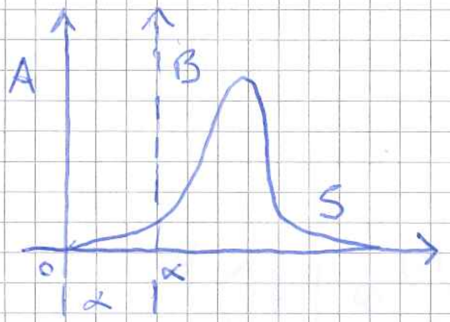
Lecture 1:

- Writing down a state uses implicitly a RF

$$|\psi\rangle = \int dx\, \psi(x)\, |x\rangle$$

position relative to RF

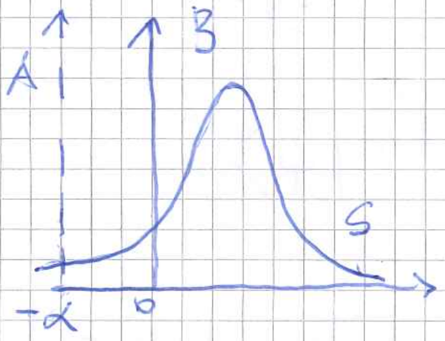


$$\psi^{(A)}(x)$$

$$|\psi\rangle = \int dx\, \psi^{(A)}(x)\, |x\rangle^{(A)} = \int dx\, \psi^{(B)}(x)\, |x+\alpha\rangle^{(B)}$$

(passive transformation)

$$\psi^{(B)}(x) = \psi^{(A)}(x+\alpha)$$

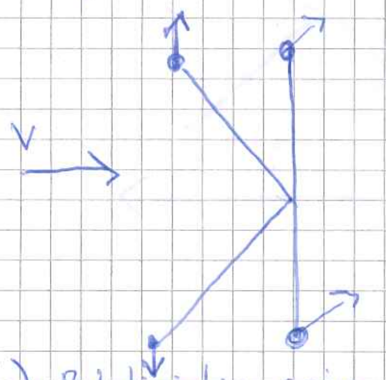


Alternatively: $|\psi\rangle \mapsto e^{i\alpha\hat{p}}\, |\psi\rangle$

(active transformation)

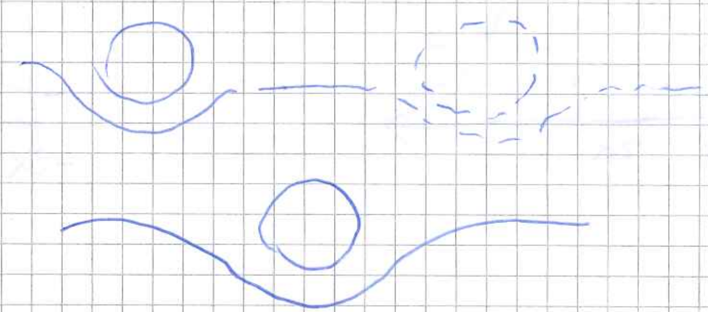
- In practice, A & B are always <u>physical</u> systems, terefore quantum. Can we "jump" into the perspective of a quantum reference frame?

- Why would we? For example, QRF transf. are useful in i) & ii)

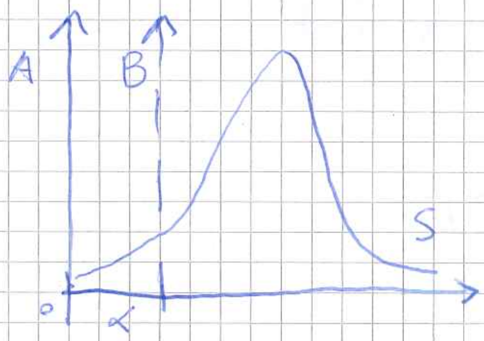


i) Relativistic spin

ii) Mass in superposition



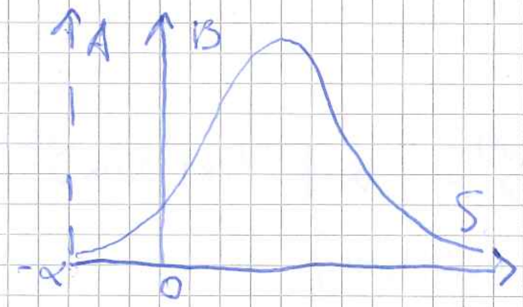
- Defining QRF transformations



A

$$|\Psi\rangle^{(A)}_{BS} = |\alpha\rangle^{(A)}_{B} \otimes |\psi\rangle^{(A)}_{S}$$

Ref [1]



B

$$|\tilde{\Psi}\rangle^{(B)}_{AS} = |-\alpha\rangle^{(B)}_{A} \otimes \underbrace{|\tilde{\psi}\rangle^{(B)}_{S}}$$

$$= e^{i\alpha \hat{p}_S} |\psi\rangle^{(A)}_{S}$$

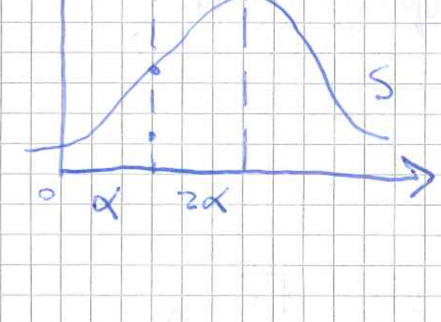
$$|\tilde{\Psi}\rangle^{(B)}_{AS} = \underbrace{\mathbb{P}_{A\to B}\, e^{i\hat{x}_B \hat{p}_S}}_{\text{QRF transformation!}} |\Psi\rangle^{(A)}_{BS} \;;\; \hat{\mathbb{P}}_{A\to B} |x\rangle_B = |-x\rangle_A$$

$$\hat{S}_{A\to B} : \mathcal{H}_B \otimes \mathcal{H}_S \longrightarrow \underbrace{\mathcal{H}_A \otimes \mathcal{H}_S}$$

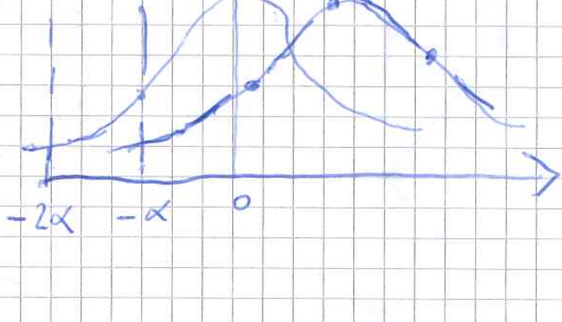
$$\boxed{\hat{S}_{A\to B} = \mathbb{P}_{A\to B}\, e^{+i\hat{x}_B \hat{p}_S}}$$

→ more precision later.

- Jumping onto a QRF in superposition



A   B



A

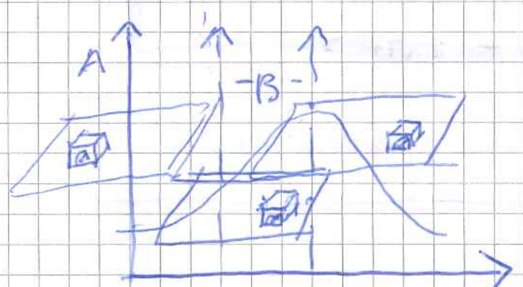
$$|\Psi\rangle^{(A)}_{BS} = \frac{1}{\sqrt{2}}\left(|\alpha\rangle^{(A)}_{B} + e^{i\phi}|2\alpha\rangle^{(A)}_{B}\right)\otimes|\psi\rangle^{(A)}_{S}$$

$$|\tilde{\Psi}\rangle^{(B)}_{AS} = P_{A\to B}\, e^{i\hat{x}_B\hat{p}_S}\, |\Psi\rangle^{(A)}_{BS}$$

$$= \frac{1}{\sqrt{2}}\left(|-\alpha\rangle^{(B)}_{A}\otimes e^{i\alpha\hat{p}_S}|\psi\rangle^{(B)}_{S} + e^{i\phi}|-2\alpha\rangle\, e^{2i\alpha\hat{p}_S}|\psi\rangle^{(B)}_{S}\right)$$

Entanglement is QRF-dependent! (more precision later)

- Two comments:
  - We assume infinite resources!



  - Nothing special about position basis

$$\hat{S}^{p}_{A\to B} = P_{A\to B}\, e^{i\hat{p}_B\hat{x}_S}$$

- States transform as

$$|\Psi\rangle \longmapsto S|\Psi\rangle$$

Operators transform as

$$M \longmapsto S M S^{\dagger}$$

Note:

$$P_{A\to B}\, e^{i\hat{x}_B\hat{p}_S}\, \mathbb{1}_B\otimes M_S\, e^{-i\hat{x}_B\hat{p}_S} P^{\dagger}_{AB}$$

is in general **not** of the form

$$\mathbb{1}_A\otimes\tilde{M}_S$$

(more on this later)

- Transformation of the Hamiltonian

Let V be a (time dependent in gral.) unitary

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$iV\frac{d}{dt}|\psi\rangle = VHV^{\dagger}V|\psi\rangle$$

$$i\frac{d}{dt}\underbrace{V|\psi\rangle}_{=:|\varphi\rangle} = \left(VHV^{\dagger} + i\frac{dV}{dt}V^{\dagger}\right)\underbrace{V|\psi\rangle}_{=:|\varphi\rangle}$$

$$\boxed{\hat{H}^{(B)}_{S} = \hat{S}_{A\to B}\,\hat{H}^{(A)}_{S}\,\hat{S}^{\dagger}_{A\to B} + i\frac{d\hat{S}_{A\to B}}{dt}\hat{S}^{\dagger}_{A\to B}}$$

- Equivalence principle for superposition of accelerations

Free particle Hamiltonian

$$H^{(A)}_{S} = \frac{\hat{p}_S^{\,2}}{2m}$$

$$H^{(A)}_{B} : \text{trivial}$$

$$\hat{S}_{A\to B} = \hat{W}(t)\hat{V}(t)$$

$$W(t) = \mathbb{P}e^{-im\left(\hat{\ddot{\xi}}_B \hat{X}_S + \frac{1}{2}\int^{t}\dot{\xi}^2_B\right)}$$

$$V(t) = \mathbb{P}e^{i\hat{\dot{\xi}}_B \hat{P}_S}$$

$$\mathbb{P}|\xi\rangle_B = |-\xi\rangle_A$$

Assume:

- B is a semiclassical state s.t. its trajectory is well defined
- B can be in a "cat state" superposition of trajectories

$$\frac{1}{\sqrt{2}}\left(|\xi_1(t)\rangle + |\xi_2(t)\rangle\right)$$

$\rightarrow$ $\xi(t)$ & $\xi(t)$ are $\underbrace{\text{well}}_{\text{approx.}}$ defined

- Promote $\xi(t)$ (+ time derivatives) to quantum operators

$$H' := VHV^\dagger + i\frac{dV}{dt}V^\dagger$$

Ref[2]

$$VHV^\dagger = H = \frac{p_s^2}{2m}$$

$$i\frac{dV}{dt}V^\dagger = -i\frac{d}{dt}i\dot{\hat{\xi}}_A\hat{p}_s = \dot{\hat{\xi}}_A\hat{p}_s$$

using $P e^{i()} = Pe^{i()}P^\dagger P = e^{iP()P^\dagger}P$

$$\boxed{H' = \frac{\hat{p}_s^2}{2m} + \dot{\hat{\xi}}_A\hat{p}_s}$$

$$H'' := WH'W^\dagger + i\frac{dW}{dt}W^\dagger$$

$$i\frac{dW}{dt}W^\dagger = i\frac{d}{dt}\left[-im\left(-\dot{\hat{\xi}}_A X_s + \frac{1}{2}\int^t\dot{\hat{\xi}}_A^2\right)\right]$$

$$= -\ddot{\hat{\xi}}_A m\hat{X}_s + \frac{m}{2}\dot{\hat{\xi}}_A^2$$

$$W\frac{p_s^2}{2m}W^\dagger = \frac{\hat{p}_s^2}{2m} + \left[im\dot{\hat{\xi}}_A X_s, \frac{p_s^2}{2m}\right] + \frac{1}{2}\left[im\dot{\hat{\xi}}_A\hat{X}_s, \left[im\dot{\hat{\xi}}_A X_s, \frac{p_s^2}{2m}\right]\right]$$

$$= \frac{\hat{p}_s^2}{2m} - \dot{\hat{\xi}}_A\hat{p}_s + \frac{1}{2}\left[im\dot{\hat{\xi}}_A\hat{X}_s, -\dot{\hat{\xi}}_A\hat{p}_s\right]$$

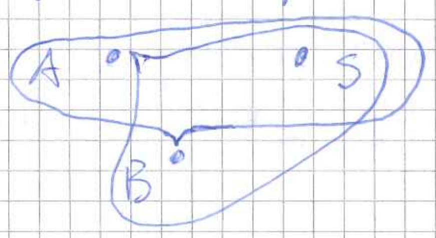
$$= \frac{\hat{p}_s^2}{2m} - \dot{\hat{\xi}}_A\hat{p}_s + \frac{m}{2}\dot{\hat{\xi}}_A^2$$

$$W\dot{\hat{\xi}}_A\hat{p}_s W^\dagger = \dot{\hat{\xi}}_A\hat{p}_s - m\dot{\hat{\xi}}_A^2$$

$$H'' = \underbrace{W\frac{\hat{p}_s^2}{2m}W^\dagger}_{\frac{p^2}{2m} - \dot{\xi}p + \frac{m}{2}\dot{\xi}^2} + \underbrace{W\dot{\xi}_A\hat{p}_s W^\dagger}_{+\dot{\xi}p - m\dot{\xi}^2} - \ddot{\hat{\xi}}_A m\hat{X}_s + \frac{m}{2}\dot{\hat{\xi}}_A^2$$

$$\boxed{\hat{H}'' = \frac{\hat{p}_s^2}{2m} - m\ddot{\hat{\xi}}_A\hat{X}_s} \longrightarrow \text{eq. principle !}$$

• Constraint systems



- 3 "players" but only 2 particles
- No external RF for translations

$$\Rightarrow \boxed{(\hat{p}_A + \hat{p}_B + \hat{p}_S)|\Psi\rangle_{ABS} = 0}$$

• Derivation of $P_{A\to B}\, e^{+i\hat{x}_B \hat{p}_S}$ from $|\Psi\rangle_{ABS}$

$$|\Psi\rangle_{ABS} = \int dp_A\, dp_B\, dp_S\, \delta(p_A+p_B+p_S)\,\Psi(p_A,p_B,p_S)\,|p_A,p_B,p_S\rangle_{ABS}$$

$$= \int dp_B\, dp_S\, \Psi(-p_B-p_S,p_B,p_S)\,|-p_B-p_S,p_B,p_S\rangle_{ABS}$$

$$U^\dagger_{BS}(\hat{x}_A) := e^{i\hat{x}_A(\hat{p}_B+\hat{p}_S)}$$

$$U^\dagger_{BS}(\hat{x}_A)|\Psi\rangle_{ABS} = \int dp_B\, dp_S\, \Psi(-''-)\,|-p_B-p_S+p_B+p_S, p_B, p_S\rangle_{ABS}$$

$$= |p=0\rangle_A \otimes \underbrace{\int dp_B\, dp_S\, \Psi(-p_B-p_S,p_B,p_S)\,|p_B,p_S\rangle_{BC}}_{=:\,|\psi\rangle^{(A)}_{BC}}$$

$$U^\dagger_{AS}(\hat{x}_B)\, U_{BS}(\hat{x}_A)\, |p=0\rangle_A \otimes |\psi\rangle^{(A)}_{BC} =$$

$$= |p=0\rangle_B \otimes \underbrace{\int dp_A\, dp_S\, \Psi(p_A,-p_A-p_S,p_S)\,|p_A,p_S\rangle_{AS}}_{=\,|\tilde{\Psi}\rangle^{(B)}_{AS}}$$

$\boxed{\text{Ref. [3]}}$

$$= \int dp_B\, dp_S\, \Psi(-p_B-p_S,\, p_B,\, p_S)\,|-p_B-p_S,\, p_S\rangle_{AS}$$

$$= e^{-i\hat{x}_B\hat{p}_S} \int dp_B\, dp_S\, \Psi(-''-)\, P_{B\to A}\,|p_B,\, p_S\rangle_{BS}$$

$$= \underbrace{P_{A\to B}\, e^{i\hat{x}_B\hat{p}_S}}_{=\,S_{A\to B}} \underbrace{\int dp_B\, dp_S\, \Psi(-p_B-p_S,p_B,p_S)\,|p_B,p_S\rangle_{BS}}_{=\,|\psi\rangle^{(A)}_{BS}}$$

$$\boxed{U^\dagger_{AS}(\hat{x}_B)\, U_{BS}(\hat{x}_A)\,|p=0\rangle_A \otimes |\psi\rangle^{(A)}_{BS} = |p=0\rangle_B \otimes S_{A\to B}\,|\psi\rangle^{(A)}_{BS}}$$

- Group theoretic generalisation

Position QRF = QRF for translation group $\longrightarrow$ QRF for group G

$$|x\rangle_A \longrightarrow |g\rangle_A \in L^2(G)$$

$|g\rangle$: $\underbrace{\text{Regular}}$ $\underbrace{\text{representation}}$

$\boxed{\text{Ref. [16]}}$

$\boxed{\text{Ref. [17]}}$

$$U_g U_h = U_{gh}$$

$$\hookrightarrow g \to |g\rangle \; ; L_g |h\rangle = |gh\rangle \, , R_g |h\rangle = |hg^{-1}\rangle$$

$$\langle g | h \rangle = \delta(g^{-1}h) \; ; [L_g, R_h] = 0$$

- Getting rid of external degrees of freedom means averaging over the (global) action of G $\boxed{\text{Refs. [5, 4]}}$

- Incoherent group averaging T_G

$$\rho \longmapsto \int dg \, U_g \rho U_g^\dagger$$

$$U_h \rho U_h = \rho$$

- Mixed state formalism (ignorance)
- More common in Q info
- There could be another RF out there

- Coherent group averaging

$$|\psi\rangle \longmapsto \int dg \, U_g |\psi\rangle$$

$$U_h |\psi\rangle = |\psi\rangle$$

$\hookrightarrow$ no phases!

- Pure state formalism
- More common in Q G
- No external RF (does not exist)

$$|g\rangle = \sum_{q x \alpha} \sqrt{\frac{\dim q}{|G|}} \, D^{(q)}_{x\alpha}(g) \, |q ; x, \alpha\rangle$$

q: charge labels i rep

x: "global" dofs destroyed by twirling

α: multiplicities left untouched by group action L_g. "Relative" dofs

General red. rep:

$$\mathcal{H} = \bigoplus_q M^q \otimes N^q$$

N^q ← multiplicities

Reg. rep. multiplicities = dim irrep

$$\rho = \begin{pmatrix} * & * & * \\ \vdots & \ddots & \vdots \\ * & * & * \end{pmatrix} \; ; \; T_G[\rho] = \begin{pmatrix} \frac{1}{d_{q_1}} \mathbb{1}_{q_1} \otimes \rho_N^{q_1} & & \\ & \ddots & \\ & & \frac{1}{d_{q_N}} \mathbb{1}_{q_N} \otimes \rho^{q_N} \end{pmatrix}$$

Projects onto charges: block diagonal

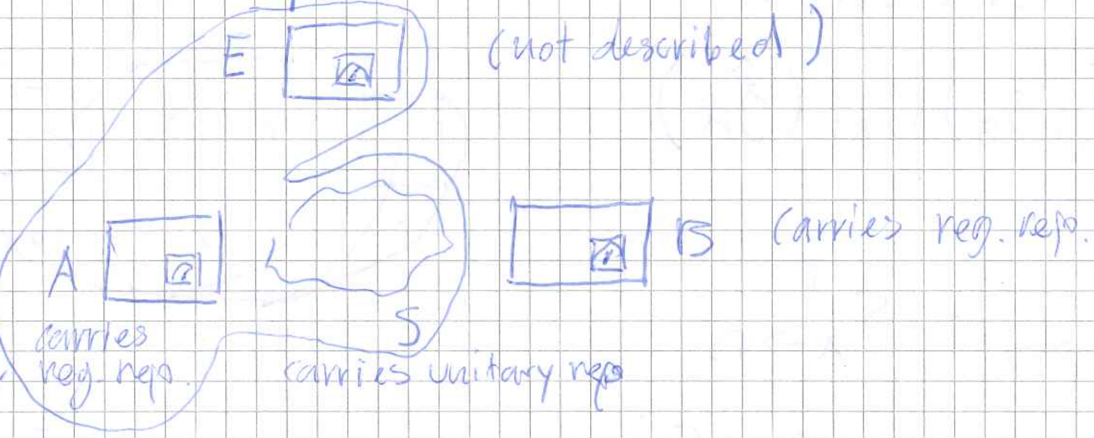
$\frac{1}{d_q}\mathbb{1}_q$ $\hookrightarrow$ randomizes M^q subsystem

ρ^{q_N} $\hookrightarrow$ does not touch multiplicities

Example: 3 spin-1/2 s

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2} \oplus \underbrace{\frac{1}{2} \oplus \frac{1}{2}}_{\text{multiplicity}}$$

- The setup



- How does A see B & S?
- How to "jump" from A to B?

- A: I measure M on the system

E: A performs $\int dg\, |g\rangle\langle g|_A \otimes U_g M_S U_g^\dagger$

Example: $\int dx\, |x\rangle\langle x|_A \otimes T_x \hat{X}_S T_x = \hat{X}_S - \hat{X}_A$

$S|A = \int dg\, |g\rangle\langle g| \otimes U_g M_S U_g^\dagger \rightarrow$ is a subalgebra of $\mathcal{H}^{(E)} := \mathcal{H}_A \otimes \mathcal{H}_S$

"S relative to A"

$S|A$ is invariant: $L_g \otimes U_g \; S|A \; L_g^\dagger \otimes U_g^\dagger$

$\boxed{\text{Ref [6]}}$

$\boxed{\text{Ref. [7]}}$

$$\mathcal{H}^{(E)}_{AS} = \bigoplus_q \mathcal{H}_{A_L} \otimes \mathcal{H}_{A_R} \otimes \mathcal{H}_S$$

Group acts

$$\text{Gauge subsystem} = \bigoplus_q T^L_q \otimes \mathbb{1}^R_q \otimes T^S_q$$

$$U_S^\dagger(\hat{g}_A) = \int dg\, |g\rangle\langle g|_A \otimes U_g^\dagger$$

"Jumping" + A's QRF.

$$\mathcal{L}(\mathcal{H}^{(A)}_{AS}) = U_S^\dagger(\hat{g}_A)\, \mathcal{L}(\mathcal{H}^{(E)}_{AS})\, U_S(\hat{g}_A)$$

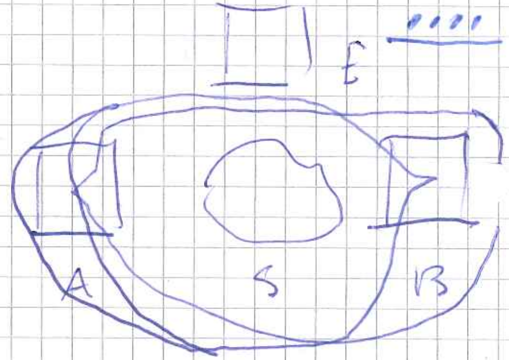
$$= \bigoplus_q \mathcal{L}(\mathcal{H}_{gauge}) \otimes \underbrace{\mathcal{L}(\mathcal{H}_{extra\ particle}) \otimes \mathcal{L}(\mathcal{H}_{S|A})}_{\text{invariant part}} =: \overline{S|A}$$

system "as seen" by A.

group acts here!

$\boxed{\text{Ref [8]}}$

PRF transformation:

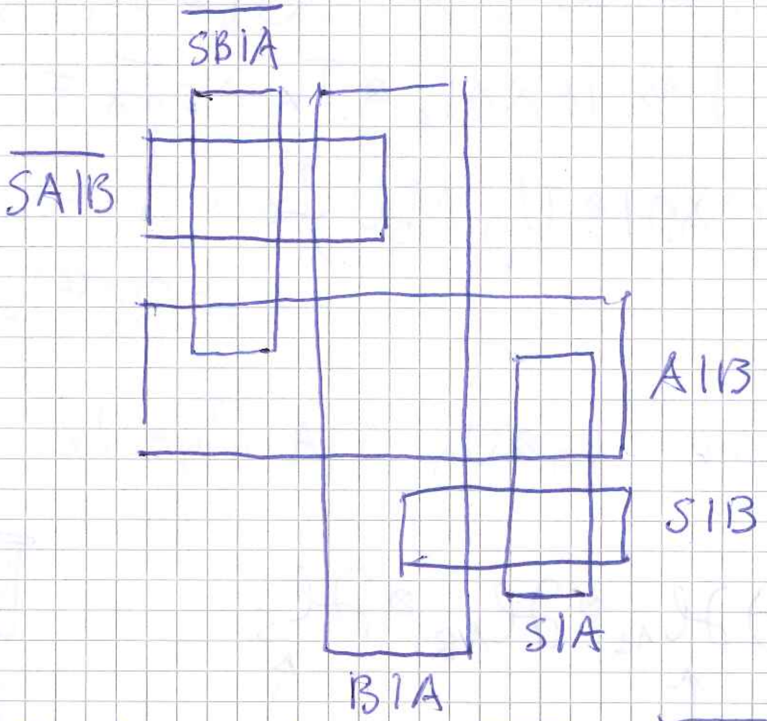


$$S_{A\to B} = U^\dagger_{AS}(\hat{g}_B)\, U_{BS}(\hat{g}_A)$$

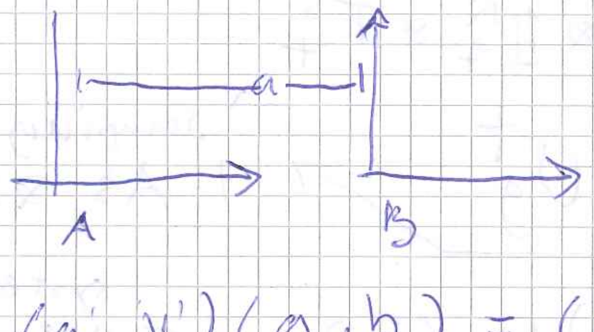
$$\mathcal{S}_{A\to B} = S_{A\to B} \cdot S^\dagger_{A\to B}$$

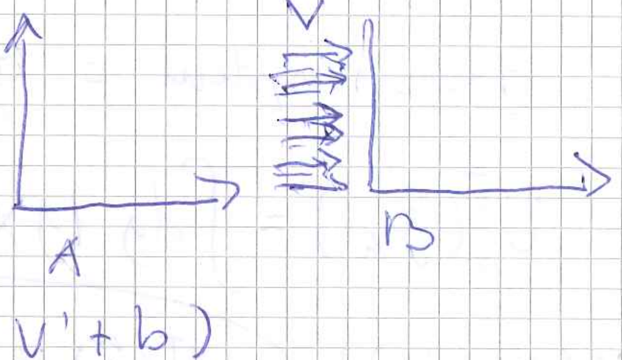
$$\mathcal{S}_{A\to B}\left[\mathbb{1}\otimes\mathbb{1}\otimes T_{S|A}\right] = \int dg\, |g\rangle\langle g| \otimes \mathbb{1} \otimes U_g T_{S|B} U_g^\dagger$$

...



Galilei group (1D)





$$(a', v')(a, b) = (a'+a, v'+b)$$

In Hilbert space

$$U(a,v) = e^{-ia\hat{p} - iv\hat{K}}$$

$$\hat{K} = \hat{p}t - m\hat{x}$$

$$U^\dagger(\hat{g})\, L_g \otimes U_g\, U(\hat{g}) =$$

$$= \int dg'\, dh'\, |g'\rangle\langle g'|\, L_g\, |h'\rangle\langle h'| \otimes U_{g'}^\dagger\, U_g\, U_{h'}$$

$$= \int dh'\, |gh'\rangle\langle h'| \otimes U_{h'^{-1}g^{-1}}\, U_g\, U_{h'}$$

$$= L_g \otimes \mathbb{1}$$

$$U^\dagger(\hat{g}) \int dg\, |g\rangle\langle g| \otimes U_g\, T\, U_g^\dagger\; U(\hat{g})$$

$$= \int df\, dg\, dh\, |f\rangle\langle f|g\rangle\langle g|h\rangle\langle h| \otimes U_f^\dagger\, U_g\, T\, U_g^\dagger\, U_h$$

$$= \mathbb{1} \otimes T$$

$$U(a',v')\,U(a,v) = \underbrace{e^{i\frac{m}{2}(av'-a'v)}}\,U(a'+a, v'+v)$$

The Galilei group has a projective representation in Hilbert space!

Define the <u>centrally extended</u> Galilei group

$$(\theta',a',v')(\theta,a,v) = \left(\theta'+\theta+\frac{1}{2}(av'-a'v), a'+a, v'+v\right)$$

Reg. rep of the group

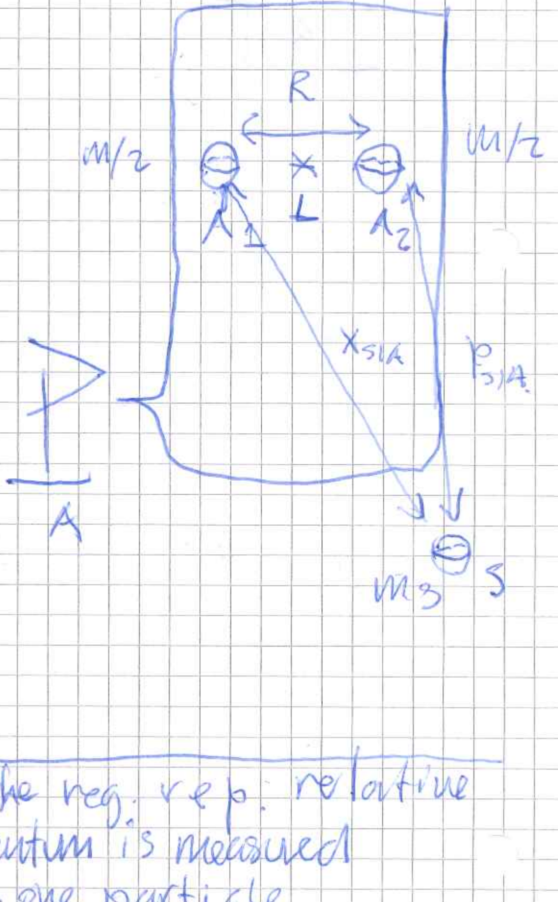
$$|(\theta',a',v')\rangle \; ; \; \langle(\theta',a',v')|(\theta,a,v)\rangle = \delta(\theta'-\theta)\delta(a'-a)\delta(v'-v)$$

$$|(\theta,a,v)\rangle = \int^{\oplus} dm \int dp \sqrt{m} \left(\tilde{U}^{(m)}(\theta,a,v)|m,p\rangle_L\right)\otimes|m,p\rangle_R$$

$$\tilde{U}^{(m)}_{(\theta,a,v)} = e^{i\theta} e^{-i(a\hat{p}+v\hat{K})}$$

$$\hat{K} = \hat{p}t - m\hat{x} \quad : \text{ boost generator}$$

$$[\hat{p},\hat{K}] = im$$



$$\hat{p}_{S|A} = U_S((\theta,\hat{a},v))\,\hat{p}_S\,U^\dagger((\theta,\hat{a},v))$$

$$= \mathbb{1}_A \otimes \hat{p}_S - m\hat{v}_A \otimes \mathbb{1}$$

$$\hat{K}_{S|A} = \mathbb{1}_A \otimes \hat{x}_S + m\hat{a}_A \otimes \mathbb{1}$$

$$\hat{v}_A = \int d\theta\, da\, dv\; a\, |(\theta,a,v)\rangle\langle(\theta,a,v)|$$

$$\hat{a}_A = \int d\theta\, da\, dv\; v\, |(\theta,a,v)\rangle\langle(\theta,a,v)|$$

$$[\hat{a}_A, \hat{v}_A] = 0 \; !$$

Interpretation:

In the reg. rep. relative momentum is measured w.r.t. one particle whereas relative position is measured wrt <u>another</u> particle!

- QRFs for time

Generator of time translations H:

$H|\Psi\rangle = 0$ ("no external time RF")

$H = H_c + H_s$ with H_c → clock, H_s → system

$\boxed{\text{Ref [9,10]}}$

$[H_c, T_c] = -i$ (perfect clock)

$0 = (H_c + H_s)|\Psi\rangle$ Page-Wootters mechanism

$0 = \langle t|H_c|\Psi\rangle + \langle t|H_s|\Psi\rangle$

$= -i\frac{d}{dt}|\psi(t)\rangle + H_s|\psi(t)\rangle$

$|\psi(t)\rangle := \langle t|\Psi\rangle$

$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$ (we recover Schrödinger eq.)

- History states

$|\Psi\rangle = \int d\alpha\, e^{-i\alpha H}|\psi\rangle$ coherent group averaging

solves

$H|\Psi\rangle = 0$

$\langle t|\Psi\rangle =: |\psi(t)\rangle$ "state of S at time t"

Probability for single measurement:

$P_a = \mathrm{Tr}\, |a\rangle\langle a|\psi(t)\rangle\langle\psi(t)|$

$= \mathrm{Tr}\, |t\rangle\langle t| \otimes |a\rangle\langle a|\Psi\rangle\langle\Psi|$

- Multi-time measurements

Problem: $P_{t,a}|\Psi\rangle$ no longer satisfies

$H|\Psi\rangle = 0$ in general!

Different interpretation of $|\Psi\rangle$ is needed

(A) Solution: Purify measurements

$$H = H_c + H_s + \underbrace{f_{aS}(\hat{T}_c)}_{\text{e.g. } K_{aS}\,\delta(\hat{T}_c - t^*)} + \hat{f}_{bS}(\hat{T}_C) + \dots$$

K_{aS} → "logarithm" of Kraus operator

$\delta(\hat{T}_c - t^*)$ → time of measurement according to clock C.

- Solving $H|\Psi\rangle = 0$ (assume $H_S = 0$ for simplicity).

$$|\Psi\rangle = \underbrace{\int e^{-i\alpha\hat{H}}\, d\alpha}_{=:\,\Pi_0}\, |\varphi\rangle \quad \text{(projector onto 0-energy subspace)}$$

$$e^{-i\alpha\hat{H}} = e^{-i\alpha(H_c + \cancel{H_s} + f_{as}(\hat{T}_c)) + \dots}$$

consider single measurement for simplicity

$$= \lim_{N\to\infty}\Big[\underbrace{e^{-\frac{i\alpha}{N}H_c}\, e^{-\frac{i\alpha}{N}f_{as}(\hat{T}) + \dots}}_{=:\,L}\Big]^N$$

$$L\,|t,s,r\rangle_{csa} = e^{-\frac{i\alpha}{N}f_{as}(t)}\,|t+\tfrac{\alpha}{N}, s, r\rangle$$

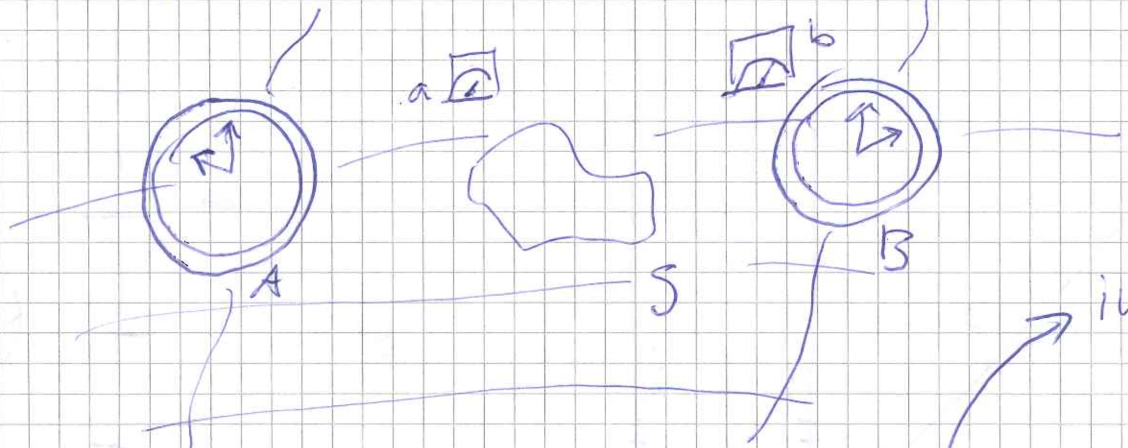
Result generalizes naturally

$$L^N|t,s,r\rangle_{csa} = e^{-i\frac{\alpha}{N}\left(f_{as}(t+\frac{N-1}{N}\alpha) + f_{as}(t+\frac{N-2}{N}\alpha) + \dots + f_{as}(t)\right)}\,\underbrace{|t+\alpha}\,, s, r\rangle$$

$$|t+\alpha, s, r\rangle = e^{-i\alpha H_c}|t\rangle$$

$$\Rightarrow e^{-i\alpha\hat{H}} = e^{-i\alpha H_c}\, \overleftarrow{T}\, e^{-i\int_0^\alpha d\lambda\left(\hat{f}_{as}(\hat{T}+\lambda) + f_{bs}(\hat{T}+\lambda) + \dots\right)}$$

• Multiple clocks: changing QRFs



$$|\Psi\rangle = \int dt_A \, |t_A\rangle_A \otimes U_{\bar{A}}(t_A)\, |\psi_0\rangle_{\bar{A}}$$

initial state

Everything that is not A e.g. S B a b

A's reference frame

Evolution wrt A

$\boxed{\text{Ref [11]}}$

$\boxed{\text{Ref. [12]}}$

$$= \int dt_B \, |t_B\rangle_B \otimes U_{\bar{B}}(t_B)\, |\psi_0\rangle_{\bar{B}}$$

Given $U_{\bar{A}}$ & $|\psi_0\rangle_{\bar{A}}$, how do we find $U_{\bar{B}}$ and $|\psi_0'\rangle_{\bar{B}}$?

$$\langle t|_A \Pi_0 |t'\rangle_A = \int d\alpha \, \underbrace{\langle t|_A \, e^{-i\alpha(H_A+H_B)} \overleftarrow{T} e^{-i\int_0^\alpha d\lambda \, \hat{f}_A(\lambda+\hat{T}_A)+\hat{f}_B(\lambda+\hat{T}_B)} |t'\rangle_A}_{\delta(t_A - t_A' - \alpha)}$$

$$= e^{-i(t-t')H_B} \overleftarrow{T} e^{-i\int_{t'}^{t} d\lambda \, f_A(\lambda)+f_B(\lambda+\hat{T}_B)}$$

$$= U_{\bar{A}}(t-t')$$

$$\Pi_0 = \int dt\, dt' \, |t\rangle\langle t'|_A \otimes U_{\bar{A}}(t-t')$$

The event in B can be quantum uncertain in time wrt A!

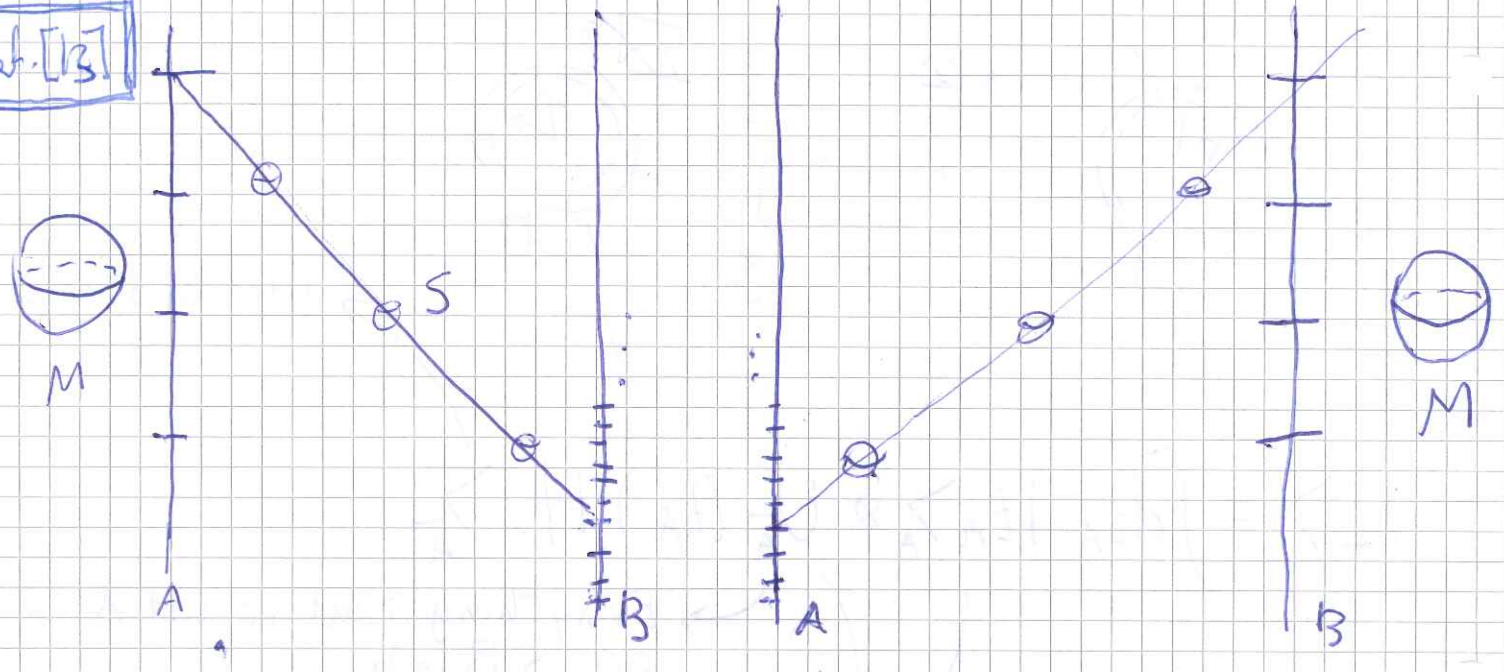
$$\Rightarrow \boxed{U_{\bar{B}}(t-t') = \int ds\, ds' \, |s\rangle\langle s'|_A \otimes \langle t|_B \, U_{\bar{A}}(s-s') \, |t'\rangle_B}$$

and

$$\boxed{|\psi_0'\rangle_{\bar{B}} = \int dt \, |t\rangle_A \otimes \langle t=0|_B \, U_{\bar{A}}(t) \, |\psi_0\rangle_{\bar{A}}}$$

• Gravitational quantum switch

$\boxed{\text{Ref. [13]}}$



C

$$\hat{H} = \sum_{I=ABC} H_I (1+\Phi_I) + \sum_{I=A,B} f_I(\hat{T}_I)(1+\Phi_I)$$

$$= -GM/(c^2 \hat{X}_{IM})$$

(gravitational potential energy at clock I).

Goal: To find history state wrt C and history state wrt A (equiv. B).

C:

$$|\Psi\rangle = \int dt\, |t\rangle_C \otimes \mathcal{T} e^{-i\sum_I \int_0^t ds \frac{1+\Phi_I}{1+\Phi_C}\left(H_I + f_I\left(s\frac{1+\Phi_I}{1+\Phi_C} + \hat{T}_I\right)\right)}$$

A:

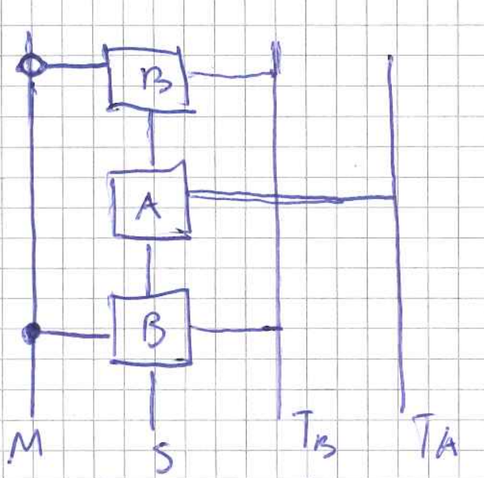
$$|\Psi\rangle = \int dt\, |t\rangle_A \otimes \mathcal{T} e^{-i\int_0^t ds \left(f_A(s) + \sum_I \frac{1+\Phi_I}{1+\Phi_A}\left(H_I + f_I\left(s\frac{1+\Phi_I}{1+\Phi_A} + \hat{T}_I\right)\right)\right)}$$

localization of events in time has quantum uncertainty wrt C

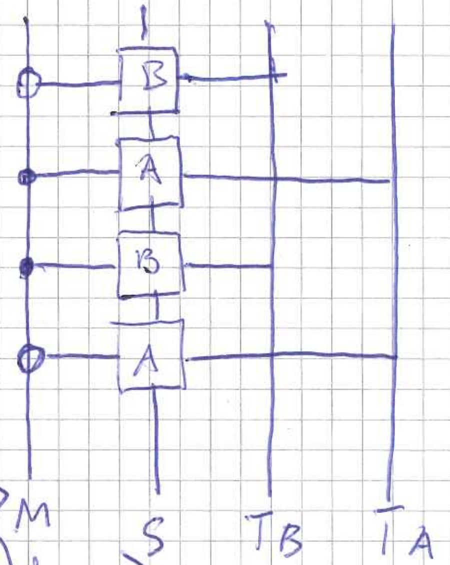
A's event is <u>always</u> localised in time wrt A.

$$|\psi(t)\rangle_{\bar{C}} = \int dt_a \,-\, \big(U_A(t+\delta)\, U_B(t-\delta)\, |M_L\rangle + U_B(t+\delta)\, U_A(t-\delta)\, |M_R\rangle \big)\, |-u-\rangle$$

A :



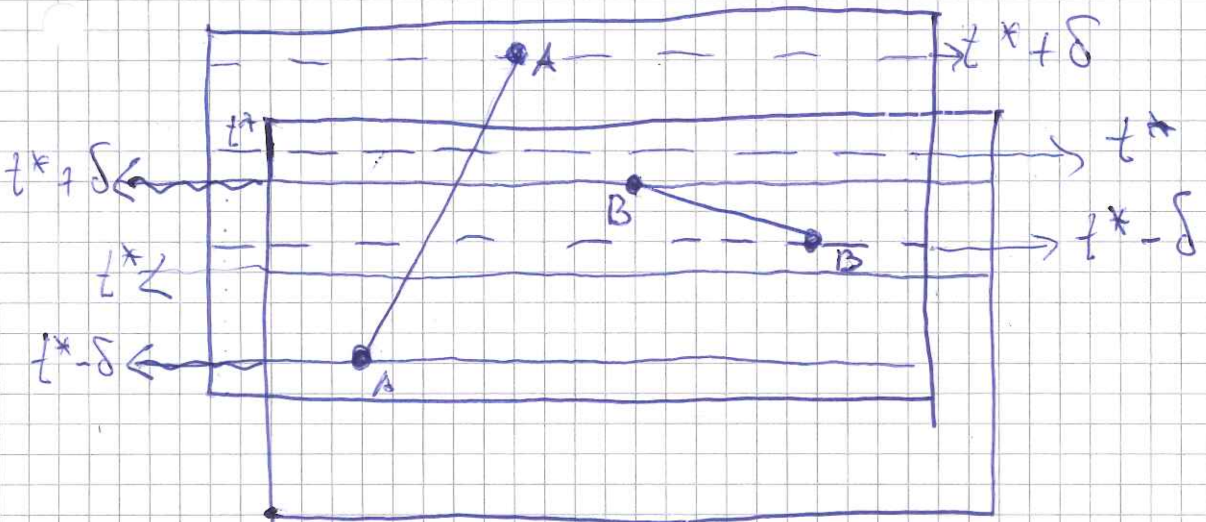
C :



Refs. [15, 14]

$$|\psi(t)\rangle_A = \int dt_c |t_c\rangle \ldots \big(U^A(t)\, U^B(\ldots) |M_L\rangle$$

C : $$+\, U^B(\ldots)\, U_A(t) \big) |M_R\rangle \big) | \ldots \rangle$$



A