Epistricted Classical Theories As Foils for Quantum Theory

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Solstice of Foundations – ETH Zurich

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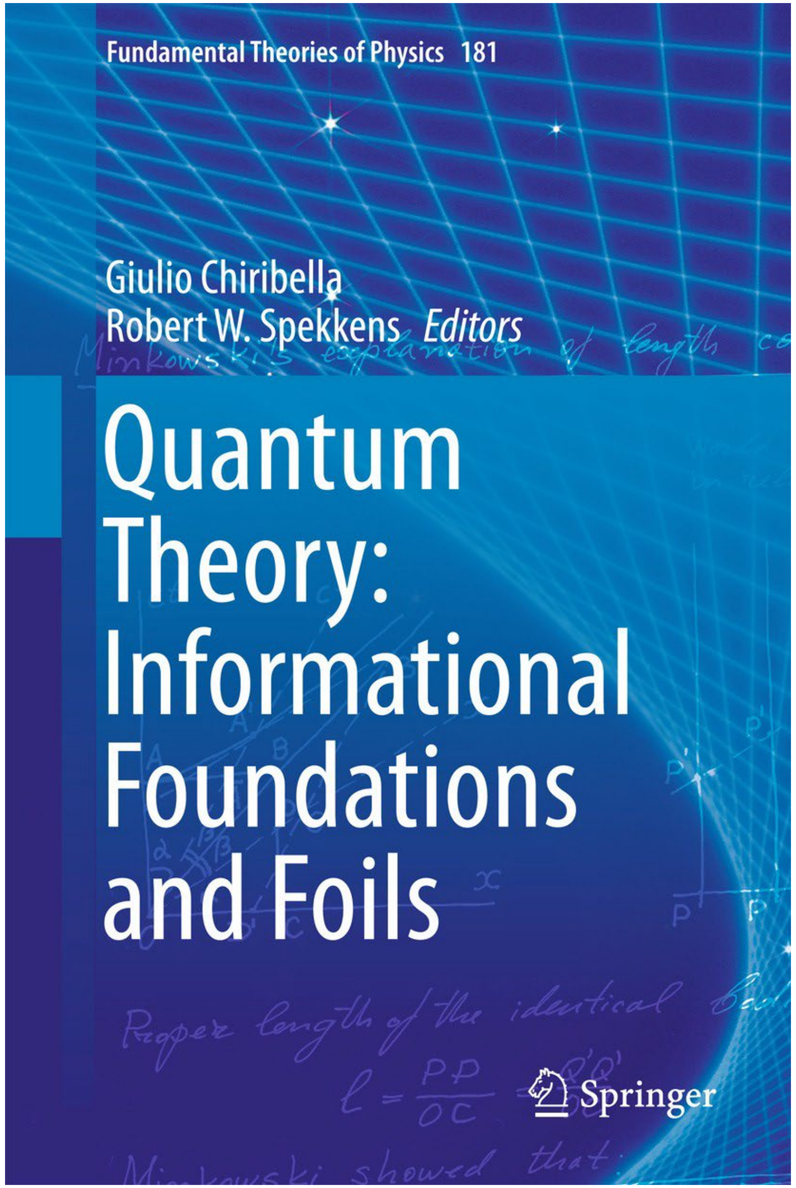
Foil Theories

foil: a person or thing that contrasts with and so emphasizes and enhances the qualities of another.

"the earthy taste of grilled vegetables is a perfect foil for the tart bite of creamy goat cheese"

From: Google Dictionary supplied by Oxford Languages.



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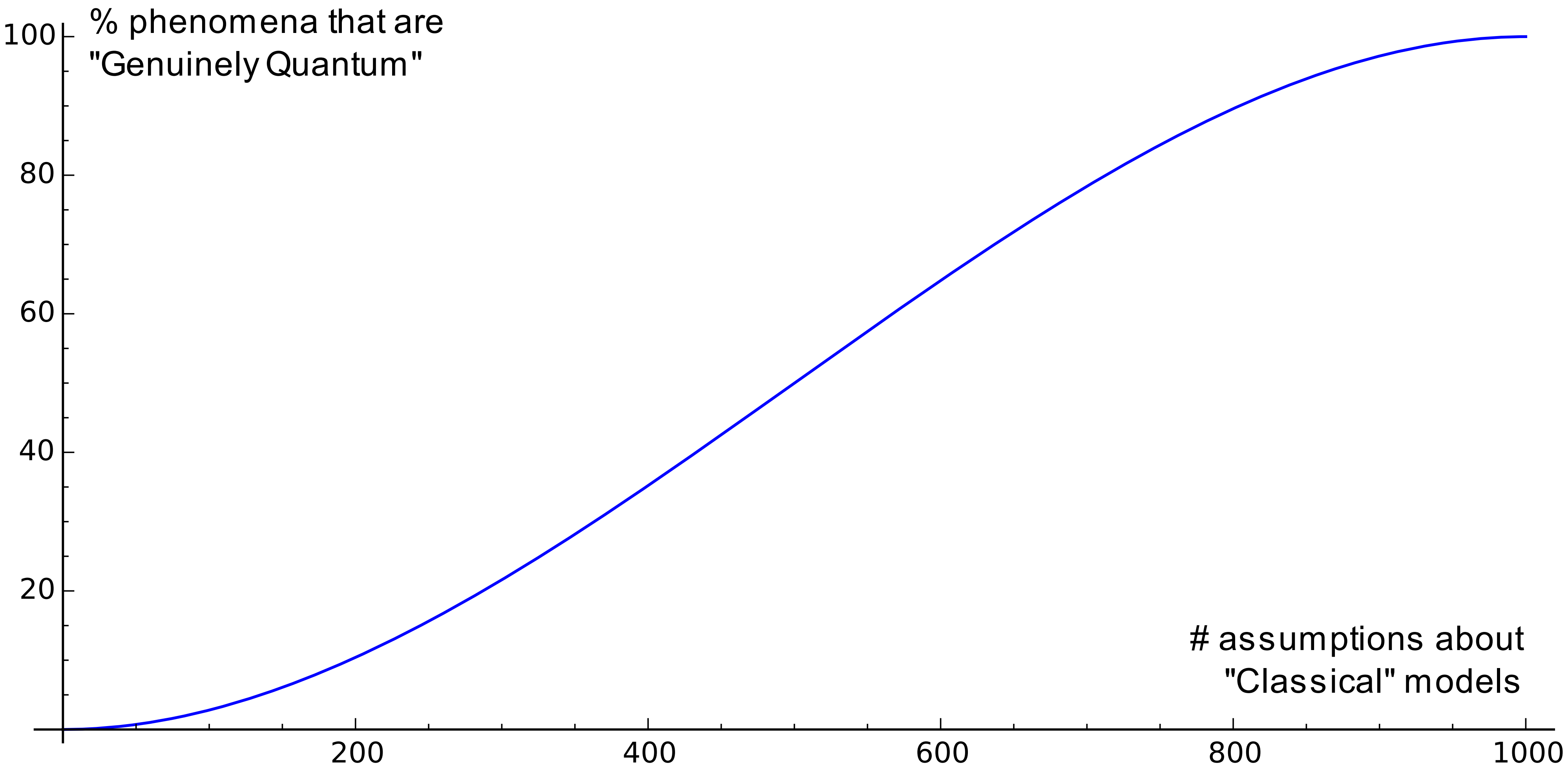
Foil Theories

- We want to answer questions like:
  - What is responsible for the information processing advantages offered by quantum theory?
  - Which features of quantum theory are “genuinely nonclassical”?
- To answer these questions, it is helpful to know what a world in which neither quantum nor classical theories.
- Two ways of doing this:
  - Operational Frameworks like GPTs
  - Ontological foil theories

But what do we mean by ”genuinely nonclassical”?
It depends on the number of assumptions you make about “classical” models



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It depends on what you call "the phenomenon"

- Example, does the state

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

have a local hidden variable model?

Experimental Setup	Only measure in the same basis	Measure in different bases
Phenomenon	EPR paradox	Bell's correlations
Local hidden variable model?	Yes	No

- Bell correlations are "nonclassical", but if someone claims that the EPR paradox has a classical explanation, and you say it does not because of Bell correlations then that would be wrong.

The Spekkens Toy Theory

- The Spekkens Toy Theory is the most interesting ontological foil theory (so far).
- It reproduces many features and phenomena of quantum theory.
- The underlying theory *is literally* classical Hamiltonian mechanics, just with a restriction on how much can be known.
  - This is called an *epistemic restriction* or *epistriction*.
- It is local, noncontextual and ψ-epistemic.
- For a contemporary review, see

L. Hausmann, N. Nurgalieva, L. del Rio arXiv:2105.03277

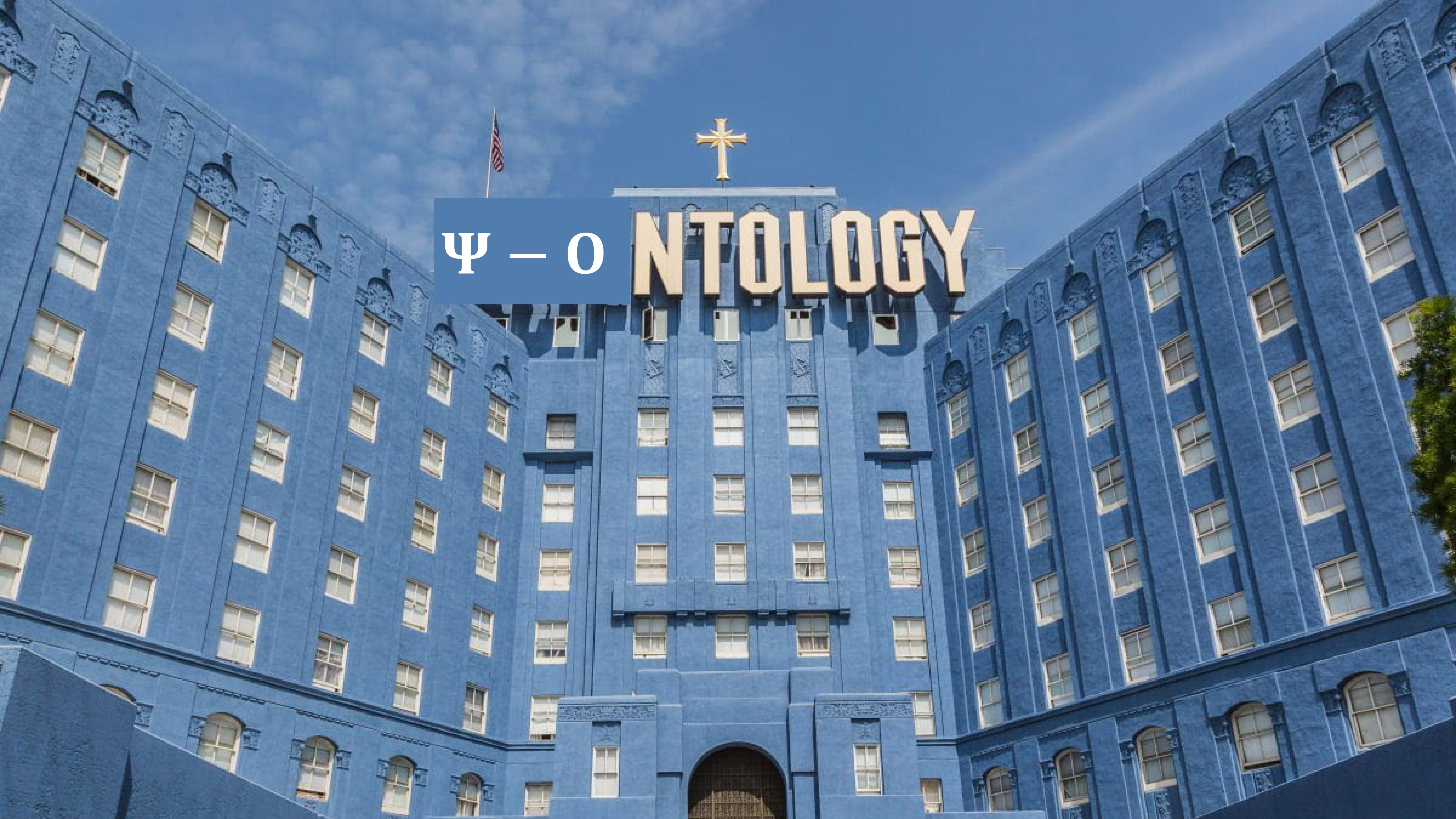
ψ-epistemic vs. ψ-ontic

- The position that quantum states are states of knowledge is called:

The ψ-*epistemic* view

- The position that quantum states are states of reality (physical states) is called:

The ψ-*ontic* view


Ψ — O
NTOLOGY

Outline

1. The Toy-Bit Theory
2. Explanations of Quantum Phenomena
3. Quantum Interference
4. Toy-Theories from Quasi-Quantization
5. Conclusions and Future Directions

1. The Toy-Bit Theory

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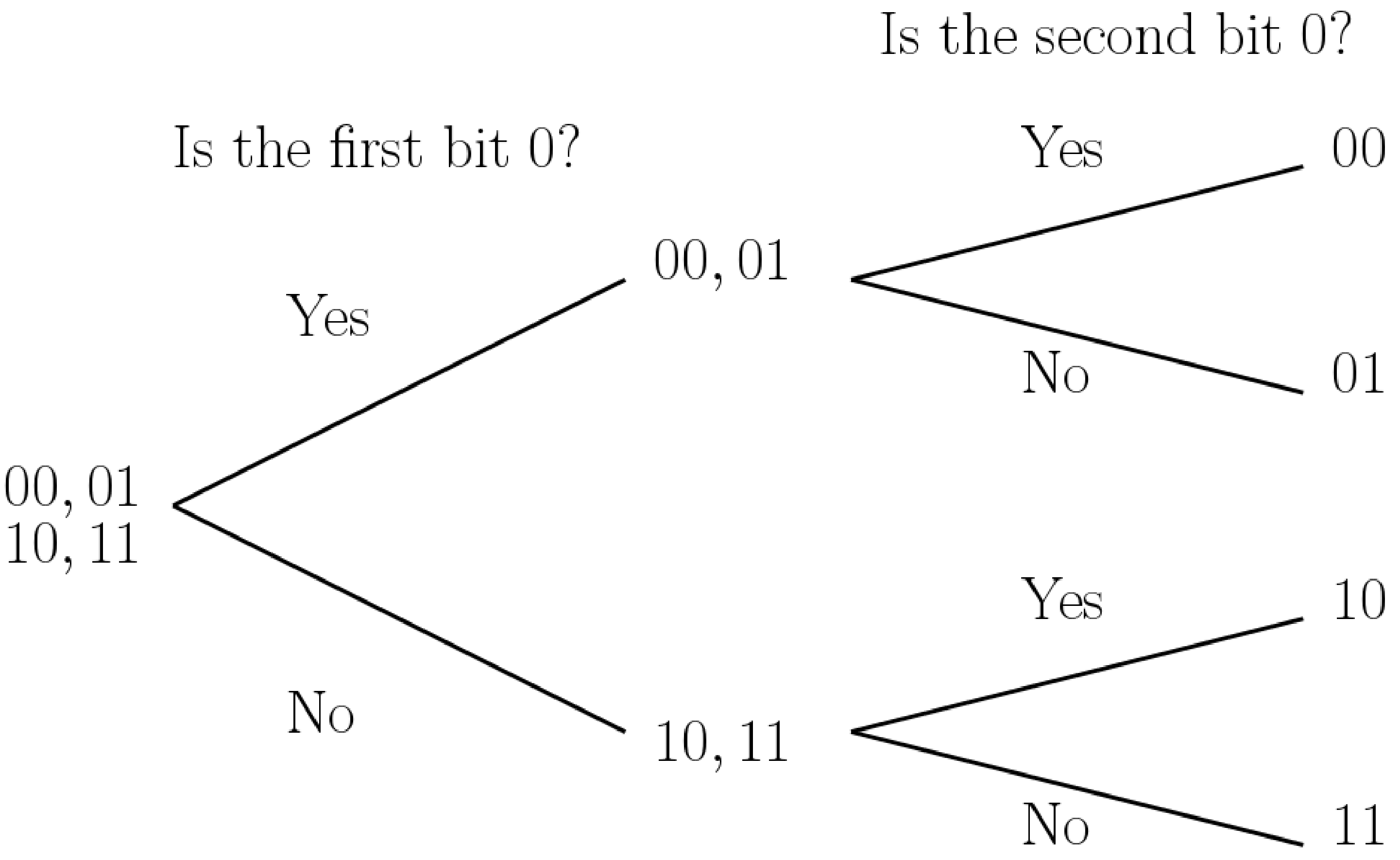
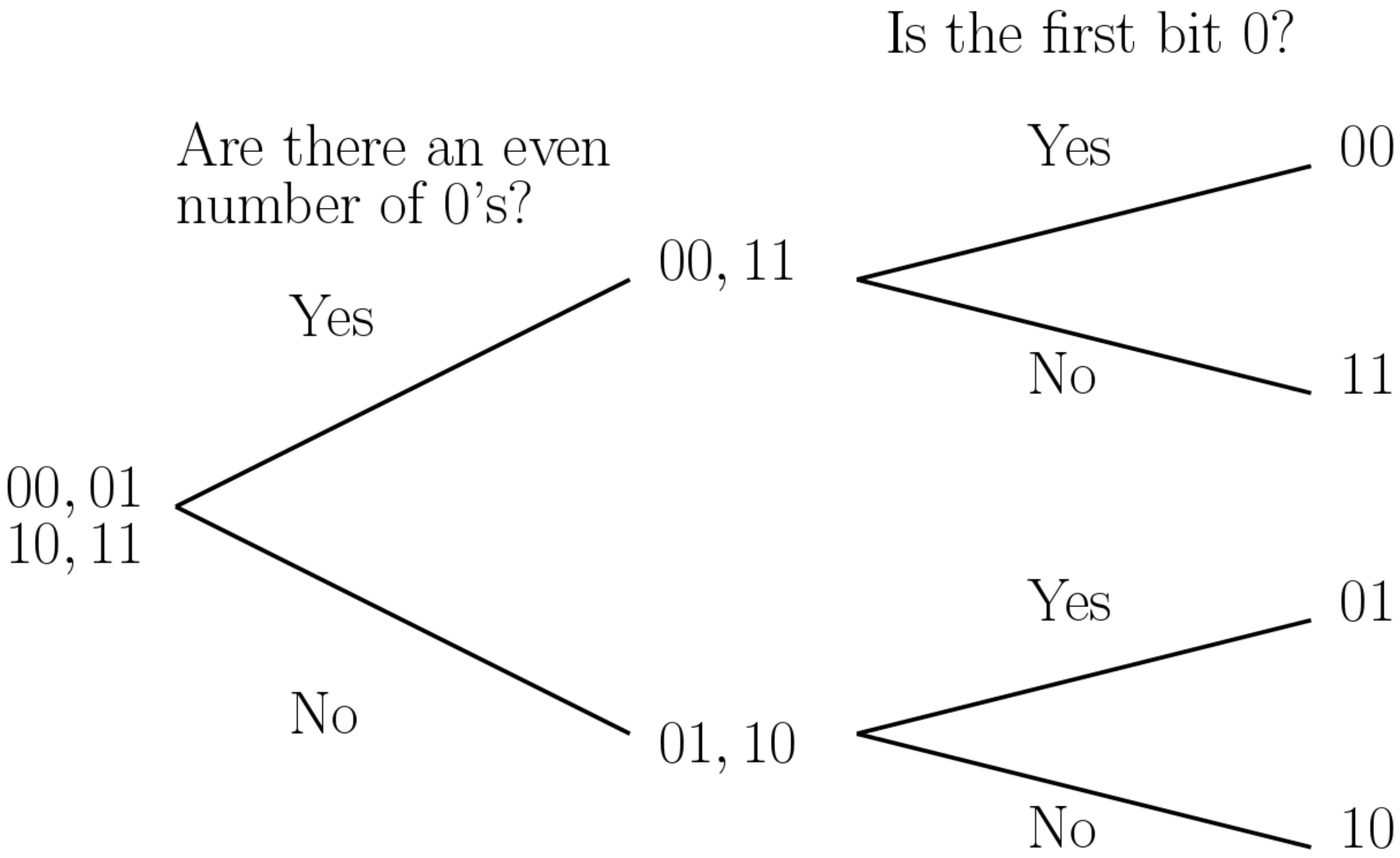
Bits in terms of questions

- We know that if a system can be in n possible states then it takes $m = \log_2 n$ bits to specify its state.
- Another way of thinking about this:
  - What is the minimum number of yes/no questions I have to ask to specify the state exactly?
  - To have the minimum number of questions, each question should cut the set of possible states in half.
  - There is more than one possible set of questions, but the minimal number is unique and equal to $m = \log_2 n$.

Example of a Minimal Set of Questions



Example of a Minimal Set of Questions

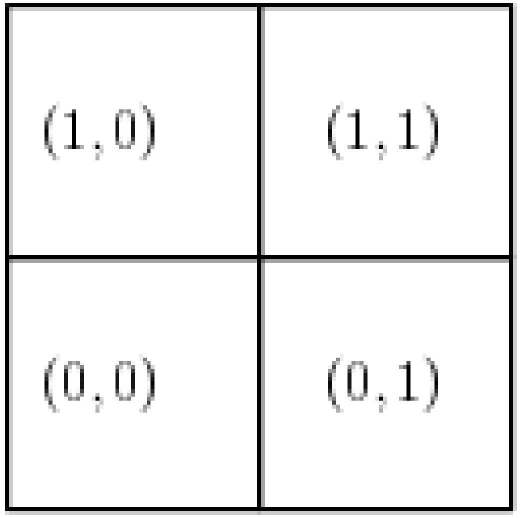


The Knowledge-Balance Principle

- Impose an “epistemic restriction” on how much we can know about a physical system, called the *knowledge-balanace principle:*
  - Of a minimal set of questions required to determine the state of a system exactly, at most we can only know the answer to half of them. We are completely uncertain about the answer to the other half.
- Simplest nontrivial case: A system that has 4 possible states.
  - It takes 2 questions to specify the state, so we can know the answer to at most one of them.
- This system contains 2 bits of information. It will be the analogue of a rebit. We call it a *toy bit*.

Ontic States

- We call the physical state of the system its *ontic state*.
- For a toy-bit, there are four possible ontic states.
  - We label them $(0,0), (0,1), (1,0), (1,1)$.
  - We can imagine a ball that can be in one of four boxes, laid out in a grid.



Epistemic States

- The *epistemic state* of a toy-bit is a probability vector over the four ontic states:

$$\begin{bmatrix} p_{00} \\ p_{01} \\ p_{10} \\ p_{11} \end{bmatrix}$$

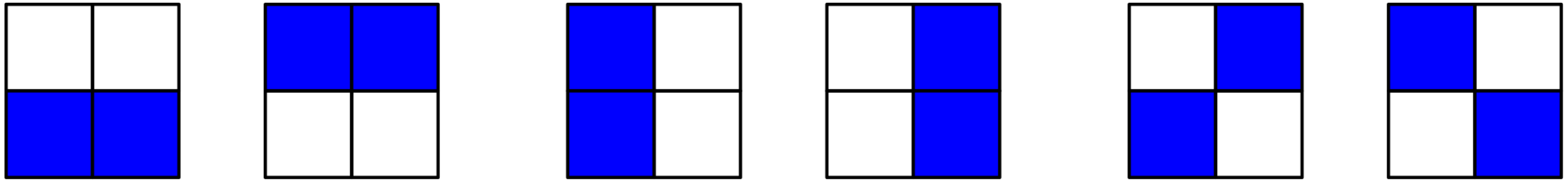
but not all possible probabilities are compatible with the knowledge-balance principle.

  - E.g. consider the question set: Is the first bit 0?, Is the second bit $+$?
  - If we know the answer to the first but not the second, we can have

$$\begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

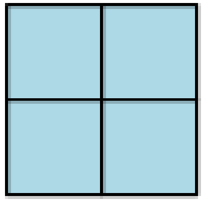
Epistemic States

- There are six epistemic states (probability distributions) compatible with the knowledge-balance principle.



$$|0) = \begin{bmatrix}0.5\\0.5\\0\\0\end{bmatrix} \quad |1) = \begin{bmatrix}0\\0\\0.5\\0.5\end{bmatrix} \quad |+) = \begin{bmatrix}0.5\\0\\0.5\\0\end{bmatrix} \quad |-) = \begin{bmatrix}0\\0.5\\0\\0.5\end{bmatrix} \quad |+i) = \begin{bmatrix}0.5\\0\\0\\0.5\end{bmatrix} \quad |-i) = \begin{bmatrix}0\\0.5\\0.5\\0\end{bmatrix}$$

- There is also a state of non-maximal knowledge



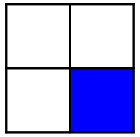
$$\begin{bmatrix}0.25\\0.25\\0.25\\0.25\end{bmatrix}$$

Measurements

- We demand that measurements on toy bits must:

  1. Be *repeatable*, i.e. yield the same result if performed twice in a row.
  2. Not violate the knowledge-balance principle, i.e. they should leave the system in a valid epistemic state.

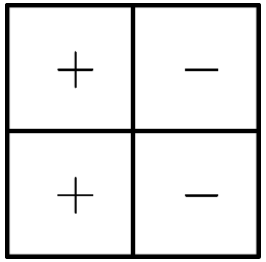
- This immediately implies that there cannot be a measurement that reveals the exact ontic state because this would have to leave us in an epistemic state like:



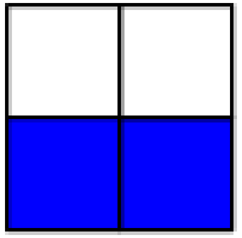
- But we can have measurements that reveal coarse grained information, provided they disturb the ontic state.

Example of a Valid Measurement

- An X measurement gives outcomes ± 1 as:

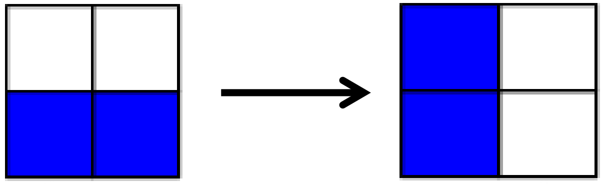


  - If we apply it to the $|0\rangle$ state

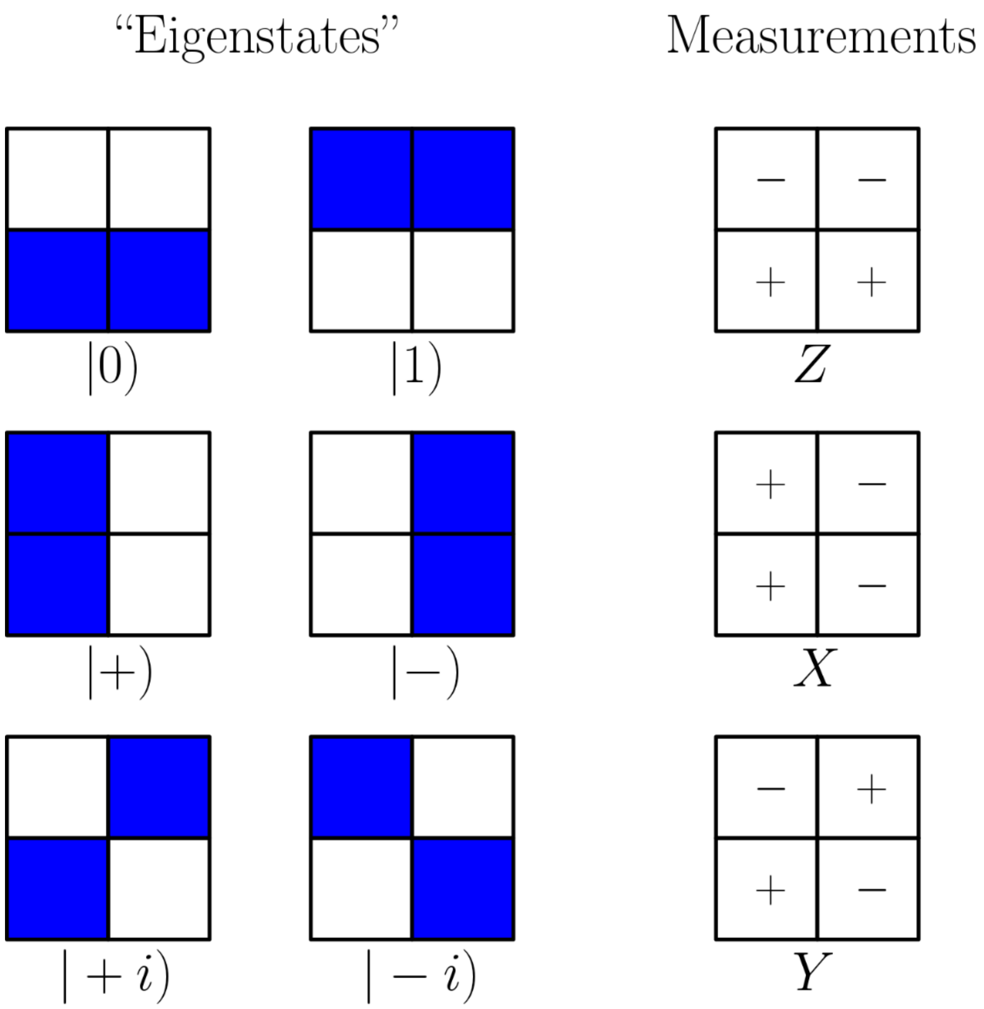


and get the $+1$ outcome, then we will know that the ontic state must have been $(0,0)$ before the measurement.

  - To preserve the knowledge-balance principle and maintain repeatability (0,0) and (0,1) must get swapped with probability ½ during the measurement.
  - Thus, after the measurement, the updated epistemic state will be $|+\rangle$.



Valid Measurements on a toy bit and their “eigenstates”



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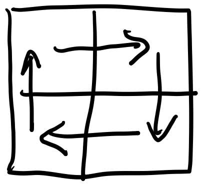
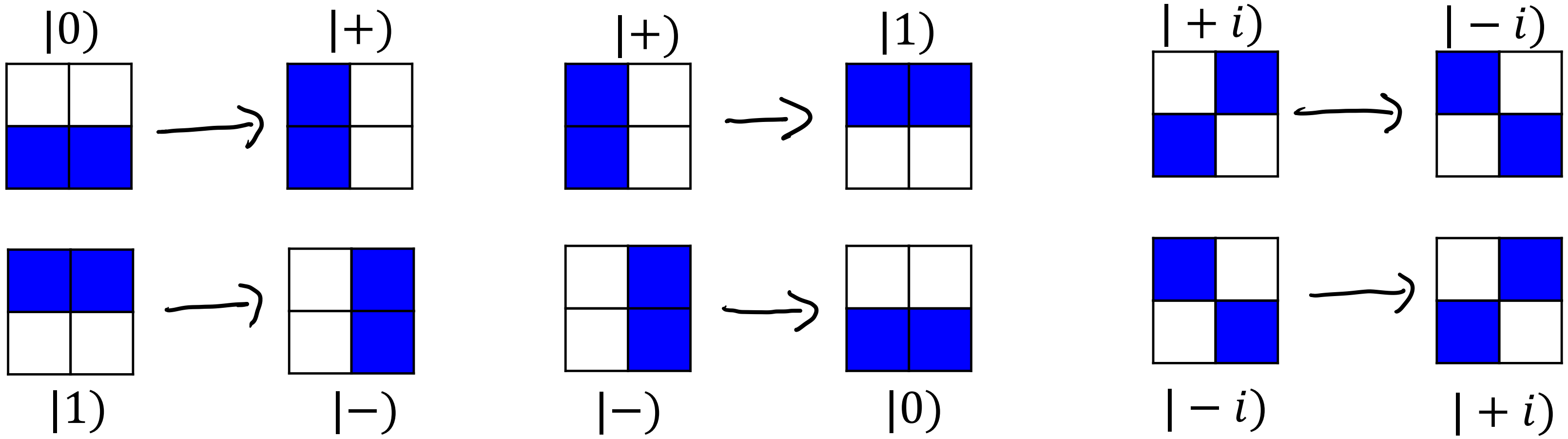
Reversible Dynamics

- Reversible dynamics (the analogue of Schrödinger equation dynamics) on a toy bit is just a permutation on the underlying ontic states. We can then compute the action on the epistemic states.
- Example:



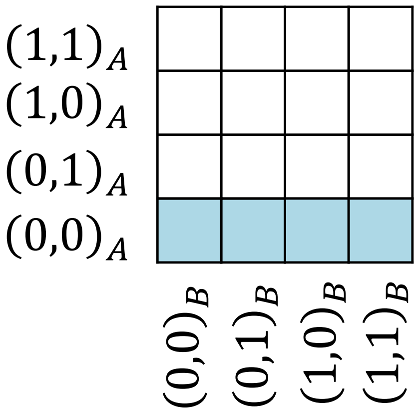


Composite systems

- When we have two toy bits, each toy bit has its own ontic state:

$$(0/1,0/1)_A\,,(0/1,0/1)_B.$$

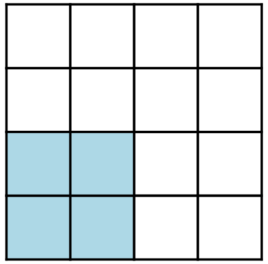
- There are $4 \times 4 = 16$ possible ontic states, so it takes 4 binary questions to specify the exact ontic state.
- By the knowledge-balance principle, we can only know the answer to 2 of them.
- Subtlety: We not only apply the knowledge-balance principle to the global system, but also to the individual subsystems.



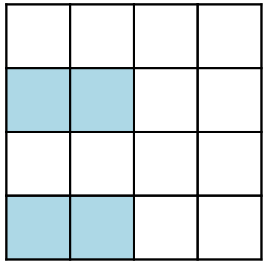
This is not a valid epistemic state because it does not satisfy the knowledge-balance principle for toy-bit A.

Product and Correlated States

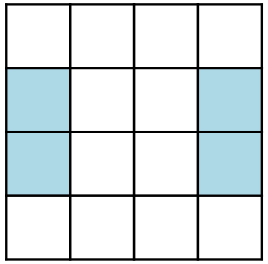
- Of the valid epistemic states, some of them are products of independent distributions of the two toy bits.



$|0)_A|0)_B$

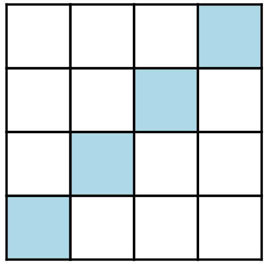


$|+)_A|0)_B$

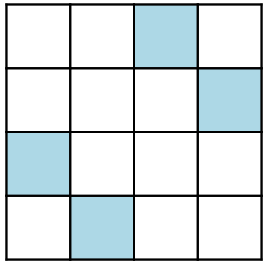


$|-i)_A|+i)_B$

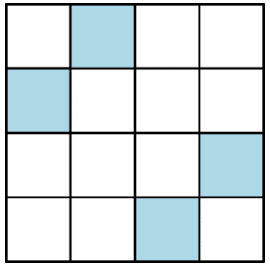
- And some of them are correlated (“entangled”)



$|\Phi^+)_{AB}$



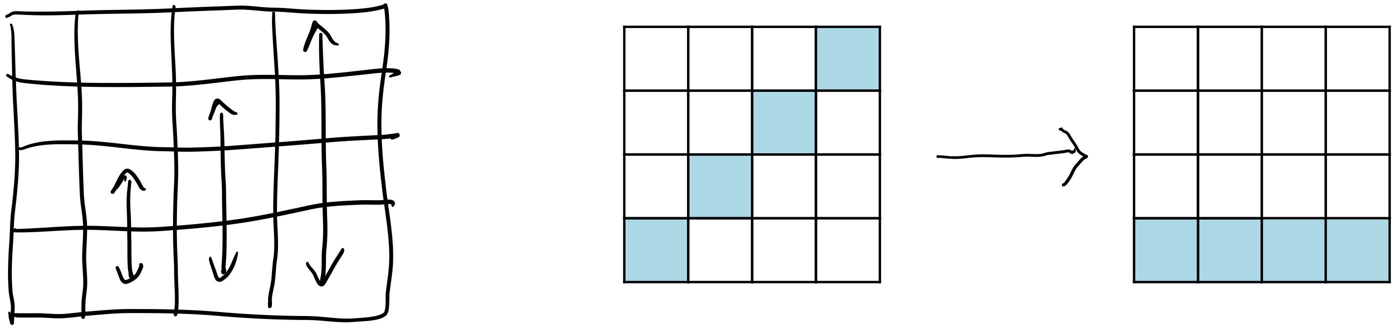
$|\Phi^-)_{AB}$



$|\Psi^+)_{AB}$

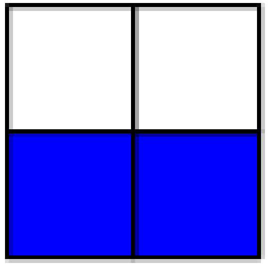
Reversible Dynamics on Composites

- Because we need to preserve the knowledge-balance principle for subsystems, not all permutations represent valid dynamics for a composite system.
- Example:

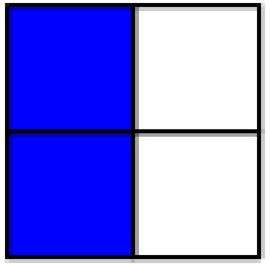


2. Explanations of Quantum Phenomena

Indistinguishability of Pure States



$|0\rangle$



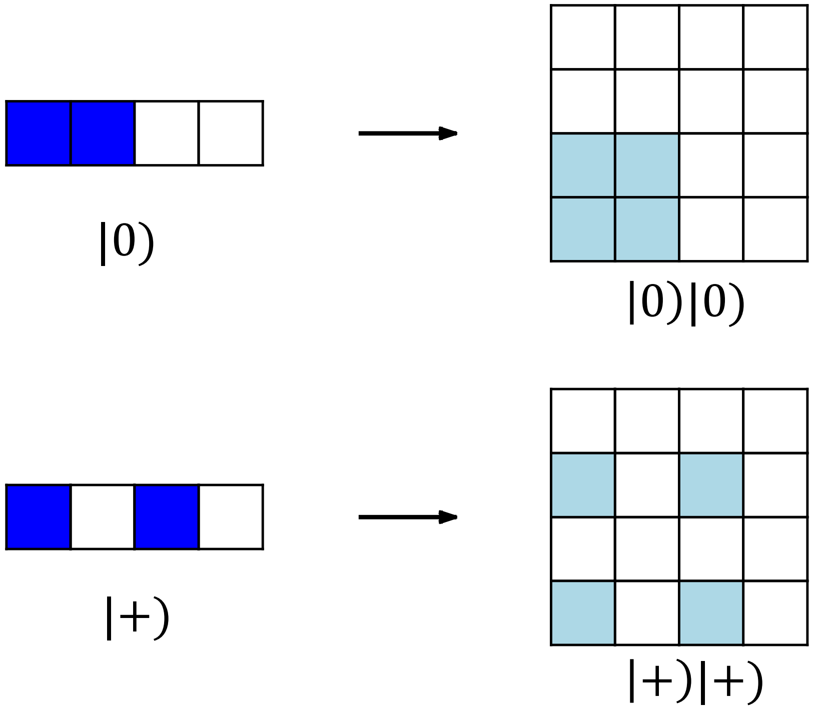
$|+\rangle$

The reason that $|0\rangle$ and $|+\rangle$ are not perfectly distinguishable is because, whichever state is prepared, there is a probability 0.5 that the ontic state is (0,+). When this happens, the physical state is exactly the same, so you cannot do any better than randomly guessing.

$$p_{\text{error}} = 0.5 \times 0.5 = 0.25$$

Note: we cannot reproduce $p_{\text{error}} = 0.146$. The optimal distinguishing measurement does not have an analogue in the toy theory.

No-Cloning Theorem



The input states are in the same ontic state with probability 0.5.

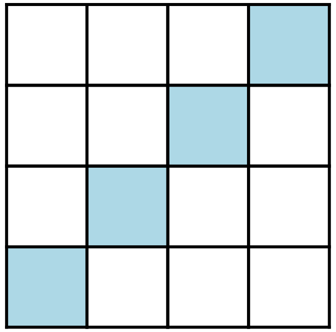
The output states are in the same ontic state with probability 0.25.

The cloning device only has access to the ontic state, so it cannot do something different to the ontic state in the overlap depending on whether $|0\rangle$ or $|+\rangle$ was prepared.

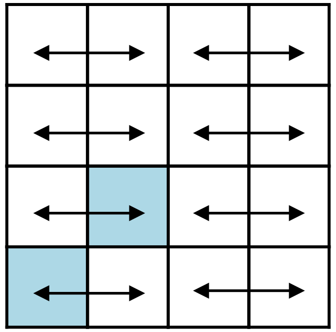
Therefore, the output states must overlap at least as much as the input states.

EPR

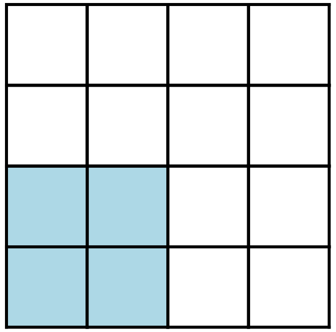
- The EPR experiment works as EPR expected in this theory:
  - The outcomes of all measurements are predetermined.
  - The two systems are initially in a correlated probability distribution.
  - The collapse is just updating information, followed by a local randomization of the system being measured.



Start in a correlated state $|\Phi^+\rangle_{AB}$



Alice measures Z and finds $+1$. There is a random local disturbance



System ends up in $|0\rangle_A|0\rangle_B$

Other Phenomena Reproduced by the Toy Theory

Phenomena arising in epistricted theories	Phenomena not arising in epistricted theories
Noncommutativity	Bell inequality violations
Coherent superposition and interference	Contextuality
Collapse of the "state"	(Exponential) Quantum computational speedup
Complementarity	Some details of items on the left
No-cloning and No-broadcasting	
Teleportation	
Remote Steering	
Key distribution	
Superdense coding	
Monogamy of entanglement	
Choi-Jamiolkowski isomorphism	
Naimark extension and Stinespring dilation	
Ambiguity of mixtures	
Locally immeasurable product bases	
Unextendable product bases	
Pre- and post-selection effects	
And many more…	

3. Interference

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Collaborators

Lorenzo Catani
TU Berlin

David Schmid
Gdansk

Rob Spekkens
Perimeter

Based on *Why interference phenomena do not capture the essence of quantum theory*
arXiv:2111.13727

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Interference $\Rightarrow$ Ψ-Ontology

- By far the most common argument given in favor of ψ-ontology is quantum interference.

"Interference is a real, observable, physical effect, and it requires a real physical cause. Such a cause must behave in a way that corresponds to how the wavefunction behaves." – T. Maudlin, Philosophy of Physics: Quantum Theory (Princeton, 2019)

Interference ⇒ Nonclassicality

"In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form.

We choose to examine a phenomenon which is impossible, *absolutely impossible*, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by "explaining" how it works. We will just *tell* you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics." - R. P. Feynman, R. B. Leighton, and M. L. Sands, The Feynman lectures on physics. (Addison-Wesley world student series, 1961-1963)

Superposition is a New Category

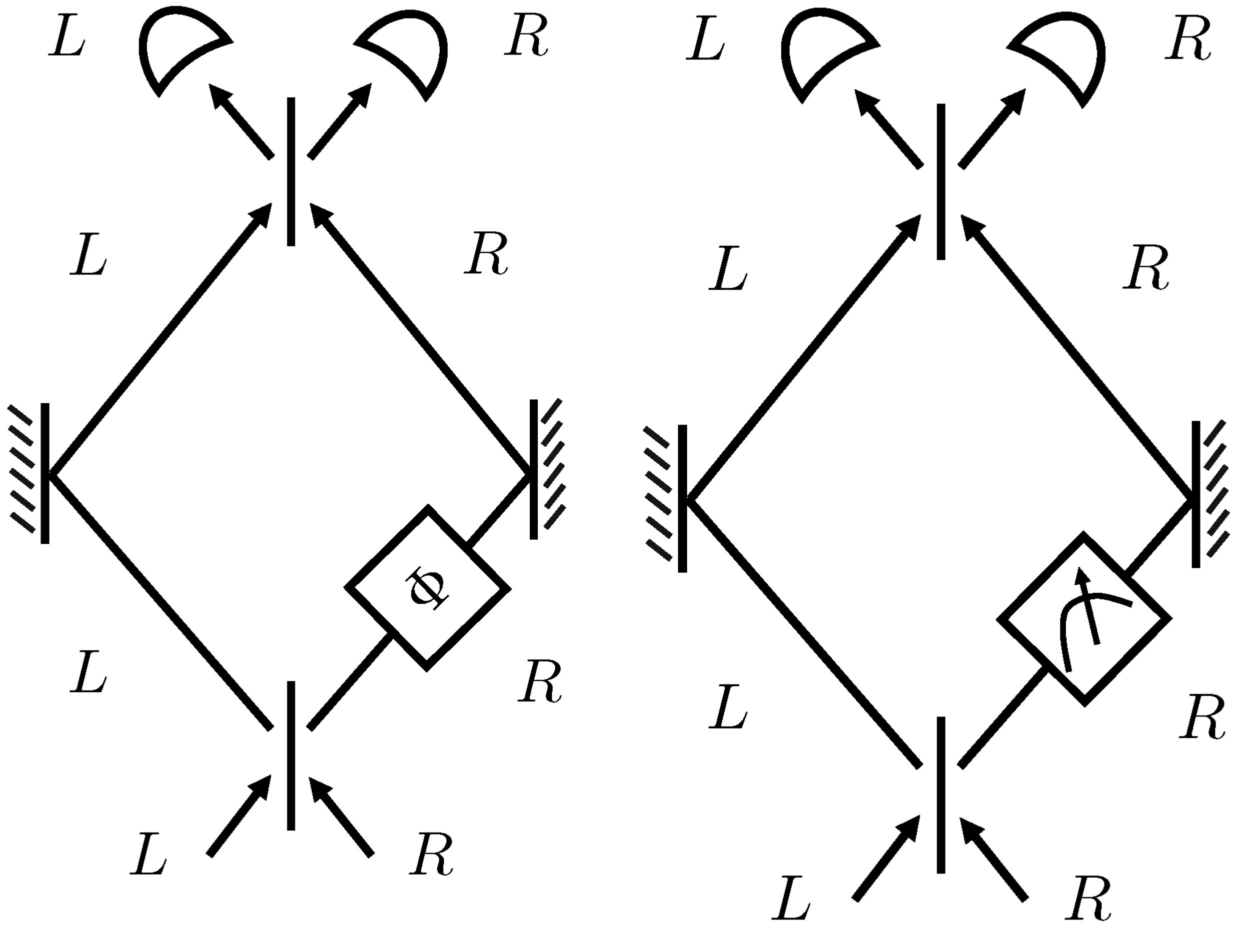
“So what we're faced with is this: Electrons passing through this apparatus, in so far as we are able to fathom the matter, do not take route h and do not take route s and do not take both of those routes and do not take neither of those routes; and the trouble is that those four possibilities are simply all of the logical possibilities that we have any notion whatever of how to entertain!

What can such electrons be doing? It seems they must be doing something which has simply never been dreamt of before (if our experiments are valid, and if our arguments are right). Electrons seem to have modes of being, or modes of moving, available to them which are quite unlike what we know how to think about.

The name of that new mode (which is just a name for something we don't understand) is superposition.” – D. Albert, Quantum Mechanics and Experience (Harvard University Press, 1992)

The TRAP Phenomena

- TRAP = Traditionally Regarded As Problematic
- We consider a single photon in a Mach-Zehnder interferometer with:
  - 50/50 beam-splitters.
  - a phase shifter that can be set to 0 or π.
  - a nondestructive detector that can be placed on one of the arms.



The TRAP Phenomena

- Suppose a single photon is inserted on path L.
- When the detector is not present:
  - the photon is always detected on path L.
  - This is interference, or wave-like behavior.
- If the detector is present, it clicks with probability $\frac{1}{2}$. Regardless of whether it clicks:
  - There is a probability $\frac{1}{2}$ of the photon being detected on each path at the end.
  - This is particle-like behavior.

What is TRAP about this?

1. Wave-particle duality
2. Observer dependence of reality
3. Failure of explanation in terms of local causes
4. Superposition is a new category

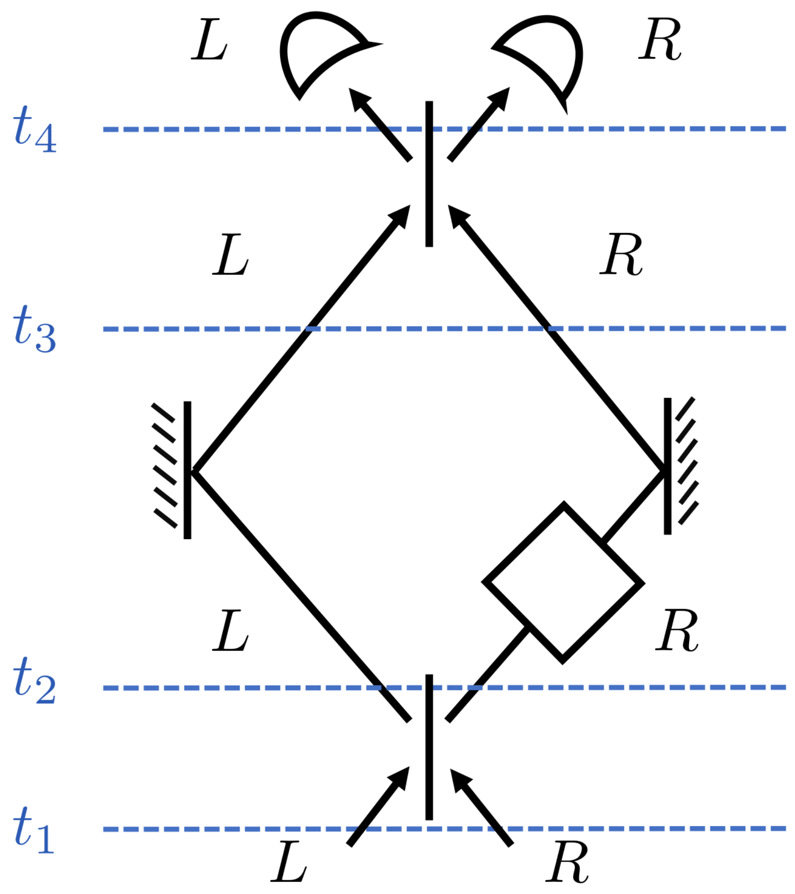
But the TRAP phenomena arise in a suitable version of Spekkens' toy theory, in which none of these are true.

Mathematical Description

- Beam-splitter:

$$|L\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle), \qquad |R\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$$

- With no detector:
  - t_1: $|L\rangle$
  - t_2: $\frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$
  - t_3: $\frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$
  - t_4: $|L\rangle$



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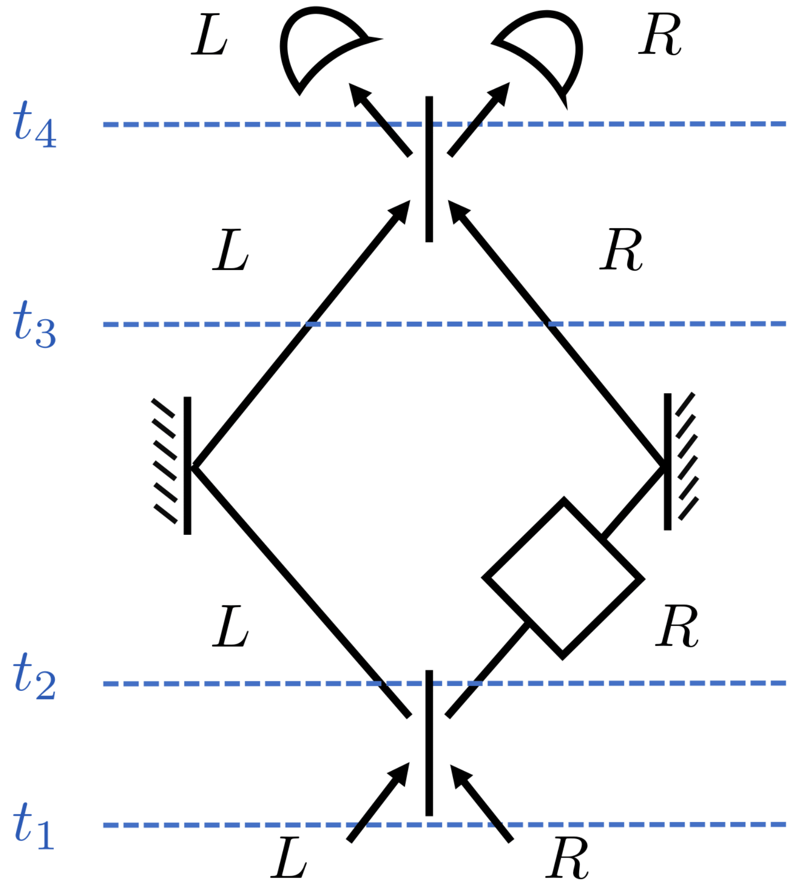
Mathematical Description

- Beam-splitter:

$$|L\rangle \to \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle), \qquad |R\rangle \to \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$$

- With a detector:
  - t_1: $|L\rangle$
  - t_2: $\frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$
  - t_3: $|L\rangle$ with prob $\frac{1}{2}$ or $|R\rangle$ with prob $\frac{1}{2}$
  - t_4: $\frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$ or $\frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$



Mathematical Description

- A phase shifter in the right path acts as:

$$|L\rangle \to |L\rangle, \qquad |R\rangle \to -|R\rangle$$

- Between t_2 and t_3 the state transforms as:

$$\frac{1}{\sqrt{2}}(|R\rangle - |L\rangle) \to -\frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$$

- And then the final beam-splitter gives $|R\rangle$ rather than $|L\rangle$.

Field theoretic description

- Consider L and R as spatial modes:
  - $|n\rangle_{L/R}$ is the state in which the left/right mode contains n photons.
  - Hilbert space is $H_L \otimes H_R$
  - Our experiment only involves the one-photon subspace.

- The reasons for using this description are:
  - It is more closely related to the structure of our model.
  - It makes it clear that the detector is a local operation, i.e. it acts on H_R only.

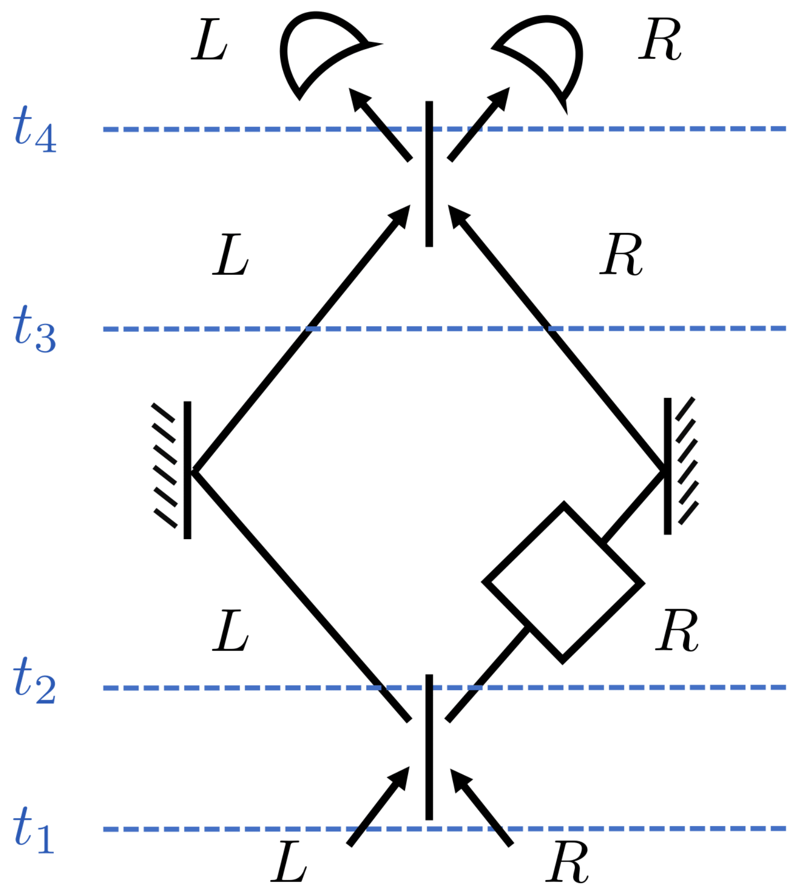
Field-Theoretic Description

- Beam-splitter:

$$|1\rangle_L|0\rangle_R \to \frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R),$$

$$|0\rangle_L|1\rangle_R \to \frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R + |1\rangle_L|0\rangle_R)$$

- With no detector:
  - t_1: $|1\rangle_L|0\rangle_R$
  - t_2: $\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R)$
  - t_3: $\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R)$
  - t_4: $|1\rangle_L|0\rangle_R$

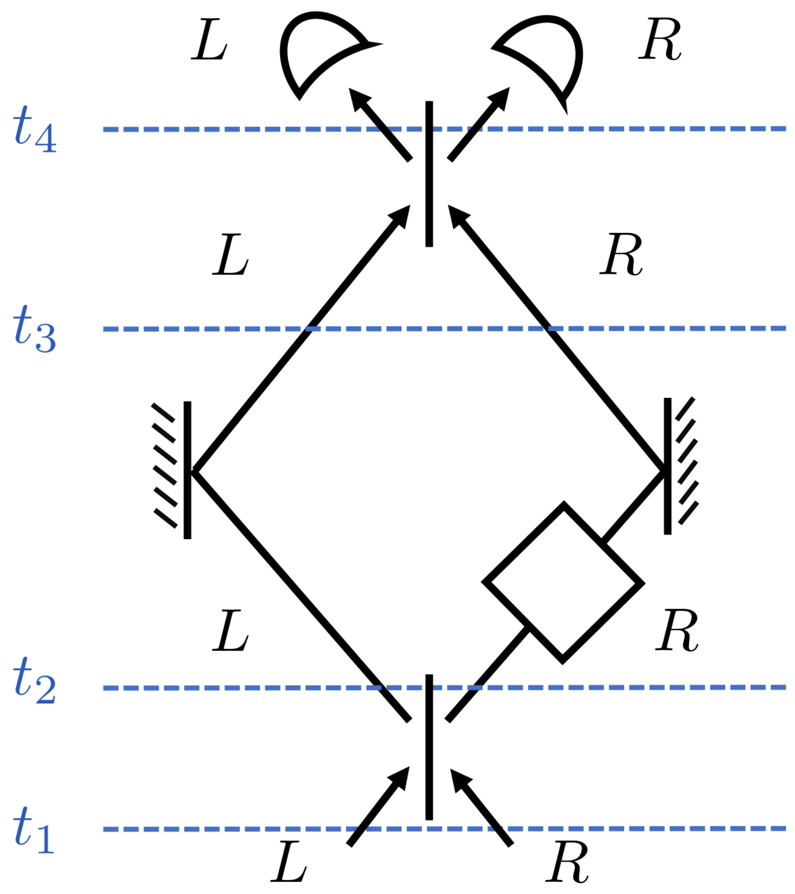


Field-Theoretic Description

- Beam-splitter:

$$|1\rangle_L|0\rangle_R \to \frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R),$$
$$|0\rangle_L|1\rangle_R \to \frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R + |1\rangle_L|0\rangle_R)$$

- With a detector:
  - t_1: $|1\rangle_L|0\rangle_R$
  - t_2: $\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R)$
  - t_3: $|1\rangle_L|0\rangle_R$ with prob $\frac{1}{2}$ or $|0\rangle_L|1\rangle_R$ with prob $\frac{1}{2}$
  - t_4: $\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R)$ or $\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R + |1\rangle_L|0\rangle_R)$



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Field-Theoretic Description

- A phase shifter in the right path acts as:

$$|0\rangle_R \to |0\rangle_R, \qquad |1\rangle_R \to -|1\rangle_R$$

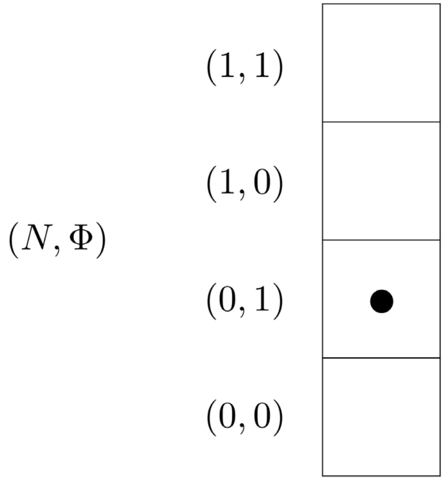
- Between t_2 and t_3 the state transforms as:

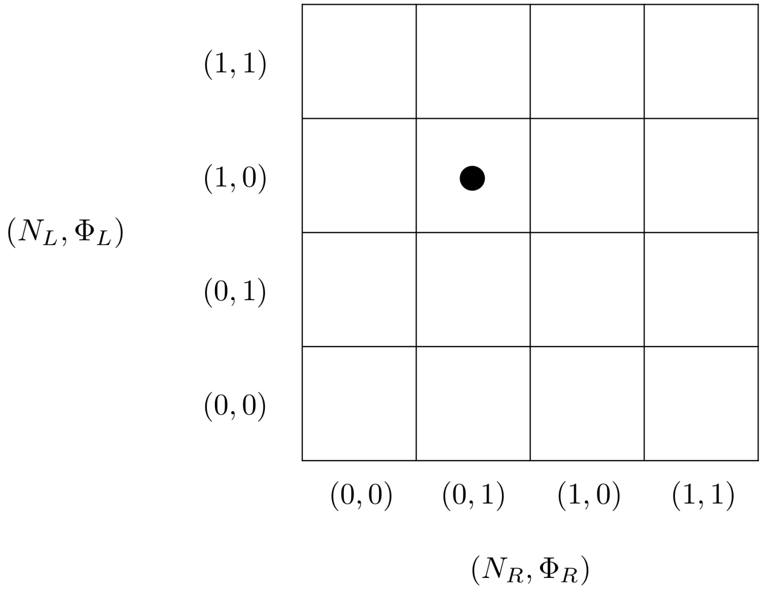
$$\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R) \to -\frac{1}{\sqrt{2}}(|0\rangle_L|1\rangle_R + |1\rangle_L|0\rangle_R)$$

- And then the final beamsplitter gives $|0\rangle_L|1\rangle_R$ rather than $|1\rangle_L|0\rangle_R$.

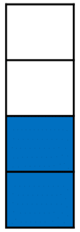
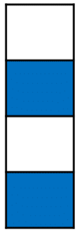
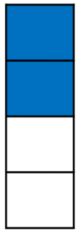
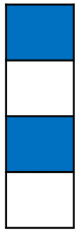
Our model: Ontology

- Spekkens' toy theory but applied to field modes rather than particles.
- A mode is assigned two binary ontic variables:
  - An occupation number N – a particle-like property
  - A "phase" Φ – a wave like property
- It has both at all times.





Single Mode Epistemic States

Quantum State	Toy State	Quantum State	Toy State
$\vert 0\rangle$	(N,Φ) (1,1) (1,0) (0,1) (0,0)  $N=0$ $(0,\phi)$	$\vert +\rangle$	(N,Φ) (1,1) (1,0) (0,1) (0,0)  $\Phi=0$ $(\eta,0)$
$\vert 1\rangle$	(N,Φ) (1,1) (1,0) (0,1) (0,0)  $N=1$ $(1,\phi)$	$\vert -\rangle$	(N,Φ) (1,1) (1,0) (0,1) (0,0)  $\Phi=1$ $(\eta,1)$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

The vacuum state is an epistemic state

- An implicit assumption in the TRAP implications is that:
  - If there is no photon in a mode then there is *nothing* in that mode.
  - i.e., there is no degree of freedom that could carry information about whether there was a detector in that mode to the second beam-splitter.
- In our model, the vacuum $|0\rangle$ is represented by an epistemic state in which $N = 0$, but Φ can be either 0 or 1.
  - Φ can convey information to the second beamsplitter.
- In a ψ-epistemic model it is natural and necessary for the vacuum to be represented by a probability distribution with support on more than one ontic state.

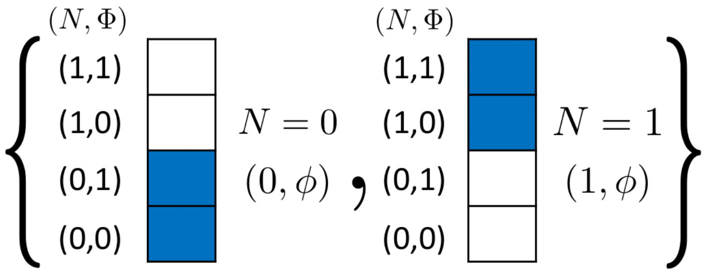
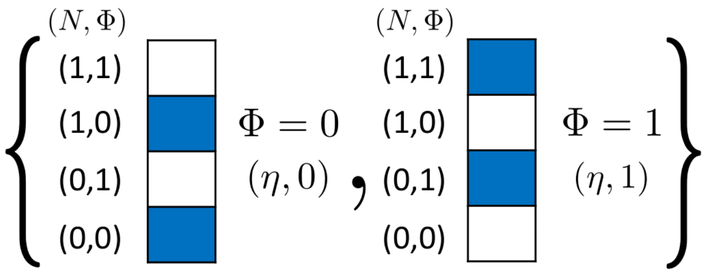
Why must the vacuum have support on more than one ontic state?

- Consider the states:

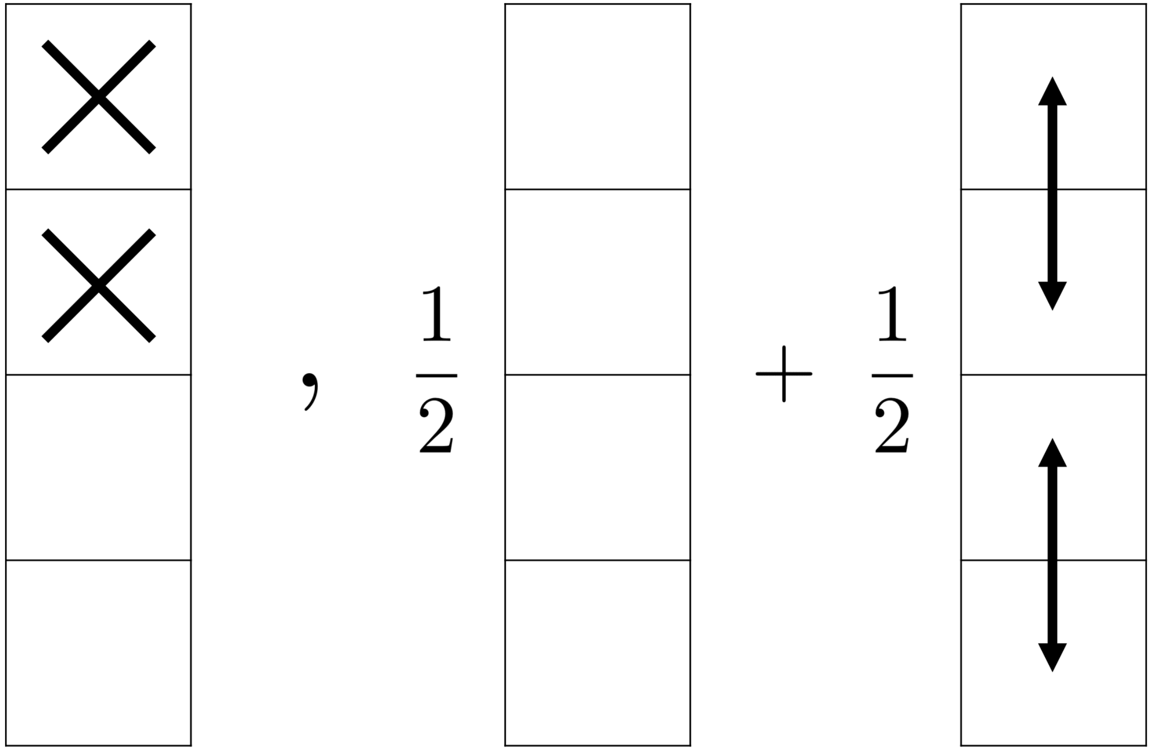
$$|0\rangle,\ |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \text{ and } |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

  and the probability distributions μ_0, μ_+, μ_- that represent them.
- $\langle -|+\rangle = 0$, so there can be no ontic states in the support of both μ_+ and μ_-.
- $|0\rangle$ and $|+\rangle$ are nonorthogonal, so there should be ontic states in the joint support of μ_0 and μ_+ in a ψ-epistemic model.
- Similarly, $|0\rangle$ and $|-\rangle$ are nonorthogonal, so there should be ontic states in the joint support of μ_0 and μ_- as well.
- $\Rightarrow$ there *must* be more than one ontic state in the support of μ_0.

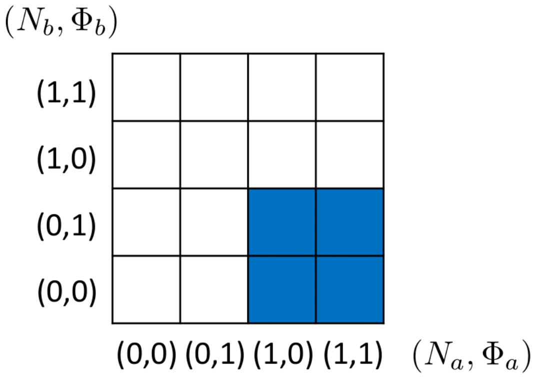
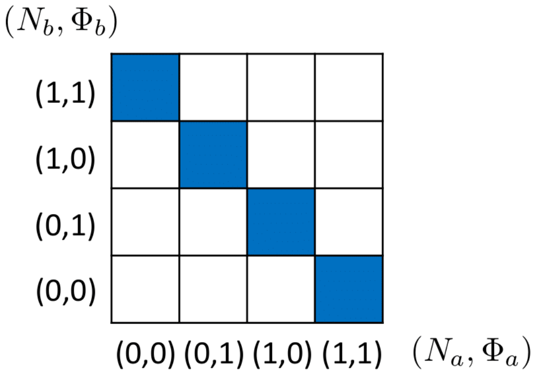
Single Mode Measurements

Observable	Toy Representation
Z	
X	

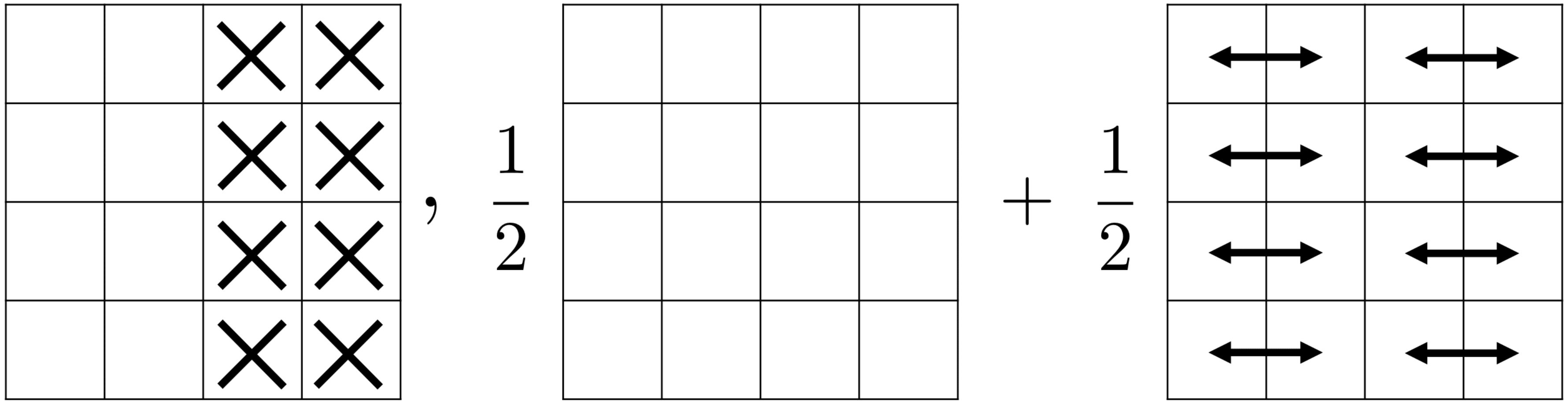
Single Mode Update Rule



Two-mode Epistemic States

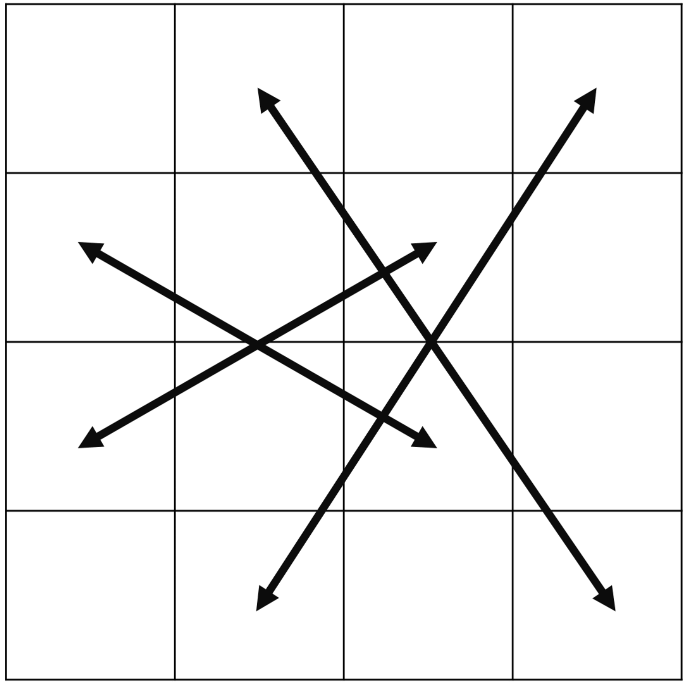
Quantum State	Toy State	
$\|10\rangle$		$N_a = 1$ $N_b = 0$ $(1, \phi_a, 0, \phi_b)$
$\frac{\|01\rangle - \|10\rangle}{\sqrt{2}}$		$N_a + N_b = 1$ $\Phi_a + \Phi_b = 1$ $(\eta_a, \phi_a, \eta_a + 1, \phi_a + 1)$

Two Mode Update Rule

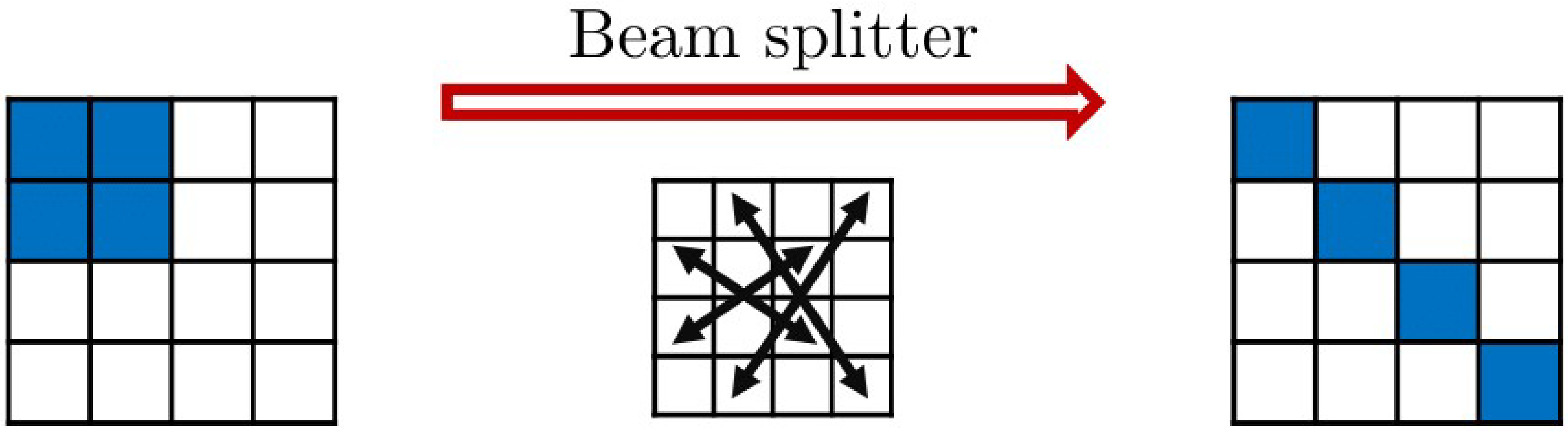


The Beam-splitter Transformation

$$N_L^{\text{out}} = \Phi_L^{\text{in}} \oplus \Phi_R^{\text{in}}$$
$$N_R^{\text{out}} = N_L^{\text{in}} \oplus N_R^{\text{in}} \oplus \Phi_L^{\text{in}} \oplus \Phi_R^{\text{in}}$$
$$\Phi_L^{\text{out}} = N_L^{\text{in}} \oplus \Phi_R^{\text{in}}$$
$$\Phi_R^{\text{out}} = \Phi_R^{\text{in}}$$



No detector $t_1 \rightarrow t_2$

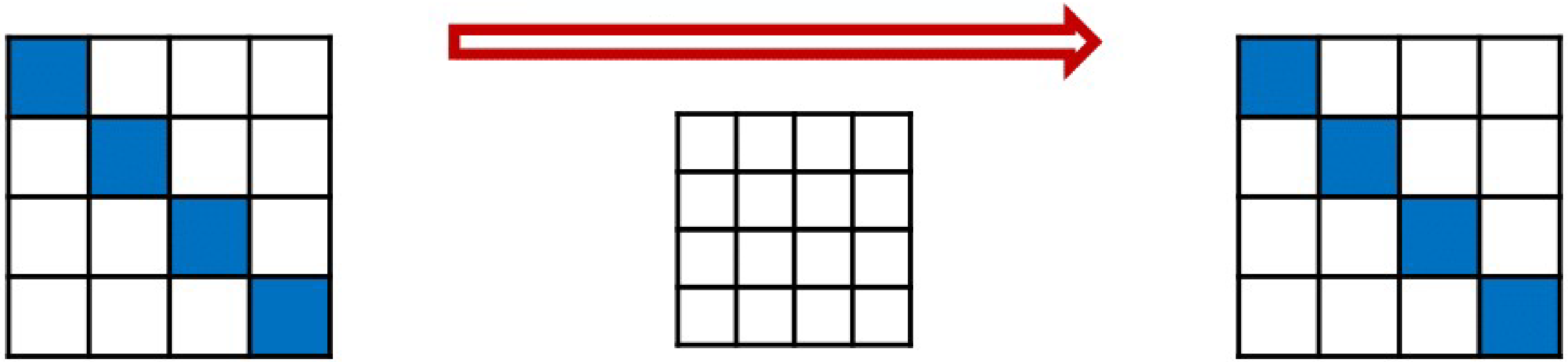


$|10\rangle$

U_{BS}

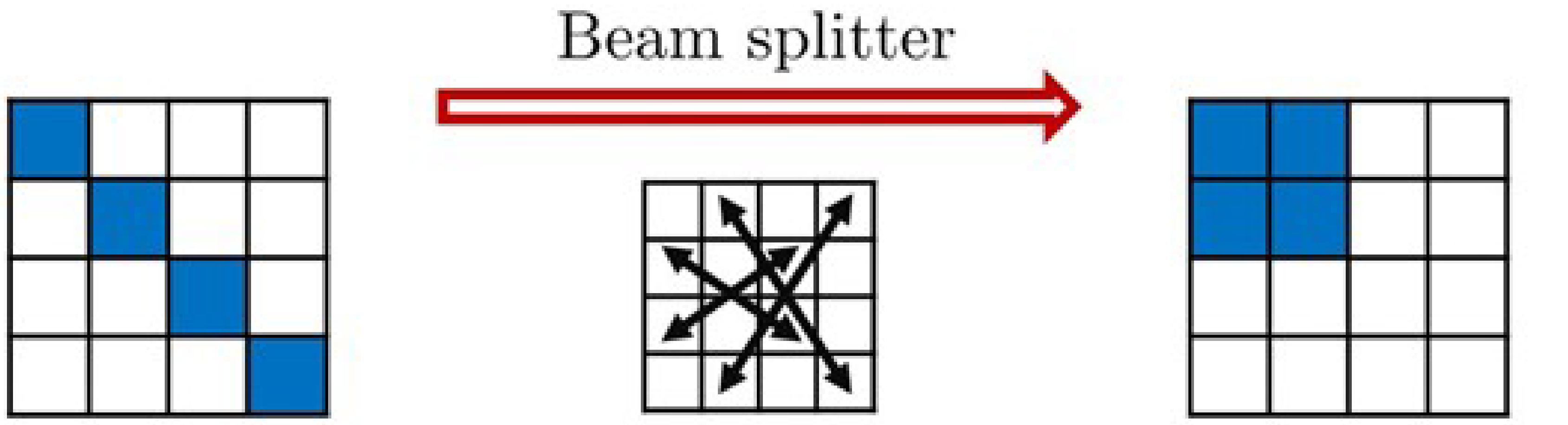
$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$

No detector $t_2 \rightarrow t_3$



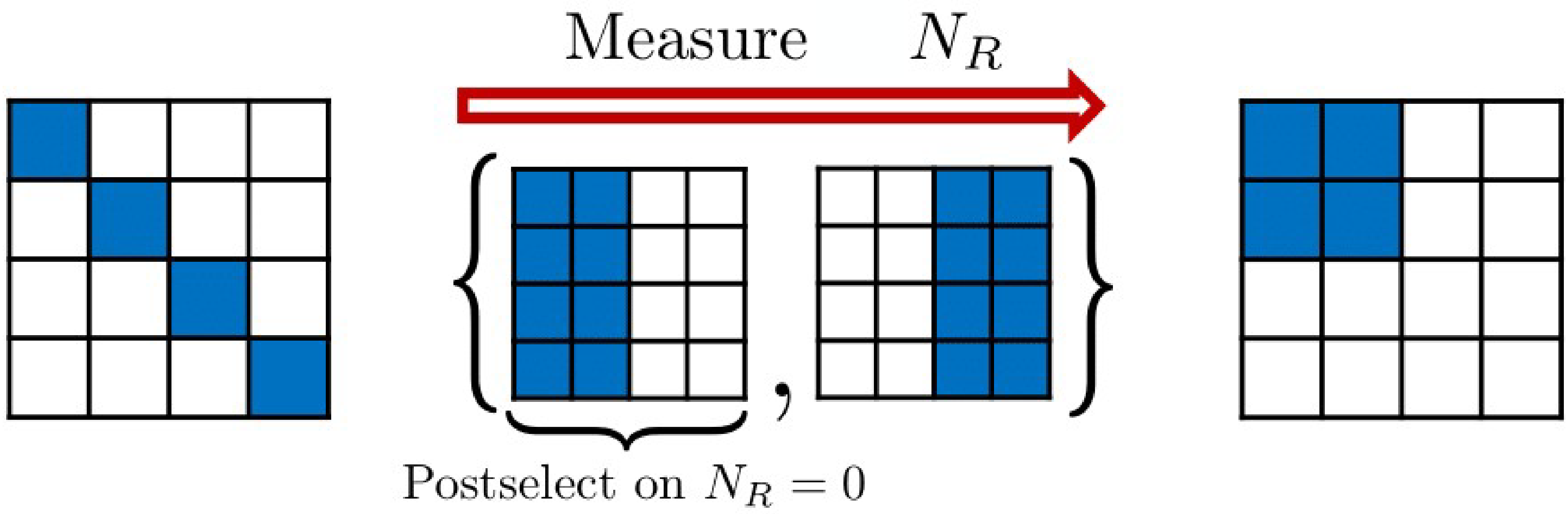
$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} \qquad \mathbb{I} \otimes \mathbb{I} \qquad \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

No detector $t_3 \rightarrow t_4$



$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$ U_{BS} $|10\rangle$

With a detector $t_2 \rightarrow t_3$

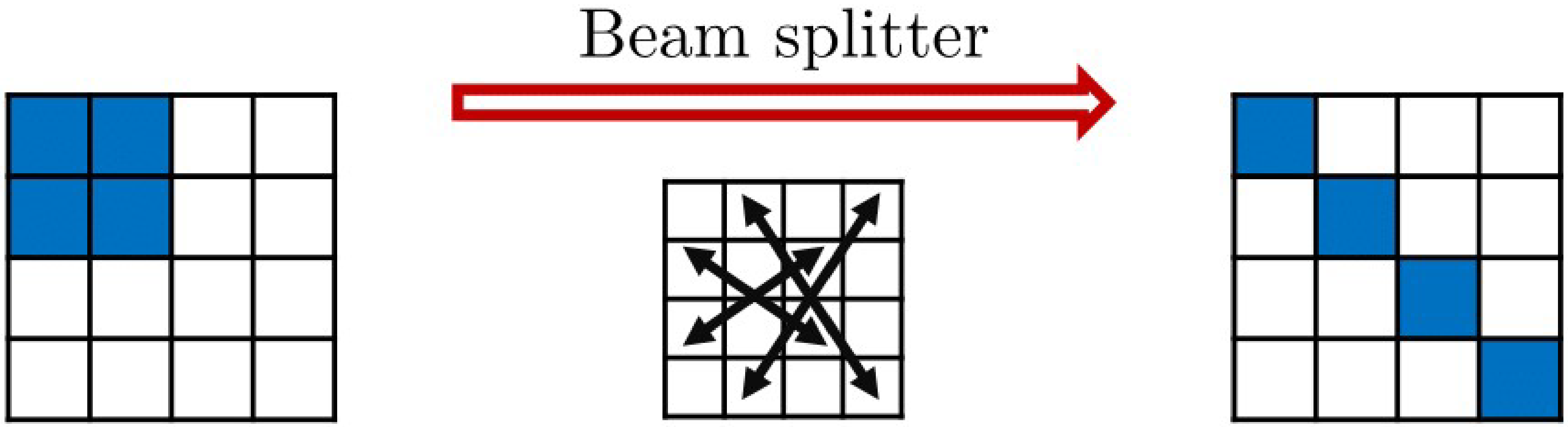


$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$|\psi\rangle \longrightarrow \frac{\mathbb{I} \otimes |0\rangle\langle 0|}{|\langle 0|\psi\rangle|} |\psi\rangle$$

$$|10\rangle$$

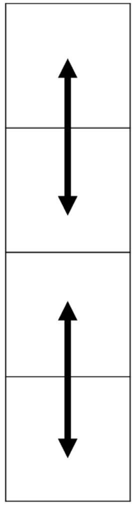
With a detector $t_3 \rightarrow t_4$



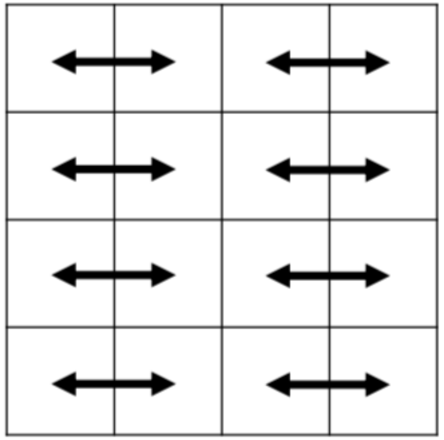
$|10\rangle$ $\qquad$ U_{BS} $\qquad$ $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$

Phase Shifter

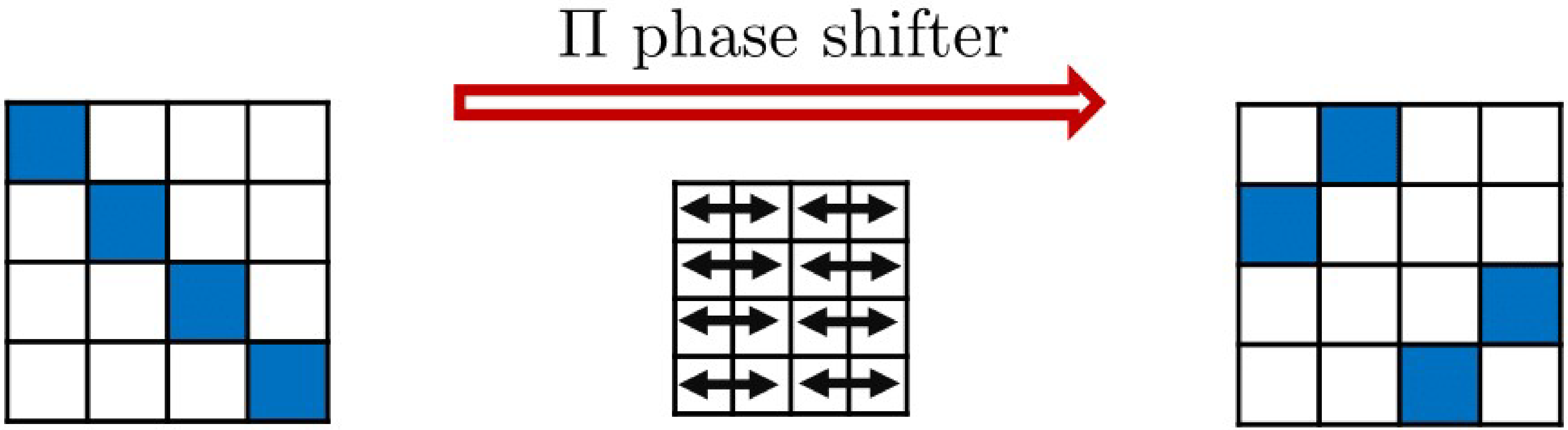
- A phase shifter on the R mode is represented by:



- On the two modes, it acts as



With a phase shifter $t_2 \rightarrow t_3$

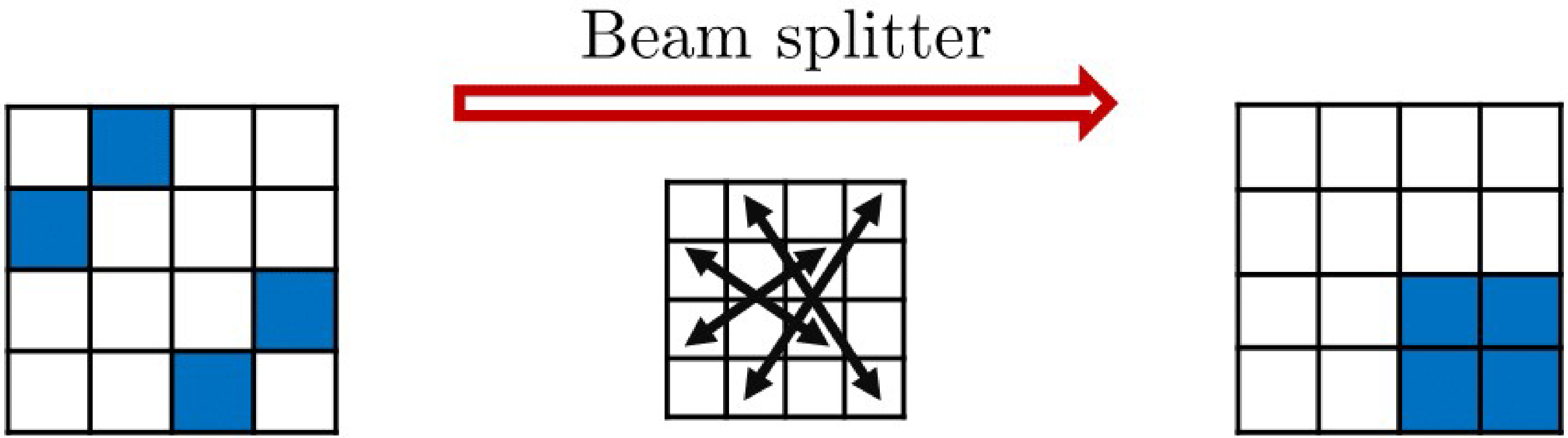


$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$\mathbb{I} \otimes Z$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

With a phase shifter $t_3 \rightarrow t_4$



$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$ $\qquad U_{BS} \qquad$ $|01\rangle$

Further Results

- With the same model, we can reproduce:
  - The Elitzur-Vaidman bomb tester.
  - Wheeler's delayed choice experiment.
- By extending the model, we can reproduce:
  - The (delayed choice) quantum eraser.
  - (Some versions of) counterfactual computation/communication.
- Many other interference phenomena claimed to be nonclassical can be reproduced in a similar way.

4. Toy-Theories from Quasi-Quantization

Quasi-Quantization

- The knowledge-balance principle is not applicable to all systems,
  - e.g. a system that requires 3 binary questions to specify the ontic state.
- It is difficult to apply and makes the relationship to classical mechanics and quantum theory obscure.
- Spekkens developed a better approach called *Quasi-Quantization* – the toy theory analogue of canonical quantization.

Quasi-Quantization

- We start by considering Hamiltonian mechanics on a phase space. Quasi-Quantization is a way of imposing an epistemic restriction on such theories.
- Guiding analogy:
  - Quantum: A set of observables is *jointly measurable* iff the observables pairwise commute according to the matrix commutator.
  - Epistricted: A set of variables is *jointly knowable* iff the variables pairwise commute according to the Poisson bracket.

Classical Complementarity

- Spekkens also restricts attention to *quadrature variables.*
  - These are linear combinations of the fundamental phase-space variables, e.g. $\alpha q + \beta p$.

- The epistemic restriction he adopts is:
  - **Classical complementarity:** The valid epistemic states are those wherein an agent knows the values of a set of quadrature variables that Poisson commute, and is maximally ignorant otherwise.

Continuous Degrees of Freedom

- Configuration space: is $\mathbb{R}^n \ni (\mathrm{q}_1, \mathrm{q}_2, \cdots, \mathrm{q}_\mathrm{n})$
- Phase space is $\Omega \equiv \mathbb{R}^{2n} \ni \mathbf{m} = (\mathrm{q}_1, \mathrm{p}_1, \mathrm{q}_2, \mathrm{p}_2, \cdots, \mathrm{q}_\mathrm{n}, \mathrm{p}_\mathrm{n})$
- Functionals on phase space: $f: \Omega \to \mathbb{R}$
- Poisson bracket:

$$\{f, g\}(\mathbf{m}) = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial q_j} \frac{\partial g}{\partial p_j} - \frac{\partial f}{\partial p_j} \frac{\partial g}{\partial q_j} \right)(\mathbf{m})$$

- Linear (quadrature) variables:

$$f = \mathrm{a}_1 q_1 + \mathrm{b}_1 p_1 + \cdots + \mathrm{a}_n q_n + \mathrm{b}_n p_n + \mathrm{c}, \qquad \mathrm{a}_1, \mathrm{b}_1, \cdots, a_n, b_n \in \mathbb{R}$$

- Associate with vector $\mathbf{f} = (\mathrm{a}_1, \mathrm{b}_1, \cdots, \mathrm{a}_n, \mathrm{b}_n) \in \mathbb{R}^{2n}$
- For these, the Poisson bracket is:

$$\{f, g\}(\mathbf{m}) = \mathbf{f}^T J \mathbf{g} \equiv \langle \mathbf{f}, \mathbf{g} \rangle, \qquad J = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots \\ -1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ 0 & 0 & -1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Discrete Degrees of Freedom

- Configuration space: is $(\mathbb{Z}_d)^n \ni (\mathrm{q}_1, \mathrm{q}_2, \cdots, \mathrm{q}_\mathrm{n})$
- Phase space is $\Omega \equiv (\mathbb{Z}_d)^{2n} \ni \mathbf{m} = (\mathrm{q}_1, \mathrm{p}_1, \mathrm{q}_2, \mathrm{p}_2, \cdots, \mathrm{q}_\mathrm{n}, \mathrm{p}_\mathrm{n})$
- Functionals on phase space: $f: \Omega \to \mathbb{Z}_d$
- Poisson bracket:

$$\{f, g\}(\mathbf{m}) = \sum_{j=1}^{n} (\quad)(\quad) - (\quad)(\quad)$$

- Linear (quadrature) variables:

$$f = \mathrm{a}_1 q_1 + \mathrm{b}_1 p_1 + \cdots + \mathrm{a}_n q_n + \mathrm{b}_n p_n + \mathrm{c}, \qquad \mathrm{a}_1, \mathrm{b}_1, \cdots, a_n, b_n \in \mathbb{Z}_d$$

- Associate with vector $\mathbf{f} = (\mathrm{a}_1, \mathrm{b}_1, \cdots, \mathrm{a}_n, \mathrm{b}_n) \in (\mathbb{Z}_d)^{2n}$
- For these, the Poisson bracket is:

$$\{f, g\}(\mathbf{m}) = \mathbf{f}^T J \mathbf{g} \equiv \langle \mathbf{f}, \mathbf{g} \rangle, \qquad J = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots \\ -1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ 0 & 0 & -1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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Quadrature Epistricted Theories as Subtheories of Quantum Mechanics

- For odd Hilbert space dimensions, Spekkens' toy theory is operationally equivalent to stabilizer quantum theory.
- For even dimensions it is not:
  - Expected, since the stabilizer qubit theory has contextuality and nonlocality, but Spekkens' theory does not.
- This is proved using the Wigner-Moyal formalism for stabilizer quantum mechanics.
- So the toy theory is not only classical in a very strong sense, but it is also equivalent to a subtheory of quantum mechanics in odd dimensions.

5. Conclusions and Future Directions

Conclusions

- We can reproduce many seemingly puzzling quantum phenomena in a theory that is *literally* classical mechanics, but with an epistemic restriction.
- Apart from the anomalous toy-bit theory, these operationally reproduce subtheories of quantum theory.
- Anything that appears in these theories cannot be a source of (exponential) quantum computational advantage.
- The main candidates that are not in the toy-theory are:

nonlocality, contextuality, ψ-ontology

Future Directions

- A toy field theory that is not restricted to one photon.
  - What is the toy theory analogue of creation and annihilation operators.
- Non quadrature epistricted theories:
  - E.g. states with a definite value of modular momentum $e^{ia\hat{p}}$
  - Status of the Gaussian toy theory in the new approach.
- Generalization to all symplectic structures on which Hamiltonian mechanics can be defined.
- Epistrictization analogues of different methods of quantization.
  - Needed for theories that do not have a symplectic structure, e.g. electromagnetism and gravity.

A Reading List for Epistricted Theories

The original toy-theory paper:

R. Spekkens, *Phys. Rev. A* 75, 032110 (2007)
https://doi.org/10.1103/PhysRevA.75.032110 https://arxiv.org/abs/quant-ph/0401052

Relevant review articles:

L. Hausmann, N. Nurgalieva, L. del Rio arXiv:2105.03277 (2021)
https://arxiv.org/abs/2105.03277

D. Jennings, M. Leifer, *Contemp. Phys.* 57:1, 60-82 (2015)
https://doi.org/10.1080/00107514.2015.1063233 https://arxiv.org/abs/1501.03202

Interference:

L. Catani, M. Leifer, D. Schmid, R. Spekkens arXiv:2111.13727 (2021)
https://arxiv.org/abs/2111.13727

A Reading List for Epistricted Theories

A stabilizer-like formalism:

M. Pusey, *Found. Phys.* 42, 688-708 (2012)
https://doi.org/10.1007/s10701-012-9639-7 https://arxiv.org/abs/1103.5037

Epistrictization:

R. Spekkens, in G. Chiribella, R. Spekkens, R. (eds) *Quantum Theory: Informational Foundations and Foils*. Fundamental Theories of Physics, vol 181. Springer, 2016)
https://doi.org/10.1007/978-94-017-7303-4_4

L. Catani, D. Browne, *New J. Phys.* 19:073035 (2017)
https://doi.org/10.1088/1367-2630/aa781c https://arxiv.org/abs/1701.07801

Gaussian Toy Theory:

S. Bartlett, T. Rudolph, R. Spekkens *Phys. Rev. A* 86:012103 (2012)
https://doi.org/10.1103/PhysRevA.86.012103 https://arxiv.org/abs/1111.5057

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A Reading List for Epistricted Theories

Applications in Quantum Computing:

L. Catani, D. Browne *Phys. Rev. A* 98:052108 (2018)
https://doi.org/10.1103/PhysRevA.98.052108 https://arxiv.org/abs/1711.08676

N. Johansson, J. Larsson, *Entropy* 21 8:800 (2019)
https://doi.org/10.3390/e21080800 https://arxiv.org/abs/1905.05082

N. Johansson, J. Larsson, *Quantum Information Processing* 16 9:223 (2017)
https://doi.org/10.1007/s11128-017-1679-7 https://arxiv.org/abs/1508.05027

Category Theory Approach:

B. Coecke, B. Edwards, arXiv:0808.1307 (2008)
https://arxiv.org/abs/0808.1037

B. Coecke, B. Edwards, arXiv:1108.1978 (2011)
https://arxiv.org/abs/1108.1978

B. Coecke, B. Edwards, R. Spekkens, Electronic Notes in Theoretical Computer Science 270 (2) 15-36 (2011)
http://dx.doi.org/10.1016/j.entcs.2011.01.021