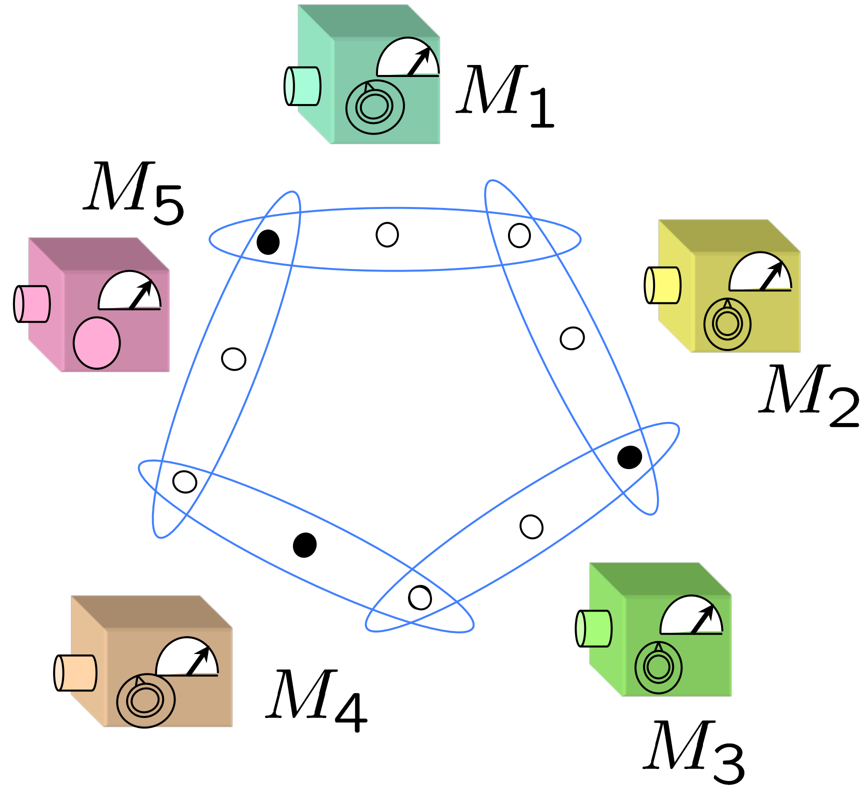
$$R \equiv \frac{1}{5}\sum_{i=1}^{5} \mathrm{pr}(m_i \in \{0,2\}|M_i, P_*)$$

Special preparation



P_*

Measurements



$$R \equiv \frac{1}{5} \sum_{i=1}^{5} \mathrm{pr}(m_i \in \{0,2\} | M_i, P_*)$$

$$R \leq \frac{4}{5}$$ KS inequality

State dependent no-go

KS-noncontextuality

+

Operational equivalences
in the set of mmts

+

Violation of KS-inequality by
statistics of mmts on special
preparation

+

Nonzero prob. of special preparation

$\Longrightarrow$ Contradiction

State dependent no-go

measurement
noncontextuality

+

Outcome determinism

+

Operational equivalences
in the set of mmts

+

Violation of KS-inequality by
statistics of mmts on special
preparation

+

Nonzero prob. of special preparation

$\Longrightarrow$ Contradiction

Justifying outcome determinism

Preparation noncontextuality

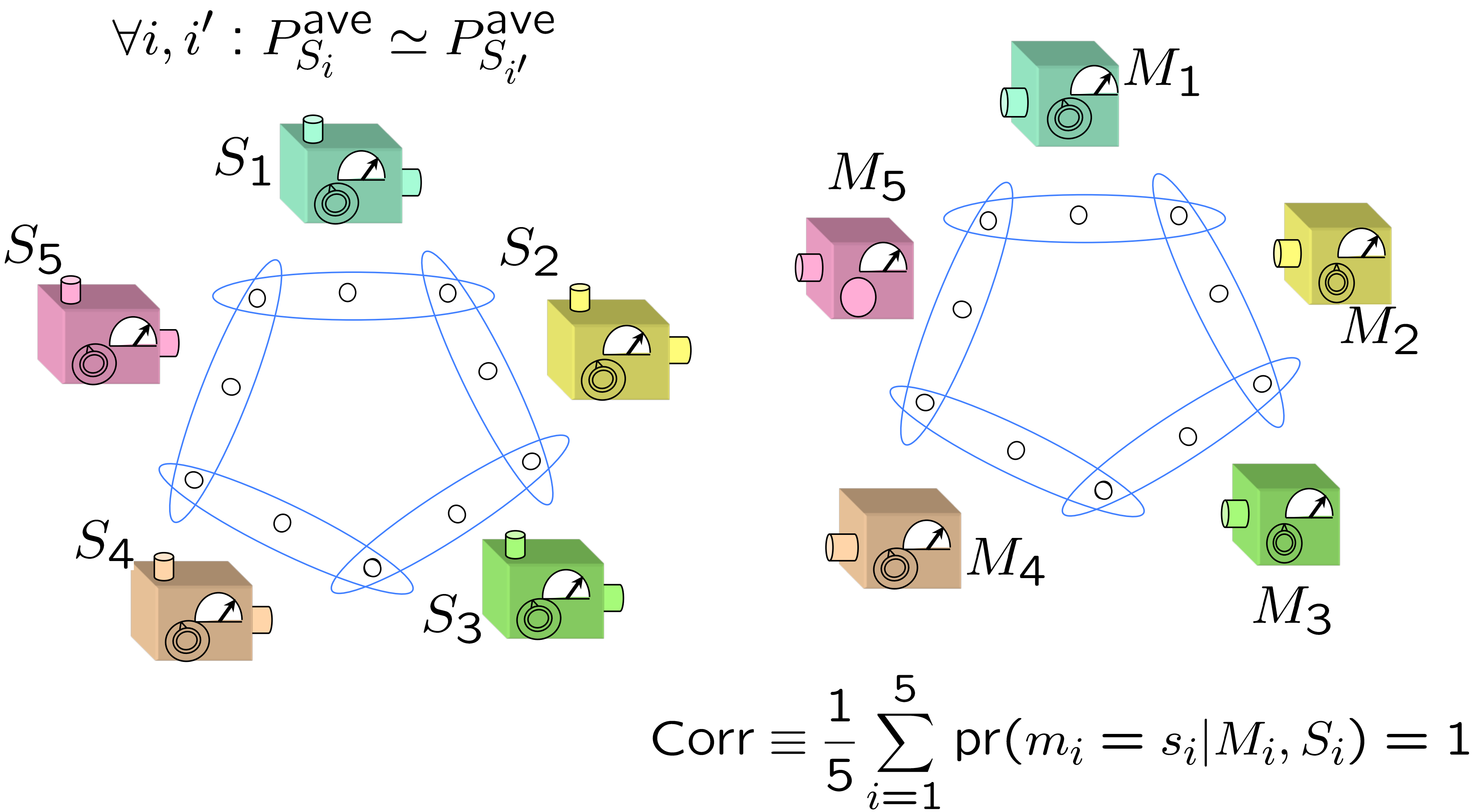
+

Operational equivalences in the set of sources

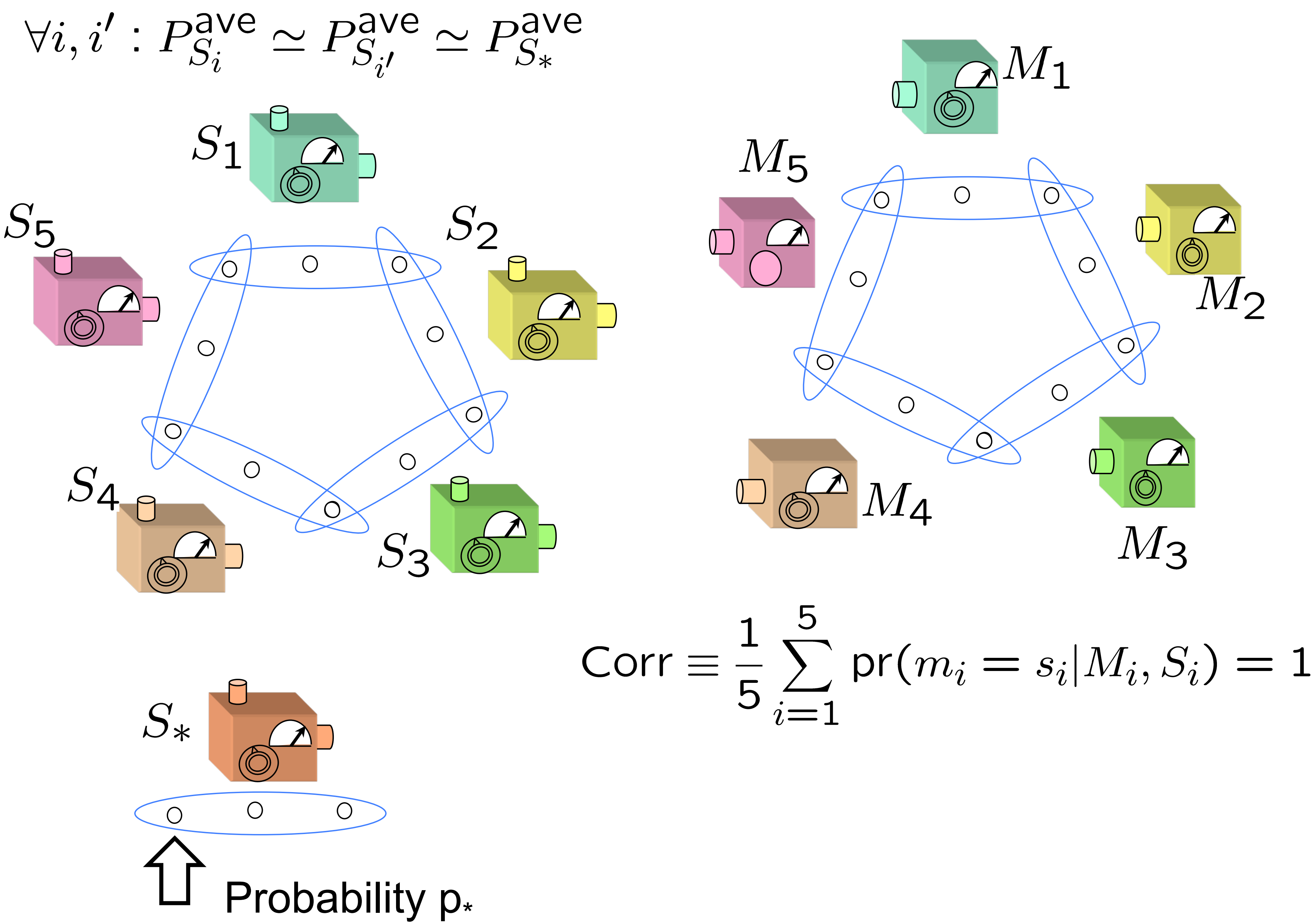
+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Outcome determinism



$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}}$
S_1
S_2
S_3
S_4
S_5
M_1
M_2
M_3
M_4
M_5
$\text{Corr} \equiv \frac{1}{5}\sum_{i=1}^{5} \text{pr}(m_i = s_i | M_i, S_i) = 1$



$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \simeq P^{\text{ave}}_{S_*}$
S_1
S_2
S_3
S_4
S_5
M_1
M_2
M_3
M_4
M_5
$\text{Corr} \equiv \frac{1}{5}\sum_{i=1}^{5} \text{pr}(m_i = s_i | M_i, S_i) = 1$
S_*
Probability p_*

State dependent no-go

measurement noncontextuality

+

Outcome determinism

+

Operational equivalences in the set of mmts

+

Violation of KS-inequality by statistics of mmts on special preparation

+

Nonzero prob. of special preparation

$\Rightarrow$ Contradiction

Justifying outcome determinism

Preparation noncontextuality

+

Operational equivalences in the set of sources

+

Perfect Correlation between outcomes of S_i and M_i for all i.

$\Rightarrow$ Outcome determinism

State dependent no-go

universal noncontextuality

+

Operational equivalences in the set of mmts and sources

+

Perfect Correlation between outcomes of S_i and M_i for all i. $\Leftrightarrow$ $\mathrm{Corr} = 1$

+

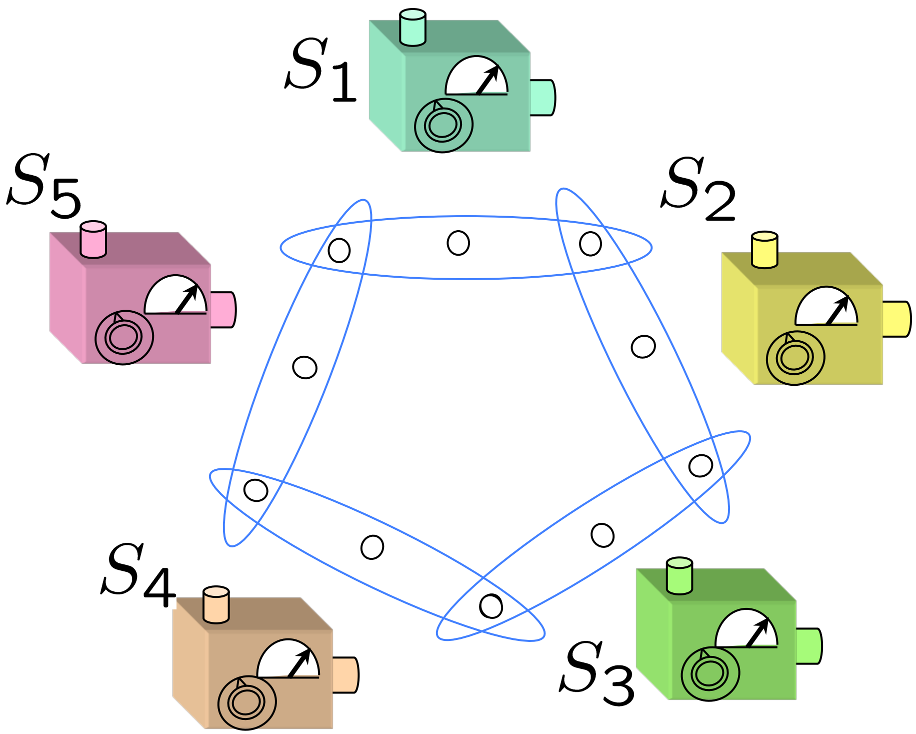
Violation of KS-inequality by statistics of mmts on special preparation $\Leftrightarrow$ $R > \frac{4}{5}$

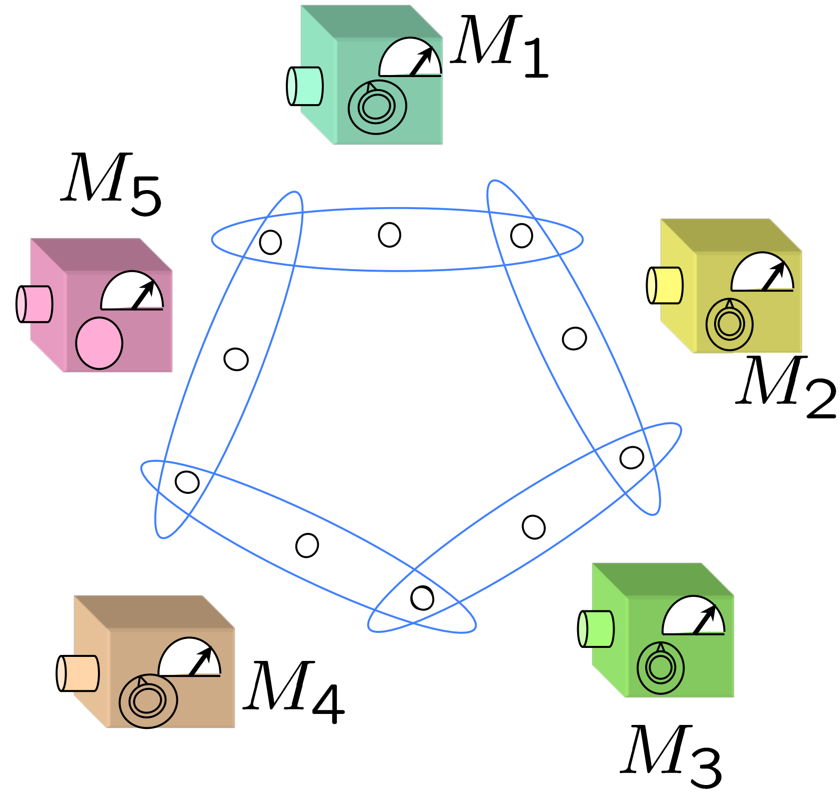
+

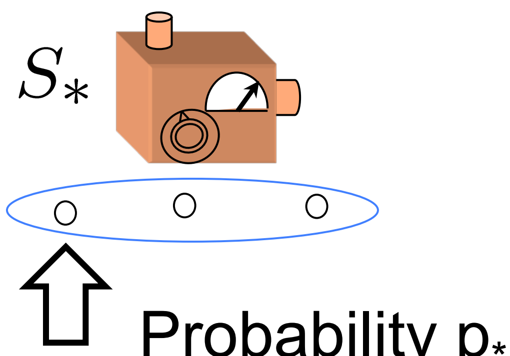
Nonzero prob. of special preparation $\Leftrightarrow$ $p_* > 0$

$\Rightarrow$ Contradiction

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \simeq P^{\text{ave}}_{S_*}$$







$$\boxed{\text{Corr} \leq 1 - p_* \frac{5}{2}\left(R - \frac{4}{5}\right)}$$

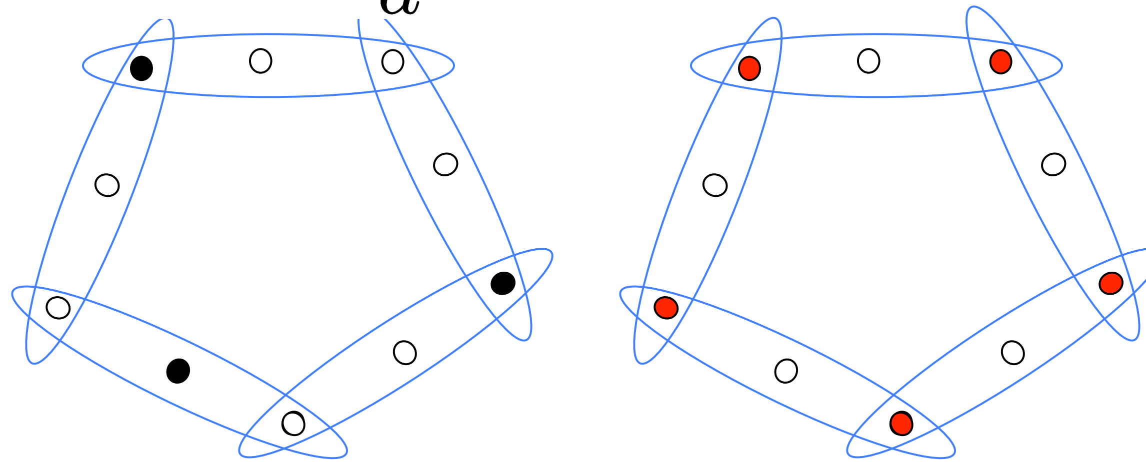
Noncontextuality inequality

Proof:

$$\boxed{\mathrm{Corr} = p_* \mathrm{C}_* + (1 - p_*)}$$

$$\mathrm{C}_* = \sum_\lambda \mathrm{Corr}(\lambda)\mu(\lambda|P_*)$$

$\lambda \in \Lambda_d$ $\lambda \in \Lambda_i$



$$\to \boxed{\mathrm{C}_* \leq 1 - \frac{5}{2}\left(R - \frac{4}{5}\right)}$$

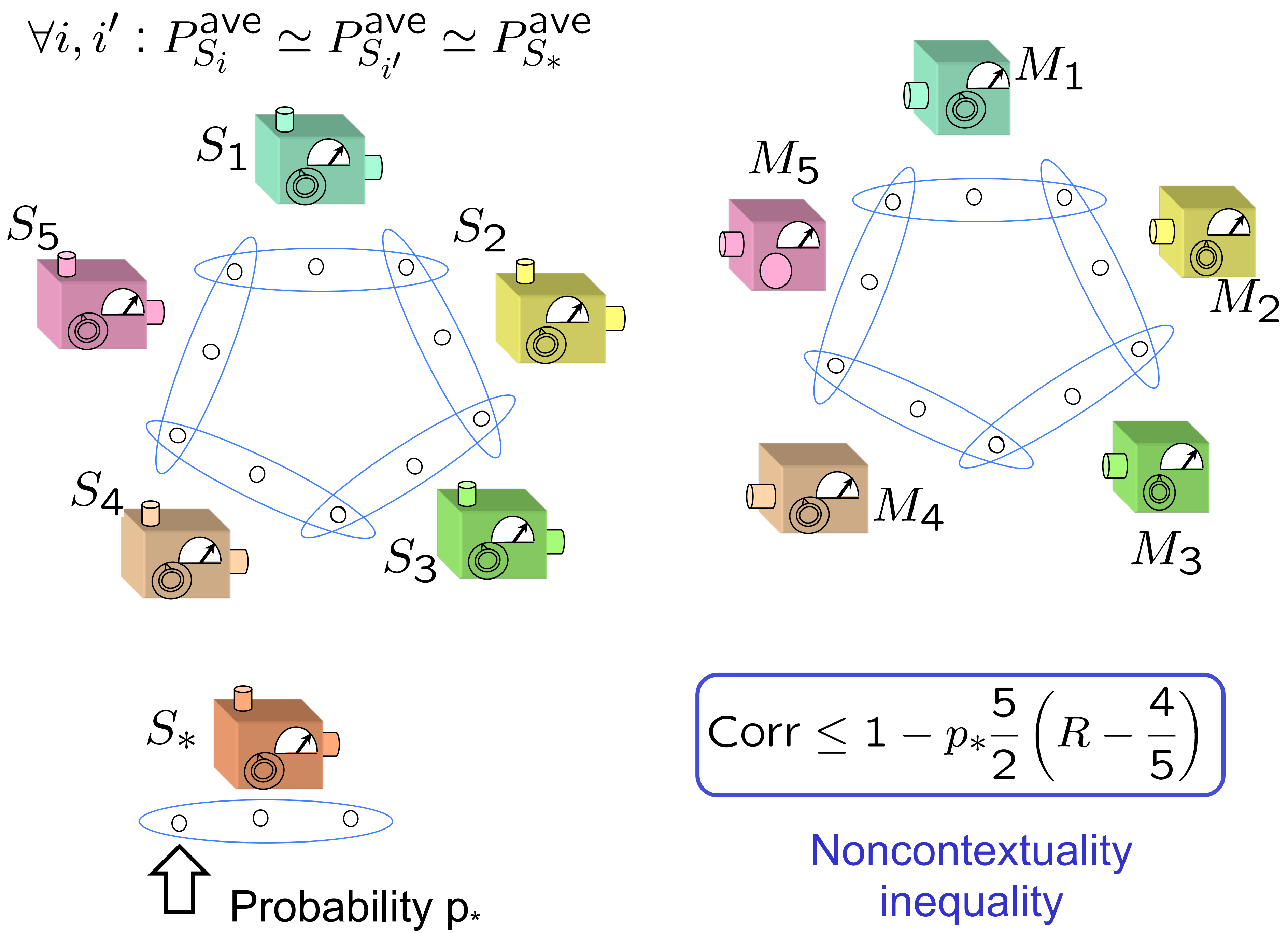
$$\mu_{\mathrm{d/i}} \equiv \sum_{\lambda \in \Lambda_{\mathrm{d/i}}} \mu(\lambda|P_*)$$

$$\mu_\mathrm{d} + \mu_\mathrm{i} = 1$$

$$R = \frac{4}{5}\mu_\mathrm{d} + \mu_\mathrm{i}$$

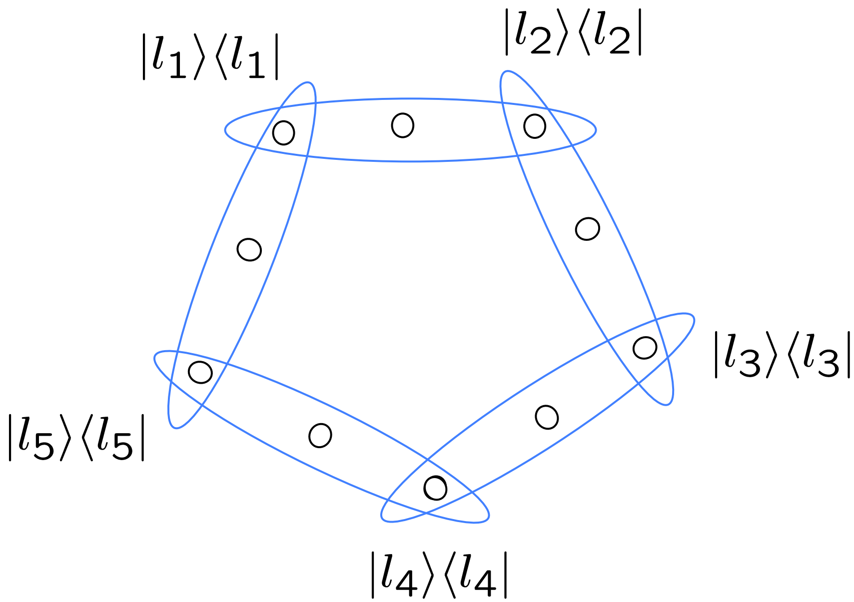
$$\mathrm{C}_* = \mu_\mathrm{d} + \frac{1}{2}\mu_\mathrm{i}$$

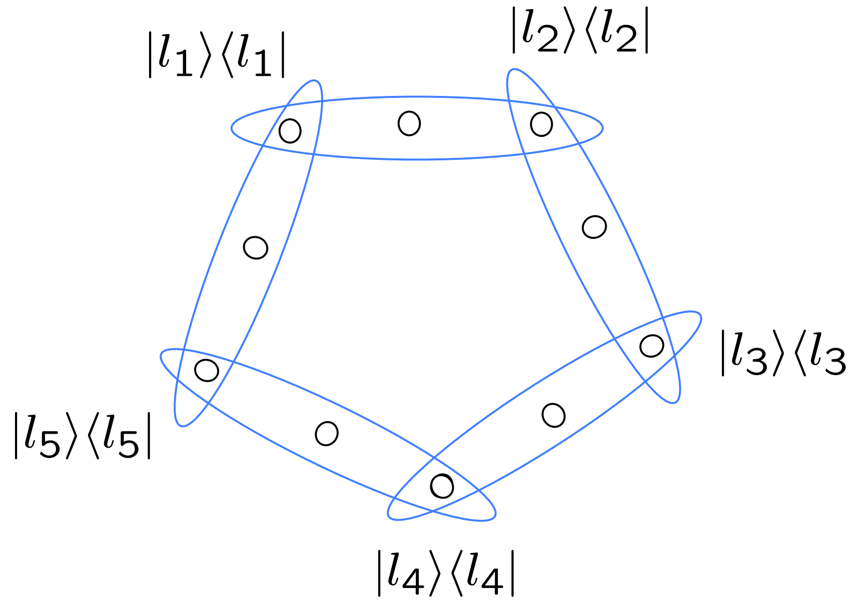
$$\to \boxed{\mathrm{Corr} \leq 1 - p_* \frac{5}{2}\left(R - \frac{4}{5}\right)}$$

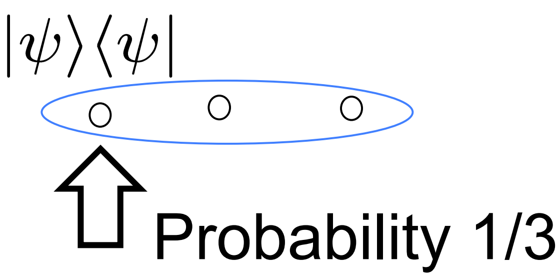


$\forall i, i' : P^{\mathrm{ave}}_{S_i} \simeq P^{\mathrm{ave}}_{S_{i'}} \simeq P^{\mathrm{ave}}_{S_*}$
S_1
S_2
S_3
S_4
S_5
M_1
M_2
M_3
M_4
M_5
S_*
Probability p_*
$\mathrm{Corr} \leq 1 - p_* \frac{5}{2}\left(R - \frac{4}{5}\right)$
Noncontextuality inequality

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \simeq P^{\text{ave}}_{S_*}$$







$$\text{Corr} = 1$$

$$R = \frac{2}{\sqrt{5}} > \frac{4}{5}$$

$$p_* = 1/3 > 0$$

Quantum violation

Noise-robust!

Conclusions

We have a general technique for inferring NC inequalities from state-independent and state-dependent proofs of KS theorem.

These are applicable to unsharp (i.e., noisy) measurements and mixed states and can be tested on experimental data

Contending with noise is critical if one hopes to use contextuality as a resource for information processing

----

One can leverage graph-theoretic results to derive inequalities for arbitrary proofs (forthcoming by Kunjwal)

It is possible to extend these techniques to derive **all** NC inequalities (see Krishna, RWS, Wolfe, arXiv:1704.01153)

References

Contextuality for preparations, transformations and unsharp measurements
RWS, Phys. Rev. A 71, 052108 (2005)

From the Kochen-Specker theorem to noncontextuality inequalities without assuming determinism
Kunjwal and RWS, PRL 115, 110403 (2015)

Robust noncontextuality inequalities from state-dependent proofs of the Kochen-Specker theorem
Kunjwal and RWS, forthcoming