# Compatibility of quantum instruments

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**Solstice of Foundations — ETH Zurich 2022**

## Abstract

Incompatibility of quantum devices is a useful resource in various quantum information theoretical tasks, and it is at the heart of some fundamental features of quantum theory. While the incompatibility of measurements and quantum channels is well-studied, the incompatibility of quantum instruments has not been explored in much detail. In this work, we revise a notion of instrument compatibility introduced in the literature that we call traditional compatibility. Then, we introduce the new notion of parallel compatibility, and show that these two notions are inequivalent. Then, we argue that the notion of traditional compatibility is incomplete, and prove that while parallel compatibility captures measurement and channel compatibility, traditional compatibility does not. Hence, we propose parallel compatibility as the conceptually complete definition of compatibility of quantum instruments. This paper is on arxiv [1].

## Introduction

Incompatibility is one of the basic features of quantum mechanics which makes it different from classical mechanics. Intuitively, two quantum devices are compatible if there exists a joint device such that implementing the joint device is equivalent to simultaneously implementing the two original devices. While incompatibility may at first sound like a drawback, in fact the incompatibility of quantum measurements leads to practical advantages in various quantum information processing tasks . From the foundational point of view, the incompatibility of quantum channels is intimately linked to the well-known no-cloning theorem, and the incompatibility of the identity channel and a non-trivial measurement is linked to the uncertainty principle.
While the incompatibility of measurements and quantum channels is well-studied, much less effort has been designated to the study of the incompatibility of quantum *instruments*, a more general class of quantum devices capturing measurement processes in their full detail. In this work, we review a definition of instrument compatibility used in the literature (which we call *traditional* compatibility), and address its conceptual adequacy and its relation to measurement and channel compatibility. We then define a new notion of instrument compatibility (which we call *parallel* compatibility) and argue that this notion is conceptually more in line with the well-established notions of measurement and channel compatibility. We further prove that parallel compatibility captures measurement and channel compatibility in a well-defined manner, while traditional compatibility cannot capture channel compatibility. We therefore propose to adapt the notion of parallel compatibility instead of traditional compatibility.

## Preliminaries

In this section we discuss the preliminaries.

### Observables and compatibility

An observable $A$ acting on Hilbert space $\mathcal{H}$ of dimension $d$, is defined as a set of positive hermitian matrices $\{A(\mathrm{x})\}$ such that $\sum_x A(x) = \mathbb{I}$. We denote the outcome set of $A$ is $\Omega_A$ and therefore, $x \in \Omega_A$. If all $A(x)$s are projectors then $A$ is a PVM and otherwise $A$ is a POVM.

### Quantum channels

Quantum channels are the CPTP maps from $\Lambda : \mathcal{S}(\mathcal{H}_1) \to \mathcal{S}(\mathcal{H}_2)$ where $\mathcal{S}(\mathcal{H})$ is the state space on Hilbert space $\mathcal{H}$. A CP unital map $\Lambda^* : \mathcal{L}(\mathcal{H}_2) \to \mathcal{L}(\mathcal{H}_1)$ is the dual channel of $\Lambda$ if $\mathrm{Tr}[\Lambda(\rho)A(x)] = \mathrm{Tr}[\rho\Lambda^*(A(x))]$ for all $x \in \Omega_A$ and for all $\rho \in \mathcal{S}(\mathcal{H})$ and for any arbitrary observable $A = \{A(x)\}$. It is well known that any quantum channel $\Lambda$ admits Krauss represenation such that $\Lambda(\rho) = \sum_x \mathcal{K}_x \rho \mathcal{K}_x^\dagger$ where $\sum_x \mathcal{K}_x^\dagger \mathcal{K}_x = \mathbb{I}$. $\mathcal{K}$s are the Krauss operators of $\Lambda$. A channel is called a unital channel if it keeps the maximally mixed state unchanged.

### Quantum Instruments

Quantum instruments simultaneously generalise measurements and quantum channels: they take a quantum state as an input and provide both a classical and a quantum output. One may think of a quantum instrument as a measurement process, by associating the classical output with the measurement outcome, and the quantum output with the post-measurement state. Mathematically, a quantum instrument $\mathcal{I}$ is defined as a set of CP maps $\{\Phi_x : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ such that $\Phi^{\mathcal{I}} \equiv \sum_x \Phi_x$ is a CPTP map. Given a quantum state $\rho$, the classical output of the instrument is $x$ and the quantum output is $\Phi_x(\rho)$, both with probability $\mathrm{Tr}[\Phi_x(\rho)]$.
Given a measurement $A$, we say that the above instrument is *$A$-compatible* if $\mathrm{Tr}[\Phi_x(\rho)] = \mathrm{Tr}[\rho A(x)]$ for all $\rho \in \mathcal{S}(\mathcal{H})$. Note that for every instrument $\mathcal{I} = \{\Phi_x\}$, there exists a unique measurement $A$, such that $\mathcal{I}$ is $A$-compatible. Indeed, we have that $\mathrm{Tr}[\Phi_x(\rho)] = \mathrm{Tr}[\rho\Phi_x^*(\mathbb{I})]$. Thus, defining $A(x) \equiv \Phi_x^*(\mathbb{I})$, we have that $\mathrm{Tr}[\Phi_x(\rho)] = \mathrm{Tr}[A(x)\rho]$, and this $A(x)$ is unique, positive semidefinite and $\sum_x A(x) = \mathbb{I}$, which follows from the fact that the dual of a CPTP map is a CP unital map.

### Three kinds of compatibility in quantum theory

One possible definition of compatibility of quantum devices is that they can be performed jointly. That is, a pair of devices is compatible if there exists a joint device, such that applying the joint device reproduces *both* of the outcomes of the compatible devices. If two devices are not compatible, we say that they are *incompatible*. Arguably, the most studied notions of compatibility in quantum theory are the following [2]:
1. *Measurement compatibility:* Two measurements $A = \{A(x)\}$ and $B = \{B(y)\}$ are compatible if there exists a measurement $\mathcal{G} = \{G(x,y)\}$ with outcome set $\Omega_{\mathcal{G}} = \Omega_A \times \Omega_B$ such that

$$A(x) = \sum_y G(x,y); \;\; B(y) = \sum_x G(x,y) \tag{1}$$

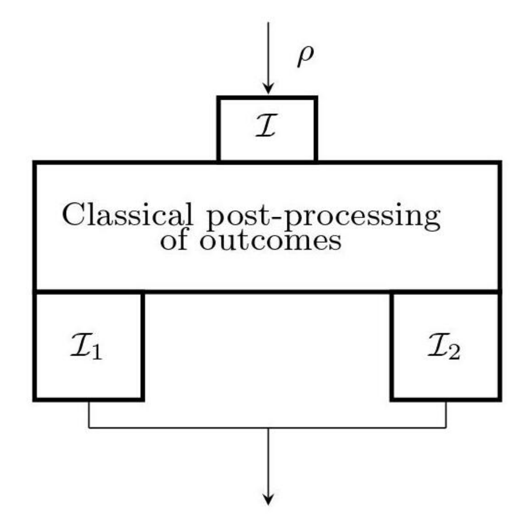
for all $x \in \Omega_A$ and $y \in \Omega_B$. Through measuring $G$, one can simultaneously recover the outputs of both $A$ and $B$. That is, the distribution $p(x,y) \equiv \mathrm{Tr}[G(x,y)\rho]$ is a joint distribution of $p(x) \equiv \mathrm{Tr}[A(x)\rho]$ and $p(y) \equiv \mathrm{Tr}[B(y)\rho]$ for all $\rho$.
2. *Channel compatibility:* Two quantum channels $\Lambda_1 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K}_1)$ and $\Lambda_2 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K}_2)$ are compatible if there exists a quantum channel $\Lambda : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K}_1 \otimes \mathcal{K}_2)$ such that $\Lambda_1(\rho) = \mathrm{Tr}_{\mathcal{K}_2}[\Lambda(\rho)]$ and $\Lambda_2(\rho) = \mathrm{Tr}_{\mathcal{K}_1}[\Lambda(\rho)]$ for all $\rho \in \mathcal{S}(\mathcal{H})$. Through implementing the channel $\Lambda$, one can simultaneously recover the outputs of both $\Lambda_1$ and $\Lambda_2$. That is, $\Lambda(\rho)$ is a joint state of $\Lambda_1(\rho)$ and $\Lambda_2(\rho)$ for all $\rho$.
3. *Measurement-channel compatibility:* A measurement $A = \{A(x)\}$ acting on the Hilbert space $\mathcal{H}$ and a quantum channel $\Lambda : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K})$ are compatible if there exists a quantum instrument $\mathcal{I} = \{\Phi_x : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ such that $\mathrm{Tr}[\Phi_x(\rho)] = \mathrm{Tr}[\rho A(x)]$ for all $x \in \Omega_A$ and $\rho \in \mathcal{S}(\mathcal{H})$ and $\sum_x \Phi_x = \Lambda$. Through implementing the quantum instrument $\mathcal{I}$, one can simultaneously recover the outputs of both $A$ and $\Lambda$.

## Main results

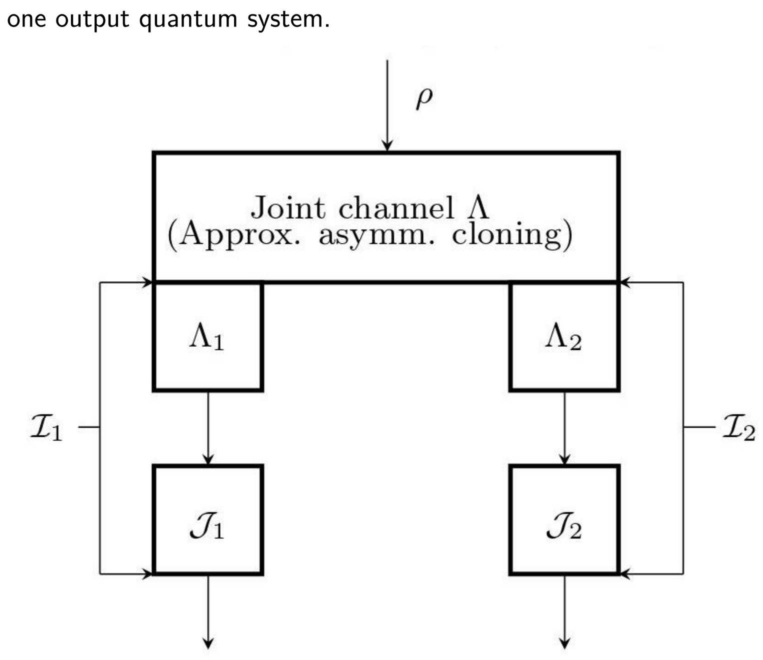
In this section, we discuss the main result.

### A. Definitions and concepts

**Definition 1:** (Traditional compatibility) Two quantum instruments $\mathcal{I}_1 = \{\Phi_x^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ and $\mathcal{I}_2 = \{\Phi_y^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ are (traditionally) compatible if there exists an instrument $\mathcal{I} = \{\Phi_{xy} : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ such that $\sum_y \Phi_{xy} = \Phi_x^1$ and $\sum_x \Phi_{xy} = \Phi_y^2$ for all $x, y$.
This definition appears in Ref. [3, Definition 3], and in Ref. [4, Definition 2.5]. The same definition is given in Ref. [5, page 15], under the name "coexistence".
**Definition 2:** (Weak compatibility) Two quantum instruments $\mathcal{I}_1 = \{\Phi_x^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ and $\mathcal{I}_2 = \{\Phi_y^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K})\}$ are weakly compatible if there exists a quantum channel $\Lambda : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K})$ such that $\sum_x \Phi_x^1 = \sum_y \Phi_y^2 = \Lambda$.
It is known that if a set of instruments is compatible then it is also weakly compatible, but it is easily seen that converse is not true in general [4].
Here, we propose a new definition of instrument compatibility, which we refer to as *parallel* compatibility.
**Definition 3:** (Parallel compatibility) Two quantum instruments $\mathcal{I}_1 = \{\Phi_x^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1)\}$ and $\mathcal{I}_2 = \{\Phi_y^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_2)\}$ are parallelly compatible if there exists an instrument $\mathcal{I} = \{\Phi_{xy} : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1 \otimes \mathcal{K}_2)\}$ such that $\sum_y \mathrm{Tr}_{\mathcal{K}_2}\Phi_{xy} = \Phi_x^1$ and $\sum_x \mathrm{Tr}_{\mathcal{K}_1}\Phi_{xy} = \Phi_y^2$ for all $x, y$.
For later convenience, we provide an alternative (but equivalent) definition of parallel compatibility.
**Definition 4:** Two quantum instruments $\mathcal{I}_1 = \{\Phi_x^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1)\}$ and $\mathcal{I}_2 = \{\Phi_x^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_2)\}$ are parallelly compatible if there exists a quantum instrument $\mathcal{I} = \{\Phi_z : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1 \otimes \mathcal{K}_2)\}$ such that $\Phi_x^1 = \sum_z p_1(x|z)\mathrm{Tr}_{\mathcal{K}_2}\Phi_z$ and $\Phi_y^2 = \sum_z p_2(y|z)\mathrm{Tr}_{\mathcal{K}_1}\Phi_z$, where $p_1$ and $p_2$ are conditional probability distributions.
**Proposition 1:**Definition 4 is equivalent to Definition 3.




(a) **Traditional compatibility:** Schematic representation of Definition 1. Recovering the quantum output of *either* $\mathcal{I}_1$ or $\mathcal{I}_2$ can be done by first implementing the joint instrument $\mathcal{I}$ on the state $\rho$ and then performing the post-processing of outcomes i.e., taking the marginal over either $x$ or $y$. The downward arrows represent quantum systems. Clearly, in this case there is only one output quantum system.




(b) **Parallel compatibility:** An example of parallel simultaneous implementation of two instruments (according to Definition 3, corresponding to Example 1. The simultaneous implementation of $\mathcal{I}_1$ and $\mathcal{I}_2$ can be done through the following steps: (i) implementing the channel $\Lambda$ on the state $\rho$ which is the joint channel of the compatible channels $\Lambda_1$ and $\Lambda_2$ (where $\Lambda_1(\rho)$ and $\Lambda_2(\rho)$ can be considered as the approximate unequal clones (unless $\Lambda_1 = \Lambda_2$) of the state $\rho$, in general and therefore, it can be considered as approximate asymmetric cloning), and then (ii) applying the instruments $\mathcal{J}_1$ and $\mathcal{J}_2$ on $\Lambda_1(\rho)$ and $\Lambda_2(\rho)$ respectively, such that $\mathcal{J}_1 \circ \Lambda_1 = \mathcal{I}_1$ and $\mathcal{J}_2 \circ \Lambda_2 = \mathcal{I}_2$. The existence of such a channel $\Lambda$ and such instruments $\mathcal{J}_1$ an $\mathcal{J}_2$ implies the parallel compatibility of the instruments $\mathcal{I}_1$ and $\mathcal{I}_2$, as explained in Example **??**. The downward arrows represent quantum systems. Clearly, in this case there are two output quantum systems.

Figure 1: Schematic representation of joint instruments for traditionally (Fig. 1a) and parallelly (Fig. 1b) compatible instruments.

**Example 1:** An example of parallelly compatible instruments Consider two compatible quantum channels, $\Lambda_1 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{H}_1)$ and $\Lambda_2 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{H}_2)$ with the joint channel $\Lambda : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$. Therefore, since from the no-signaling principle, implementation of local quantum channel on one side does not change the density matrix on the other side, for any arbitrary quantum channel $\Gamma_1 : \mathcal{S}(\mathcal{H}_1) \to \mathcal{S}(\mathcal{K}_1)$ and $\Gamma_2 : \mathcal{S}(\mathcal{H}_2) \to \mathcal{S}(\mathcal{K}_2)$, we have that $\mathrm{Tr}_{\mathcal{K}_2}(\mathbb{I} \otimes \Gamma_2) \circ \Lambda = \Lambda_1$ and $\mathrm{Tr}_{\mathcal{K}_1}(\Gamma_1 \otimes \mathbb{I}) \circ \Lambda = \Lambda_2$. Now consider a pair of arbitrary quantum instruments $\mathcal{J}_1 = \{\Phi_x'^1 : \mathcal{S}(\mathcal{H}_1) \to \mathcal{L}^+(\mathcal{K}_1) : \sum_x \Phi_x'^1 = \Gamma_1\}$ and $\mathcal{J}_2 = \{\Phi_y'^2 : \mathcal{S}(\mathcal{H}_2) \to \mathcal{L}^+(\mathcal{K}_2);\ \sum_y \Phi_y'^2 = \Gamma_2\}$. Consider another pair of instruments $\mathcal{I}_1 = \mathcal{J}_1 \circ \Lambda_1 = \{\Phi_x^1 = \Phi_x'^1 \circ \Lambda_1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1)\}$ and $\mathcal{I}_2 = \mathcal{J}_2 \circ \Lambda_2 = \{\Phi_y^2 = \Phi_y'^2 \circ \Lambda_2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_2)\}$. Then, we show that the instrument $\mathcal{I} = \{\Phi_{xy} = (\Phi_x'^1 \otimes \Phi_y'^2) \circ \Lambda : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1 \otimes \mathcal{K}_2)\}$ is a joint instrument of $\mathcal{I}_1$ and $\mathcal{I}_2$. Clearly, for all $x$

$$\begin{aligned}\Phi_x^1 &= \Phi_x'^1 \circ \Lambda_1 \\ &= \Phi_x'^1 \circ \mathrm{Tr}_{\mathcal{K}_2}(\mathbb{I} \otimes \Gamma_2) \circ \Lambda \\ &= \Phi_x'^1 \circ \mathrm{Tr}_{\mathcal{K}_2}(\mathbb{I} \otimes \sum_y \Phi_y'^2) \circ \Lambda \\ &= \mathrm{Tr}_{\mathcal{K}_2} \sum_y (\Phi_x'^1 \otimes \Phi_y'^2) \circ \Lambda \\ &= \sum_y \mathrm{Tr}_{\mathcal{K}_2} \Phi_{xy}\end{aligned} \tag{2}$$

Similarly, $\Phi_y^2 = \sum_x \mathrm{Tr}_{\mathcal{K}_1}\Phi_{xy}$ for all $x$. Hence, $\mathcal{I}_1$ and $\mathcal{I}_2$ are parallelly compatible with the joint instrument $\mathcal{I}$.
Next, we show that the notion of traditional compatibility of instruments and that of parallel compatibility of instruments are conceptually different.
**Proposition 2:** There exist pairs of quantum instruments which are parallelly compatible, but not traditionally compatible.
**Proposition 3:** There exist pairs of quantum instruments which are traditionally compatible, but not parallelly compatible.

### B. Arguments against traditional compatibility

In the previous section, we have introduced two notions of instrument compatibility and showed that these notions are conceptually different (neither of them implies the other). Here, we argue that the traditional notion has significant drawbacks.
Let us recall that measurements are devices with a quantum input and a classical output, while channels are devices with a quantum input and a quantum output. Furthermore, we say that a pair of such devices is compatible if there exists a joint device that upon taking a quantum input, reproduces *both* of the outputs of the original devices. For measurements, this means that the joint measurement produces a classical output that is the joint measurement outcome of the two compatible measurements. For channels, this means that the joint channel produces a quantum output that is the joint state of the outputs of the compatible channels. According to this principle, when one is looking for a definition of compatibility of instruments, one should look for a joint instrument that reproduces *both* the joint classical and the joint quantum output of the compatible instruments.
It is clear from Definition 1 that the traditional notion of instrument compatibility provides a joint instrument with *a single quantum output*. Thus, by design, the traditional definition does not allow for producing a *simultaneous* quantum output of *both* of the compatible quantum instruments. Furthermore, this definition only applies to instruments with the same output Hilbert space. Note that for traditionally compatible instruments, one can only recover a single quantum output via classical post-processing. This is not the case for parallel compatibility, where the joint instrument produces a joint state, whose marginals coincide with the quantum outputs of the compatible instruments. Indeed, after performing the joint instrument, one has access to *both* of the quantum outputs, and one can perform further operations on both of them simultaneously. Thus, we argue that the traditional notion of instrument compatibility does not capture compatibility in the same way as the well-established notions of measurement and channel compatibility do.
As further justification for our argument, the following two propositions show that while traditional compatibility of instruments captures measurement compatibility, it can never capture channel compatibility.
**Proposition 4:** Two measurements $A$ and $B$ are compatible if and only if there exist an $A$-compatible instrument $\mathcal{I}_A$ and a $B$-compatible instrument $\mathcal{I}_B$ such that $\mathcal{I}_A$ and $\mathcal{I}_B$ are traditionally compatible.
**Proposition 5:** The traditional compatibility of quantum instruments cannot capture the compatibility of quantum channels.

### C. Arguments for parallel compatibility

In this section, we argue that parallel compatibility does not have the flaws of traditional compatibility. In the previous section, we already argued for this from the conceptual viewpoint—that is, parallel compatibility allows for the simultaneous recovery of both of the quantum outputs of the compatible instruments. Here, we further justify the adequacy of parallel compatibility by showing that this notion captures the idea of measurement compatibility, channel compatibility and measurement-channel compatibility. We summarise these findings in the following theorem.
**Theorem 1:** Parallel compatibility of instruments captures measurement compatibility, measurement compatibility and measurement-channel compatibility.
1. Two measurements, $A$ and $B$, are compatible if and only if there exist an $A$-compatible instrument $\mathcal{I}_A$ and a $B$-compatible instrument $\mathcal{I}_B$ such that $\mathcal{I}_A$ and $\mathcal{I}_B$ are parallelly compatible.
2. Two quantum channels, $\Phi^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K}_1)$ and $\Phi^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{K}_2)$, are compatible if and only if there exist two parallelly compatible instruments $\mathcal{I}_1 = \{\Phi_x^1 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1)\}$ and $\mathcal{I}_2 = \{\Phi_y^2 : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_2)\}$ such that $\sum_x \Phi_x^1 = \Phi^1$ and $\sum_y \Phi_y^2 = \Phi^2$.
3. If an $A$-compatible instrument $\mathcal{I}_A = \{\Phi_x^A : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_1)\}$ and a $B$-compatible instrument $\mathcal{I}_B = \{\Phi_y^B : \mathcal{S}(\mathcal{H}) \to \mathcal{L}^+(\mathcal{K}_2)\}$, such that $\sum_x \Phi_x^A = \Phi^A$ and $\sum_y \Phi_y^B = \Phi^B$, are parallelly compatible, then $A$ and $B$ are both compatible with both $\Phi^A$ and $\Phi^B$.

Next, we show that even for a single measurement, two different instruments that are compatible with it, may not be parallelly compatible, as the following example shows:
**Example 2:** (Two parallelly incompatible instruments associated with the same measurement) A trivial measurement $J = \{J(x) = p_x\mathbb{I}\}$ is compatible with any quantum channel $\Lambda$ through the instrument $\mathcal{I}_{J,\Lambda} = \{p_x\Lambda\}$ [2]. Let us consider a channel $\Gamma$, which is incompatible with $\Lambda$. Clearly, $J$ is also compatible with the quantum channel $\Gamma$ through the instrument $\mathcal{I}_{J,\Gamma} = \{p_x\Gamma\}$. From Theorem **??**, we know that if two instruments are parallelly compatible, then their corresponding channels are compatible. Then, since $\Gamma$ and $\Lambda$ were chosen to be incompatible, the instruments $\mathcal{I}_{J,\Gamma}$ and $\mathcal{I}_{J,\Lambda}$ cannot be parallelly compatible.

***Note:*** Arindam Mitra (underlined) is the presenting author in QIP 2022.

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